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Classical Aspects of Ohm's Law

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Abstract. An intermediate approach to electron transport theory is outlined using a relatively simple classical model. The rate of energy loss by electrons to phonons is readily computed from the model as well as the normal mobility of electrons and the mean energy and mobility of hot electrons. The quantum aspects are satisfied by recognizing the uncertainty radius of the electron and the discrete energies of the phonons.

Introduction

Ohm's law is one of the first concepts that a student of physics encounters. It is presented as a kind of pin ball machine. The drifting electron repeatedly collides with the atoms of the solid after traversing a fixed mean free path. The mean free path is converted into a mean free time via the thermal velocity of the electron. From the mean free time and Newton's laws of motion the mobility

$$\mu = \frac{\tau_c \, e}{m} \tag{1}$$

is derived.

After this brief encounter with Ohm's law in his freshman year, the student is not likely to explore its mysteries again until some time in his graduate studies and only then if he undertakes a course in transport theory. At this time he learns that the electrons don't really collide with the atoms of the solid but rather that an electron in some energy state, characterized by the Schrödinger wave function Ψ_1, is perturbed by a phonon (also a wave motion) to make a transition to another energy state characterized by Ψ_2. The mean free time of the electron is obtained only after summing over all of the possible perturbations due to the spectrum of phonons. Moreover, the mean free time is no longer a constant of the material but depends on the energy of the electron. And, finally, the energy of the electron is a function of the applied field so that Ohm's law is only an approximation. Under certain conditions, major departures from Ohm's law can lead to both scientific as well as technological novelty.

Between the pin ball machine at one end and the formal sophistication of perturbation theory at the other, there is a time gap of some six years and a conceptual gap that is likely to be bridged only by a select group of solid state theorists. Certainly, there is a large and growing body of engineers and applied physicists who are intimately concerned with departures from Ohm's law in such phenomena as solid state microwave oscillators, solid state photo multipliers, dielectric breakdown and electro-

luminescence and whose exposure to Ohm's law falls considerably short of a graduate course in transport theory.

This discussion attempts to outline an intermediate approach to Ohm's law which might well be part of an undergraduate course of study. The approach is couched in real space (as opposed to phase space) and largely in classical concepts. Quantum aspects of the problem are brought in at the end as constraints on the classical solution. At the expense of certain computational approximations, I believe that the approach yields a clear picture of the physics of electron phonon interactions.

Outline of Argument

The momentum of an electron interacting with the phonons of a solid is randomized in a time τ_c, called the mean collision time. While the electron absorbs and emits a spectrum of phonons, its collision time is dominated by the shortest wavelength phonons with which it can interact since the density of phonons increases as λ^{-3} and since the coupling to the shorter wavelength phonons is, in general, stronger.

The electron absorbs these phonons at a rate proportional to their density n_{ph} and emits them at a rate proportional to $n_{ph} + 1$. Also, each emission or absorption is sufficient to randomize the momentum of the electron. The term unity in the emission factor is called the spontaneous emission in contrast to that induced by the thermally generated density n_{ph}. The spontaneous emission is the rate at which an electron of finite energy loses energy via phonon emission in a zero temperature ($n_{ph} = 0$) lattice. The time to emit one phonon is then related to the average rate of loss of energy by:

$$\frac{\hbar\omega}{\tau_e} = \frac{dE}{dt} \tag{2}$$

And the collision and emission times are related by:

$$\tau_e = (2\,n_{ph} + 1)\,\tau_c \doteq \frac{2\,k\,T}{\hbar\omega}\,\tau_c \text{ for } k\,T > \hbar\omega \tag{3}$$

At this point it is clear that the mobility of an electron can be computed directly by computing τ_c as is done in perturbation theory or by computing τ_e from Equation (2) and converting to τ_c via Equation (3); namely:

$$\tau_c = \frac{(\hbar\omega)^2}{2\,k\,T}\Big/\frac{dE}{dt} \tag{4}$$

The second approach to mobility depends essentially on being able to compute the average rate of energy loss dE/dt by an electron in a zero temperature lattice. A semiclassical method for carrying out this computation is outlined in the next section.

A knowledge of dE/dt also allows one to compute the mean energy (sometimes loosely called the temperature) of electrons that are heated above the lattice temperature by the applied field. The mean energy follows from the energy balance:

$$\frac{dE}{dt} = \mathcal{E}\,e\,v_d = \mathcal{E}^2\,e^2\,\frac{\tau_c}{m} = \frac{(\mathcal{E}\,e\,\hbar\omega)^2}{2\,k\,T\,m}\Big/\frac{dE}{dt}$$

or

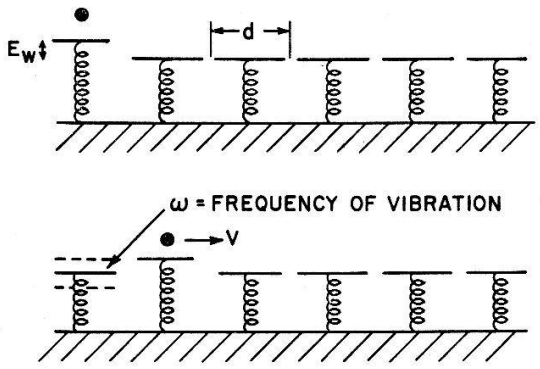
$$\frac{dE}{dt} = \frac{\mathcal{E}\, e\, \hbar\, \omega}{m\, v_t} \tag{5}$$

Here dE/dt depends on the mean energy of the heated electron, $\mathcal{E}$ is the applied field and v_t the thermal velocity of an electron at the lattice temperature.

Spontaneous Emission

Consider an electron immersed in a solid. Its coulomb field polarizes the solid medium so that the energy of the electron is lowered. For example, the energy per unit volume in the coulomb field in the solid is $1/(8\pi K)\ (e/r^2)^2$ while the same energy in vacuum was $1/8\pi\ (e/r^2)^2$. The difference $(K - 1/K)\ 1/8\pi\ (e/r^2)^2$ is a measure of the work required to remove the electron from the solid to vacuum. It is also a measure of the energy-well the electron digs for itself by polarization of the solid.

If the electron moves sufficiently slowly, it carries this polarization along with it without loss of energy. At higher electron velocities the response of the medium is not fast enough to keep pace with the electron and hence the polarization is left behind as a trail or wake of polarization energy [1]. This is, in essence, the classical physics of spontaneous emission. The magnitude of the rate of energy loss will be computed classically and certain quantum constraints will be imposed upon the solution to reconcile it with the laws of quantum physics.



Model for rate of loss of energy by electrons in a solid.

The Figure shows the classical model on which the energy-loss computation is based. A particle moves past an array of elements with a velocity v. Each element has a dimension d and is attached to a spring such that it has a characteristic frequency ω. There is a force of attraction between the particle and each element such that when the particle is stationary an energy E_w^0 is stored in the element opposite the particle. In this model, the particle represents the electron, the array of elements represent the medium, and the energy E_w^0 corresponds to the energy-well formed by the interaction of electron and medium.

We can by inspection of the Figure approximate the maximum rate of loss of energy by the particle to the array of elements. Let the particle remain opposite an element for a time ω^{-1}, that is, for a time just sufficient to allow the element to respond and

to store an energy E_w^0 in its spring. Now move the particle abruptly to the next element. This will leave behind the energy E_w^0. The particle remains on the second element for a time ω^{-1} sufficient to allow it to respond and is then abruptly moved to the third element and so on. The rate of loss of energy is then:

$$\frac{dE}{dt} \doteq E_w^0 \, \omega \tag{6}$$

and the average velocity of the particle is $v = d\,\omega$. It is clear that, within a factor of about two, the same rate of loss of energy will be incurred if the step wise motion is replaced by the average velocity $d\,\omega$.

We wish now to compute the rate of loss of energy for velocities greater than $d\omega$. Under these conditions the particle passes each element in a time short compared with its response time and imparts a momentum to it proportional to its transit time. The energy imparted will be proportional to the square of the momentum or to the square of its transit time. Thus, the energy-well will be proportional to the same factor:

$$E_w = E_w^0 (T\,\omega)^2 = E_w^0 \left(\frac{d\omega}{v}\right)^2 \tag{7}$$

The factor ω is a normalizing factor such that when the transit time $T \to \omega^{-1}$, the energy well $E_w \to E_w^0$ consistent with Equation (6). To compute the time rate of loss of energy, we need to multiply Equation (7) by the number of energy-wells traced out per second; namely:

$$\frac{dE}{dt} = E_w \frac{v}{d} = E_w^0 \, d \, \frac{\omega^2}{v} \tag{8}$$

It remains to evaluate E_w^0 for an electron in a solid. To do so we recognize that the maximum energy the electron can impart to the medium is its coulomb energy e^2/d. Moreover, since we are concerned here with the interaction between the electron and the lattice ions or atoms, the available coulomb energy is reduced or screened by the electronic part of the dielectric constant to a value $e^2/(Kd)$ where K is the electronic part of the dielectric constant. The electron, for velocities not exceeding about ten volts, carries this electronic polarization with it so that the lattice sees only the reduced coulomb field or energy. Since $e^2/(Kd)$ is the maximum energy the electron can impart to the lattice, we multiply it by a factor β, where $0 \leqq \beta \leqq 1$, to obtain E_w^0. By definition, β is the fraction of available coulomb energy of the electron that can be imparted to the lattice. With this substitution for E_w^0, Equation (7) becomes:

$$\frac{dE}{dt} = \beta \, \frac{e^2 \, \omega^2}{K \, v} \tag{9}$$

Equation (9) is essentially the classical result for the rate of energy loss or the spontaneous emission by an electron of velocity v to phonons of frequency ω. The factor β is a coupling constant that is still to be evaluated and that is a function of the type of phonon as well as the frequency of the phonon. Equation (9) was evaluated for the dimension d or rather for a spherical shell of dimensions $d \pm d/2$ surrounding the electron. A more complete analysis (see Ref. [2]) sums up the contributions from other shells so that Equation (9) is to be multiplied by a numerical factor in the neighborhood of unity.

The major quantum constraint on Equation (9) is that imposed by the quantum rule of k-conservation. The shortest wavelength phonon an electron of velocity v can emit is given by:

$$\lambda\!\!\!^{-} \equiv \frac{\lambda}{2\pi} = \frac{\hbar}{2 m v} \tag{10}$$

This constraint can also be looked upon semiclassically as saying that the uncertainty radius of the electron should not exceed the wavelength of the phonon it emits. In the case of acoustic phonons Equation (9) can be rewritten as

$$\omega = \frac{2 m v v_s}{\hbar} \tag{11}$$

where v_s is the velocity of sound. By Equation (11) the appropriate ω in Equation (9) is defined for each velocity v of the electron.

It remains to clarify the meaning of the coupling constant β. This has been done in detail in References [2] and [3]. We quote here the results. On the one hand, β was defined as the fraction of available coulomb energy of the electron that can be transferred to the medium or, more particularly, to the mode of vibration of the medium through which the electron loses energy. On the other hand, it can be shown [2, 3] that this definition of β is equivalent to the ratio of electrical to total energy of the macroscopic sound wave corresponding to the type of phonon or electron-phonon coupling under consideration. For example, in the case of acoustic phonons coupled to electrons via their piezoelectric fields, β is essentially the well-known quantity, the square of the electromechanical coupling coefficient, long used to analyse crystal oscillators. In general, β has an easily visualizable physical meaning for the several types of phonons and electron-phonon couplings. Its values are listed in Table 1. This form of coupling constant has the additional virtues that it is valid also for electron-electron interactions and for identifying the several types of acoustoelectric interactions. In brief, it unifies an extensive body of otherwise disconnected literature.

Sample Calculations

The utility of Table 1 together with Equations (2)-(5), (10) and (11) is illustrated by the following sample calculations.

Equations (2)–(5) can be written as:

$$\frac{dE}{dt} = \frac{\hbar\omega}{\tau_e} = \frac{(\hbar\omega)^2}{2 k T \tau_c} = \frac{\mathcal{E} e \hbar \omega}{m v_t}$$

and Equations (10) and (11) as:

$$\hbar\omega = 2 m v v_s = 2 m v \lambda\!\!\!^{-} \omega$$

In thermal equilibrium $v = v_t$ and the temperature dependence of mobility for interaction with acoustic phonons becomes:

$$\mu = \frac{\tau_c e}{m} \propto \frac{\omega^2}{v_t^2} \Big/ \frac{dE}{dt}$$

$$\propto \begin{cases} v_t^{-3} \propto T^{-3/2} & \text{deformation potential coupling} \\ v_t^{-1} \propto T^{-1/2} & \text{piezoelectric coupling} \end{cases}$$

Table I

Phonons	Coupling	β	v-dependence of dE/dt
Polar optical	Polarization field	$\varepsilon_0 - \varepsilon_\infty/\varepsilon_0$	v^{-1}
Non-polar optical	Deformation potential (D)	$K D^2/(4\pi\varrho e^2\omega^2\lambda^2)$	v
Acoustic	Piezoelectric field	$\varepsilon_p^2/(K C)$	v
Acoustic	Deformation potential (B)	$K B^2\omega^2/(4\pi\varrho e^2 v_s^4)$	v^3

Definitions:
ε_0 low frequency dielectric constant
ε_∞ high frequency (optical) dielectric constant
K dielectric constant at frequencies $\gtrsim \omega$
ε_p piezoelectric constant
C elastic constant
ϱ density (grams/cm^3)
v_s velocity of sound

The dependence on electric field of the mean energy E of hot electrons is obtained by:

$$\frac{1}{\omega}\frac{dE}{dt} \propto \mathcal{E}$$

or

$$\begin{cases} v^2 \propto E \propto \mathcal{E} & \text{deformation potential coupling to acoustic phonons} \\ v \propto E^{1/2} \propto \mathcal{E} & \text{deformation potential coupling to optical phonons} \end{cases}$$

The threshold field for generating hot electrons when the electrons interact with acoustic phonons is obtained from:

$$\frac{(\hbar\omega)^2}{2kT\tau_c} = \frac{\mathcal{E} e \hbar\omega}{m v_t}$$

or

$$\frac{\mathcal{E} e \tau_c}{m} = v_{drift} = \frac{\hbar\omega}{m v_t} = 2 v_s .$$

References

[1] H. Frohlich and E. Pelzer, ERA Report # L/T, 184 (1948).
[2] A. Rose, RCA Rev. *27*, 600 (1966).
[3] A. Rose, RCA Rev. *27*, 98 (1966).