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Simulation of the Total Heat Transfer of Spherical Hailstones

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Abstract. An effective Nusselt number is introduced which, in combination with the Reynolds number, may be sufficient to explain how different icing conditions can lead to statistically equal ice and air bubble structures within hailstone shells. Two cases were treated: one explains the autosimulation of the total heat transfer and the icing conditions within a hail cloud model; and the other allows a comparison of experiments in an icing tunnel with atmospheric conditions.

This approach can be of importance in any treatment of combined heat or mass transfer processes.

1. Introduction

One of the problems in the search for an understanding of the growth of hailstones – which is essential for the understanding and prevention of hailstorms – is the interpretation of their structure, i.e. the size, shape, and orientation of the ice single-crystals, and the arrangement and size distribution of the air bubbles. These features, differing from shell to shell, are the result of the icing conditions under which they grew. The heat exchange plays a decisive role in the creation of this structure, controlling the type of ice deposits on the growing hailstones; it determines – once factors such as air temperature, cloud liquid water content, and particle diameter and shape are given – whether porous, solid or spongy ice (with liquid water pockets) is going to be formed, what characteristics the air bubbles will assume, and what temperature these deposits will have. These processes are more or less understood [1]; however, one problem arises. Is the relationship between icing conditions and resulting ice deposits unique? Can it be assumed that a particular ice deposit grows under only one set of icing conditions, or are there more possibilities? On the basis of known hailstone structures, ambiguous answers might be expected. This means that various sets of icing conditions should be found which produce the same type of deposit. An approach to solve this problem shall be made using similarity theory and the hypothesis that a similarity of total heat exchange is closely related to sets of icing conditions which can produce identical ice structures.

2. The Effective Nusselt Number

The heat exchange by conduction and convection of spherical hailstones with a diameter $>$ 0.5 cm can be described by the Nusselt number $Nu = 0.54\ Re^{1/2}$, where Re stands for the Reynolds number [2]. The contribution by evaporating, condensing or sublimating water molecules is described by the Sherwood number $Sh = 0.51\ Re^{1/2}$, which contains the mass transfer coefficient; this quantity, when multiplied by the latent heat of vaporization, condensation, or sublimation yields the heat transfer by the contribution of gaseous H_2O. Because accreted, supercooled water drops also contribute to the heat transfer, another similarity number, E, can be formulated, which is, in effect, a Nusselt number for heat transferred to the hailstone by the collected cloud droplets. E is essentially identical with the collection efficiency. It is mainly a function of the Reynolds number; however, its variations are limited approximately to a range between 0.6 and 1.0. Radiation is always less than 5% of the total heat exchange for growing hailstones and is, therefore, neglected.

In order to have similarity of heat exchange, there must be geometrical similarity and similarity of flow; i.e., the Reynolds number must be the same. However, this means that individual heat transfer contributions must be similar, since Nu, Sh and E would remain constant also.

LIST et al. [2] showed that the Reynolds number of a hailstone in typical hailclouds [3] is a function of the particle diameter, D, only, namely $Re \cong 7.3 \times 10^3\, D^{3/2} \pm 10\%$; at higher levels in the clouds, increases in terminal speeds of hailstones of given diameter due to lower values of pressure and temperature are compensated by increases of the kinematic viscosity.

This means that simulation of the individual transfers is achieved at any level in a hailcloud as long as particles with equal diameters are compared. However, such similarity is not very useful for correlating ice structures and sets of icing conditions because any type of shell of given dimensions grows or grew under similar heat transfer conditions. But how can the requirements be made more restrictive?

If a Nusselt number is constructed like the regular one, containing not only the heat transfer by conduction and convection, but also the other contributions due to the effects of the gaseous and liquid water phase, then a new expression is obtained:

$$Nu_{eff} = \frac{k\, Nu\, \Delta T + D_{wa}\, Sh\, \Delta c\, L + 0.25\, \nu\, Re\, E\, w_f\, \bar{c}_w\, \Delta T}{k\, \Delta T + D_{wa}\, \Delta c\, L + 0.25\, \nu\, Re\, w_f\, \bar{c}_w\, \Delta T}\,.$$

This expression is called the effective Nusselt number, where k is the thermal conductivity, ΔT and Δc are the temperature and concentration differences between the particle surface and the 'undisturbed' air, respectively. D_{wa} is the diffusivity of water vapor in air, L the latent heat of the phase change from liquid or solid to vapor, or vice versa, ν the kinematic viscosity, w_f the liquid water content of the cloud, and $\bar{c}_w$ is the mean heat capacity of water averaged over the temperature difference. The factor 0.25 is introduced to account for the distribution of the accreted sensible heat over the whole (spherical) surface. It is essential for the proper weighting of the different components.

By setting two pairs of the related transfer and diffusivity components equal to zero, a ratio is obtained which is a 'traditional' number. In this way, the effective Nusselt number separates and weights the different transfer and transport processes and deals with their contribution to the overall ratio. The character of the basic Nusselt number is conserved: we still compare transfer and transport quantities. To obtain these quantities in compatible dimensions, new transfer terms ($k\, Nu\, \Delta T\, D^{-1}$, $D_{wa}\, Sh\, \Delta c\, L\, D^{-1}$, $0.25\, \nu\, Re\, E\, w_f\, \bar{c}_w\, \Delta T\, D^{-1}$) and diffusivity terms ($k\, \Delta T$, $D_{wa}\, \Delta c\, L$, $0.25\, \nu\, Re\, w_f\, \bar{c}_w\, \Delta T$) have to be formed.

The new Nusselt number, Nu_{eff}, being more general and containing more parameters than the regular Nusselt number, has a series of special characteristics: (i) Nu_{eff} increases with Re only when the relative contribution to the heat transfer by the accreted drops is small, i.e. when w_f is small. If w_f is big, then Nu_{eff} approaches E as w_f approaches infinity or as Re approaches infinity; although the latter fact is not entirely applicable since the equations for Nu and Sh are only valid for subcritical Reynolds numbers of spheres, i.e. for $Re < 2 - 4 \times 10^5$. This means that it is the heat transfer by accreted particles which dominates at high Reynolds numbers. (ii) If the total transfer rate of heat through the surface is equal to zero, but not the transport, then $Nu_{eff} = 0$; this occurs if a particle is in heat transfer equilibrium with its surroundings. This means that the magnitude of the effective Nusselt number is a measure of the amount of heat released by the fraction of accreted water which is freezing. Production of spongy ice generally means small imbalance and low effective Nusselt numbers. (iii) If the temperature is decreasing, an increased contribution of the accretion to the heat exchange causes a decrease in the effective Nusselt number. The effect is enhanced if a pressure decrease accompanies the temperature decrease.

3. Auto Simulation of the Total Heat Transfer in Clouds

If the effective Nusselt numbers are calculated for 2 cm hailstones in a hail cloud model according to Beckwith [3] and with $E = 1$, conditions are obtained as function of temperature according to Figure 1. The different curves are valid for different values of the free water content of the cloud. The curve $I = 1$, where I is the fraction

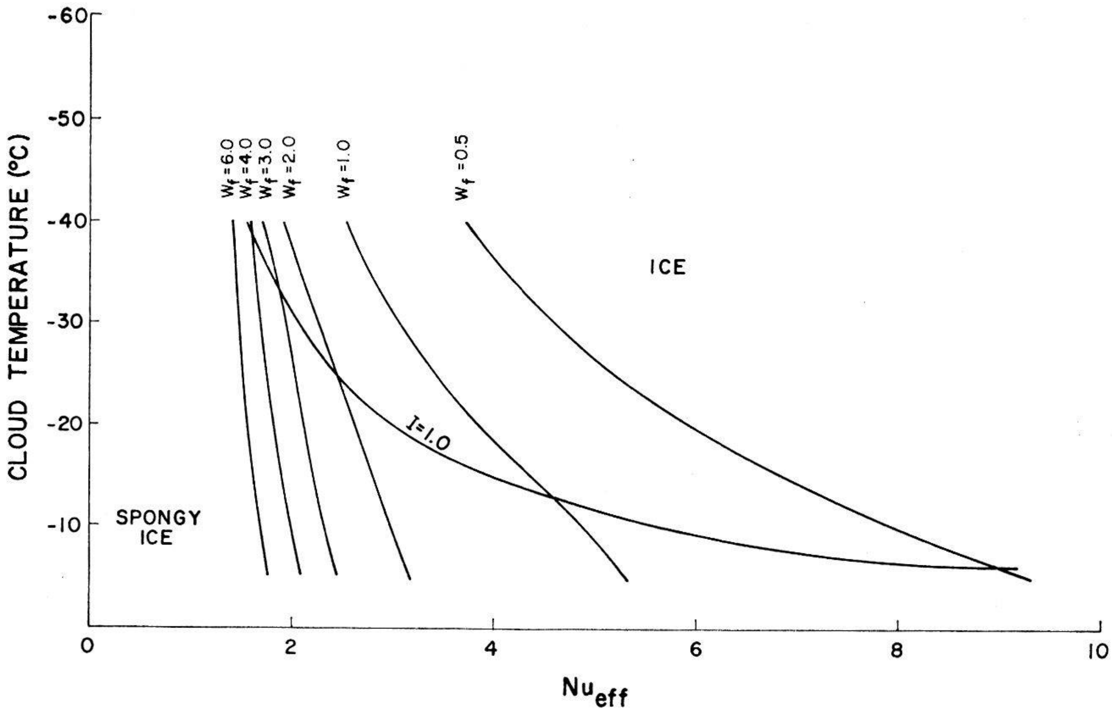

Figure 1

Auto-simulation of icing conditions of hailstones in hailclouds on the basis of effective Nusselt numbers, at different height levels indicated by air temperatures and for different liquid water contents; the hailstone diameter is assumed to be 2 cm. Equal effective Nusselt numbers are assumed to essentially produce equal ice deposits.

of accreted water which freezes, divides the icing conditions into two areas, one leading to spongy ice, the other leading to solid or porous ice. It can be found, for example, that a two centimeter hailstone at a height level in the cloud according to a temperature of − 38 °C has the same effective Nusselt number of about 3.42 at a liquid water content of 1 g m^{-3} as an equally sized hailstone at the − 20 °C level but with a liquid water content of twice the above amount, namely 2 g m^{-3}.

If the effective Nusselt number is indeed strongly connected with the production of similar ice structures – only experiments can provide a final proof – then Figure 1 would show the conditions under which shells of equal characteristics could be formed on hailstones of given size but at different cloud water contents and temperatures. One restriction should be made: simulation should only be expected for either spongy ice or solid and porous ice.

4. Simulation of Icing Conditions in the Laboratory

It was pointed out that *Re* in the atmosphere is principally determined by the diameter only (for spherical or ellipsoidal shapes). In the case of a laboratory experiment where the diameter can be varied independently of the velocity if the particles are suspended then a new degree of freedom can be obtained; hailstones growing in the atmosphere (chosen to have a diameter of 4 cm) can be compared with any size of particle as long as *Re* remains constant.

Effective Nusselt numbers are shown versus temperature in Figure 2. In the case of the solid lines, representing atmospheric conditions for 4 cm hailstones, the pressure varies with temperature according to Beckwith's model, whereas the broken lines represent conditions with the temperature varying at constant pressure (p = 1015 mb) in an icing tunnel. The parameter for the different curves is again the liquid water

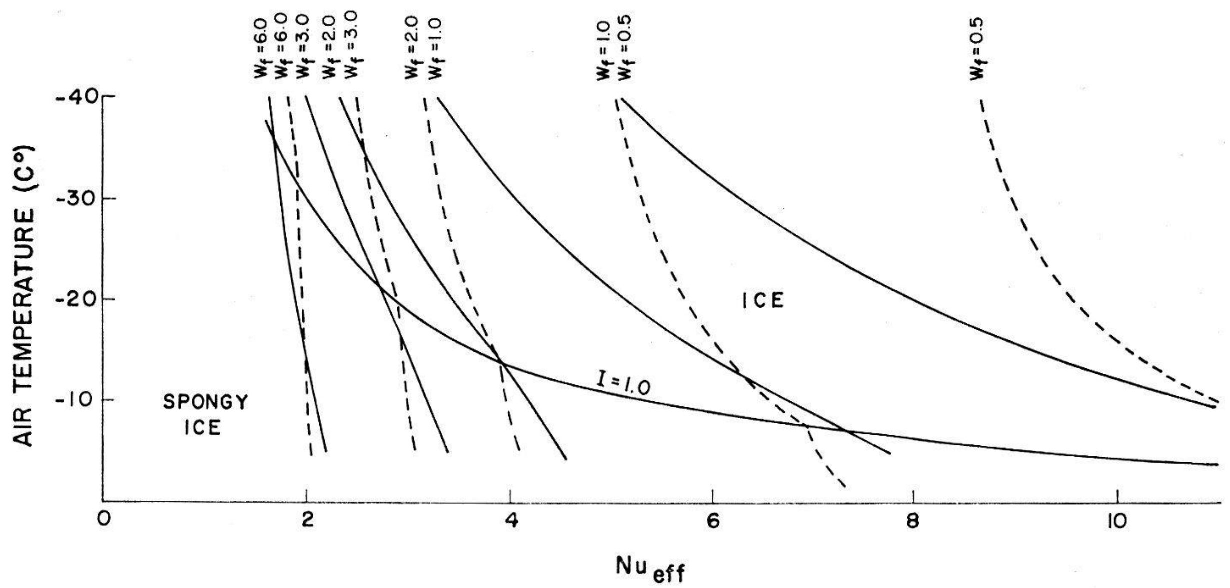

Figure 2

Effective Nusselt numbers versus air temperatures, at different liquid water contents: for laboratory (broken lines) and cloud conditions (solid lines); for a Reynolds number of Re = 58,200, representing a 4 cm hailstone freely falling in a hailcloud or a laboratory model with D determined by the wind-speed, the kinematic viscosity and Re.

content at which the real and the artificial hailstones grow. An interesting point is that the separation condition, $I = 1$, can be represented by one single line for both laboratory and atmospheric conditions. Sets of icing conditions, having equal effective Nusselt numbers, can be obtained from Figure 2 and may be related to identical ice structures.

A comparison of Figures 1 and 2 shows that equal effective Nusselt numbers can be obtained in the laboratory with bigger variations of the liquid water content.

Summary

The expected ambiguity that different sets of icing conditions lead to identical ice structures (in the statistical sense), led to the hypothesis that such sets may be correlated by the requirement of equal Reynolds numbers and of equal, newly defined effective Nusselt numbers. Such a Nusselt number considering all the heat transfer processes was derived from a dimensional viewpoint and numerical examples show under what conditions similarity can be obtained. Similar approaches could be tried for other processes where combined transfers occur.

The authors believe that this approach, i.e. the simulation of the total heat transfer, hopefully leading to the same ice structures, is to be preferred to a previous attempt [2], where – as another possible approach – equivalent heat transfer ratios were assumed to have the same effect.

In this treatment the solubility of air in water was assumed to be of minor importance. However, it is obvious that such a factor cannot be entirely neglected, and thus would cause some deviations from the given results.

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