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Coulomb and Mass Difference Corrections to K^-p Scattering

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Abstract. This paper presents a K-matrix formalism for low energy K^-p reactions which in a non-relativistic model includes electromagnetic mass difference effects and the effect of the Coulomb potential. In extension of the usual K-matrix treatment both inner and outer corrections are included in a first-order perturbation treatment, in the elastic as well as in the reaction channels. The new results are compared with those of the previous K-matrix treatments.

1. Introduction

A formalism for the inclusion of Coulomb and mass difference corrections to the scattering matrix for a many-channel system has recently been published by Oades and Rasche [1, 2]. This work is particularly relevant to the $\bar{K}N$ system where there are already six open channels just above threshold. Dalitz and Tuan [3] pointed out that the real K-matrix elements are convenient parameters to describe such a scattering process, the relation between the K and T matrices, the Heitler equation, then guaranteeing a unitary scattering matrix. For this reason it is of use to apply the formalism of Oades and Rasche to calculate the Coulomb and mass difference corrections to the K-matrix rather than to the scattering matrix.

It should be pointed out that all numerical analyses of the low-energy $\bar{K}N$ multichannel system have only partially included the Coulomb and mass difference corrections. (For a review of such $\bar{K}N$ analyses see [4].) The method used is that of Dalitz and Tuan [3] where the Coulomb force is only included in the K^-p channel and there only outside the range of the strong interaction (outer Coulomb corrections). The neglect of the Coulomb forces in the $\pi\Sigma$ channels may be justified due to the much smaller Coulomb parameter in these channels but the influence of the Coulomb force in the K^-p channel within the strong interaction region (inner Coulomb corrections) cannot be neglected. In this paper both inner and outer Coulomb corrections are included in all charged channels.

Dalitz and Tuan also only include the $\bar{K}^0 n - K^-p$ mass difference and treat only the kinematic effects, i.e. by including different phase space densities. In a non-relativistic potential model the dynamical mass difference effects due to changes of the Schrödinger equation can also be calculated and in this paper such effects are calculated for all channels.

This non-relativistic formalism is only intended to be applied to the low-energy region (see [1] for comments on this point) where K^-p scattering is well described by only s-waves. For this practical reason and for the sake of simplicity the formalism is only given for the case $l=0$. Thus the basic equation is

$$\left(\frac{d^2}{dr^2}+k_i^2\right)|R\}_i=\begin{cases}\sum_j 2m_i(U_{ij}+V_i\delta_{ij})|R\}_j & r\leqslant r_0\\ 2m_i V_i|R\}_i & r\geqslant r_0\end{cases} \tag{1}$$

where k_i is the c.m. momentum in channel $|i\rangle$, $|R\}_i$ is the ith component of the radial part of the state vector, m_i is the reduced mass for channel $|i\rangle$, U_{ij} is the strong interaction potential $\langle i|U|j\rangle$, V_i is the Coulomb potential $\langle i|V|i\rangle$ and r_0 is some range beyond which the strong interaction can be neglected (units are chosen such that $\hbar=c=1$).

It should be pointed out that an explicit form for the strong potential is not required in the following work, the $l=0$ radial wave function being approximated by a second-order polynomial in the numerical calculations. It should also be noted that γ processes such as radiative capture and bremsstrahlung as well as the open $\Lambda\pi\pi$ channel are neglected since the relevant cross-sections are very small compared to the other channels. For further comments on the question of neglecting the radiative capture processes see [5].

In Section 2 the general K-matrix formalism is outlined and in Section 3 it is applied to charge independent K^-p scattering. In Section 4 the inner corrections are included and in Section 5 the full corrections are derived. Finally, in Section 6, the K-matrix parameters of Martin and Ross [6] are used to compare physical quantities calculated using the Dalitz–Tuan corrections and using the full corrections.

2. K Matrix Formalism

We start from the normal Schrödinger equation

$$(H_0+V)|\psi\rangle=E|\psi\rangle \tag{2}$$

where H_0 is the Hamiltonian of the free particles. The solutions $|\psi^+\rangle$ and $|\psi^1\rangle$ are defined by the following integral equations

$$|\psi_a^+\rangle=|\varphi_a\rangle+(E-H_0+i\epsilon)^{-1}V|\psi_a^+\rangle, \tag{3}$$

$$|\psi_a^1\rangle=|\varphi_a\rangle+\tfrac{1}{2}\sum_{\pm}(E-H_0\pm i\epsilon)^{-1}V|\psi_a^1\rangle, \tag{4}$$

where $|\varphi_a\rangle$ is a solution of (2) for $V=0$. The K and T matrices are now defined by these solutions.

$$T_{ba}=\langle\varphi_b|V|\psi_a^+\rangle, \tag{5}$$

$$K_{ba}=\langle\varphi_b|V|\psi_a^1\rangle. \tag{6}$$

From the relation between solutions (3) and (4) one obtains the relation between the K and T matrices [7]

$$T+i\pi K\delta(E-H_0)\,T=K. \tag{7}$$

If $|\varphi_a\rangle$ represents a plane wave in channel α with wave vector $\vec{k}^\alpha$

$$|\varphi_a(\vec{x})\rangle=(2\pi)^{-3/2}\exp(i\vec{k}_\alpha\vec{x})|\alpha\rangle \tag{8}$$

we get from equations (3) and (4), by inserting $\Sigma_a |\varphi_a\rangle\langle\varphi_a|$,

$$|\psi_a^+(\vec{x})\rangle = (2\pi)^{-3/2} \exp(i\vec{k}_\alpha \vec{x})\,|\alpha\rangle + \sum_\beta \int d^3\vec{k}'_\beta (2\pi)^{-3/2} \exp(i\vec{k}'_\beta \vec{x})$$
$$*[\vec{k}_\alpha^2/(2m_\alpha) - \vec{k}_\beta'^2/(2m_\beta) + i\epsilon]^{-1}\, T_{ba}|\beta\rangle \tag{9}$$

$$|\psi_a^1(\vec{x})\rangle = (2\pi)^{-3/2} \exp(i\vec{k}_\alpha \vec{x})\,|\alpha\rangle + \tfrac{1}{2}\sum_{\beta,\pm} \int d^3\vec{k}'_\beta (2\pi)^{-3/2} \exp(i\vec{k}'_\beta \vec{x})$$
$$*[\vec{k}_\alpha^2/(2m_\alpha) - \vec{k}_\beta'^2/(2m_\beta) \pm i\epsilon]^{-1}\, K_{ba}|\beta\rangle. \tag{10}$$

In the limit $|\vec{x}| \to \infty$ the integration over $d^3\vec{k}'_\beta$ can be carried out to give

$$|\psi_a^+(\vec{x})\rangle \underset{x\to\infty}{\sim} \exp(i\vec{k}_\alpha \vec{x})\,|\alpha\rangle - \sum_\beta (2\pi)^2 m_\beta x^{-1} \exp(ik_\beta x)\, T_{ba}|\beta\rangle, \tag{11}$$

$$|\psi^1(\vec{x})\rangle \underset{x\to\infty}{\sim} \exp(i\vec{k}_\alpha \vec{x})\,|\alpha\rangle - \tfrac{1}{2}\sum_\beta (2\pi)^2 m_\beta x^{-1}[\exp(ik_\beta x)\, K_{ba}$$
$$+ \exp(-ik_\beta x)\, K_{-ba}]\,|\beta\rangle, \tag{12}$$

where

$$|b\rangle = |k_\beta \hat{\vec{x}}, \beta\rangle,$$
$$|-b\rangle = |-k_\beta \hat{\vec{x}}, \beta\rangle,$$
$$E_b = E_a \quad \text{or} \quad k_\beta^2 = m_\beta/m_\alpha\, k_\alpha^2.$$

To obtain the connection between the K-matrix elements and the solution of the radial Schrödinger equation, one expands both the solution and the K-matrix in a series of Legendre polynomials[1])

$$|\psi_a^1(\vec{x})\rangle = (2\pi)^{-3/2} \sum_l i^l (2l+1)\, P_l(\hat{\vec{k}}_\alpha, \hat{\vec{x}})\, |\psi_a^{1l}(x)\rangle, \tag{13}$$

$$K_{ba} = -(2\pi)^{-2}(m_\alpha m_\beta)^{-1/2} \sum_l (2l+1)\, P_l(\hat{\vec{k}}_\beta, \hat{\vec{k}}_\alpha)\, K_{\beta\alpha}^l, \tag{14}$$

$$K_{-ba} = -(2\pi)^{-2}(m_\alpha m_\beta)^{-1/2} \sum_l (2l+1)(-1)^l\, P_l(\hat{\vec{k}}_\beta, \hat{\vec{k}}_\alpha)\, K_{\beta\alpha}^l. \tag{15}$$

For fixed l we have from (12)

$$|x\psi_a^{1l}(x)\rangle \underset{x\to\infty}{\sim} (v_\alpha)^{-1/2} \sin(k_\alpha x - l\pi/2)\,|\alpha\rangle + \sum_\beta k_\beta^{1/2} K_{\beta\alpha}^l k_\alpha^{1/2}$$
$$*(v_\beta)^{-1/2} \cos(k_\beta x - l\pi/2)\,|\beta\rangle \tag{16}$$

where $v_\alpha = k_\alpha/m_\alpha$. With the definitions

$$\bar{K}^l = P^{1/2} K^l P^{1/2} \quad \text{and} \quad (P^{1/2})_{\alpha\beta} = \delta_{\alpha\beta}\, k_\alpha^{1/2} \tag{17}$$

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[1]) These expansions are only this simple for the scattering of two spin 0 particles or, as in our example, for one spin 0 and one spin $\frac{1}{2}$ particle where only the s waves are important and the higher partial waves are neglected.

we can write for any solution of the radial Schrödinger equation

$$|x\psi^l(x)\rangle \underset{x\to\infty}{\sim} \sum_\alpha A_\alpha (v_\alpha)^{-1/2} \sin(k_\alpha x - l\pi/2)\,|\alpha\rangle$$
$$+ \sum_{\alpha,\beta} \bar{K}^l_{\beta\alpha} A_\alpha (v_\beta)^{-1/2} \cos(k_\beta x - l\pi/2)\,|\beta\rangle. \tag{18}$$

This means that the $\bar{K}$-matrix provides the transformation which connects the coefficients of the sine and cosine terms in asymptotic form of the radial wave function.

From equation (11) one sees that the usual scattering amplitude for the transition $a \to b$ is

$$f_{ba} = -m_\beta (2\pi)^2\, T_{ba}. \tag{19}$$

The differential cross-section is obtained by using (19) together with the expansion

$$T_{ba} = -(2\pi)^{-2} (m_\alpha m_\beta)^{-1/2} \sum_l (2l+1)\, P_l(\hat{\bar{k}}_\beta, \hat{\bar{k}}_\alpha)\, T^l_{\beta\alpha} \tag{20}$$

to give

$$\frac{d\sigma}{d\Omega}(a \to b) = k_\beta/k_\alpha \left| \sum_l (2l+1)\, P_l(\hat{\bar{k}}_\beta, \hat{\bar{k}}_\alpha)\, T^l_{\beta\alpha}\right|^2. \tag{21}$$

One obtains $T^l_{\beta\alpha}$ from $K^l_{\beta\alpha}$ by relation (7) using expansions (14) and (20)

$$(T^l)^{-1} = (K^l)^{-1} - iP \tag{22}$$

or, solving for T^l,

$$\bar{T}^l = (1 - i\bar{K}^l)^{-1}\, \bar{K}^l, \tag{23}$$

where in general

$$\bar{A} = P^{1/2} A P^{1/2}. \tag{24}$$

In the presence of Coulomb forces the expansion of the radial wave function (18) must be modified to an expansion in terms of the regular and irregular Coulomb wave functions

$$|x\psi^l(x)\rangle \underset{x\to\infty}{\sim} \sum_\alpha A_\alpha (v_\alpha)^{-1/2} F_l(k_\alpha x, \eta_\alpha)\,|\alpha\rangle + \sum_{\alpha,\beta} \bar{K}^l_{\beta\alpha} A_\alpha (v_\beta)^{-1/2}$$
$$* G_l(k_\beta x, \eta_\beta)\,|\beta\rangle \tag{25}$$

where

$$\eta_\alpha = Z_1 Z_2 e^2 v_\alpha^{-1}.$$

The connection to the differential cross-section proceeds in an analogous manner to give

$$\frac{d\sigma}{d\Omega}(a \to b) = k_\beta/k_\alpha \left| f^{\text{Coul}}_{aa}\, \delta_{\alpha\beta} + \sum_l (2l+1)\, P_l(\hat{\bar{k}}_\beta, \hat{\bar{k}}_\alpha)\, T^l_{\beta\alpha} \exp(i\sigma^l_\alpha + i\sigma^l_\beta)\right|^2 \tag{26}$$

where σ_α^l and σ_β^l are the Coulomb phases for angular momentum l and Coulomb parameters η_α and η_β respectively and where f_{aa}^{Coul} is the pure Coulomb scattering amplitude in the absence of strong interactions.

3. Charge Independent K^-p Scattering

a) General notation

The nomenclature of [1] is here repeated very briefly. $|i\rangle$ and $|j)$ are the states in the charge and isospin basis.

$$\begin{aligned}
|1\rangle &= |pK^-\rangle & |1) &= |I=2, \Sigma\pi\rangle \\
|2\rangle &= |n\bar{K}^0\rangle & |2) &= |I=1, N\bar{K}\rangle \\
|3\rangle &= |\Sigma^+\pi^-\rangle & |3) &= I=1, \Sigma\pi\rangle \\
|4\rangle &= \Sigma^0\pi^0\rangle & |4) &= |I=1, \Lambda\pi\rangle \\
|5\rangle &= |\Sigma^-\pi^+\rangle & |5) &= |I=0, N\bar{K}\rangle \\
|6\rangle &= |\Lambda\pi^0\rangle & |6) &= |I=0, \Sigma\pi\rangle
\end{aligned} \tag{27}$$

The unitary matrix for the basis transformation consists of the following Clebsch–Gordan coefficients

$$|i) = \sum_{i=1}^{6} |i\rangle\langle i|j) \tag{28}$$

$$\langle i|j) = \begin{pmatrix} 0 & \sqrt{1/2} & 0 & 0 & \sqrt{1/2} & 0 \\ 0 & \sqrt{1/2} & 0 & 0 & -\sqrt{1/2} & 0 \\ \sqrt{1/6} & 0 & \sqrt{1/2} & 0 & 0 & \sqrt{1/3} \\ \sqrt{2/3} & 0 & 0 & 0 & 0 & -\sqrt{1/3} \\ \sqrt{1/6} & 0 & -\sqrt{1/2} & 0 & 0 & \sqrt{1/3} \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}.$$

The following matrices are used:

$$\begin{aligned}
\langle i|M|j\rangle &= m_i\,\delta_{ij} \\
\langle i|P^2|j\rangle &= k_i^2\,\delta_{ij} \\
\langle i|P^{1/2}|j\rangle &= k_i^{1/2}\,\delta_{ij} \\
\langle i|D|j\rangle &= (d^2/dr^2 + k_i^2)\,\delta_{ij}.
\end{aligned} \tag{29}$$

To distinguish between quantities in the charge and isospin basis matrix elements and components are written with superscripts $\wedge$ or $\frown$ respectively. For example a general state vector is written as

$$|R\} = \sum_i \hat{R}_i |i\rangle = \sum_j \widehat{R}_j |j) \tag{30}$$

in the two bases.

b) Exact charge independence for $l = 0$

Neglecting mass differences and the electromagnetic interactions one has exact charge independence. In this case variables are denoted with a superscript N, e.g.

$$\begin{aligned} \hat{m}_1^N &= \hat{m}_2^N = \hat{m}_1, \\ \hat{m}_3^N &= \hat{m}_4^N = \hat{m}_5^N = \hat{m}_3, \\ \hat{m}_6^N &= \hat{m}_6 \end{aligned} \tag{31}$$

For this charge independent case the mass matrix M^N is also diagonal in the isospin basis. The same is true for the matrices P^{2N} and D^N. With $V = 0$ equation (1) becomes

$$D^N |R^N\} = \begin{cases} 2M^N\, U |R^N\} & r \leqslant r_0 \\ 0 & r \geqslant r_0. \end{cases} \tag{32}$$

From Ref. [1] we know that we can write a regular solution of (32) for $r \geqslant r_0$ as follows:

$$R_i^N = \sum_\alpha A_\alpha R_{i\alpha}^N = \sum_\alpha A_\alpha t_{i\alpha}^N (v_i^N)^{-1/2} \sin (k_i^N r + \delta_\alpha^N) \quad r \geqslant r_0 \quad i = 1 \ldots 6, \tag{33}$$

i and α number the six components and six independent regular solutions respectively and A_α are six arbitrary constants. t^N is a real unitary matrix, i.e.

$$(t^N)^+ t^N = 1. \tag{34}$$

t^N decomposes into a one-, a three- and a two-dimensional submatrix corresponding to $I = 2$, 1 and 0, because the strong interaction potential U has non-vanishing matrix elements only between states of the same isospin.

The relation (18) gives us the connection between t^N, the phases δ_α^N and the matrix K^N, when we expand the radial wave function $|R^N\}$ in terms of sines and cosines for $r \to \infty$

$$\widehat{R}_i^N \underset{r\to\infty}{\sim} \widehat{B}_i (\widehat{v}_i^N)^{-1/2} \sin (\widehat{k}_i^N r) + \widehat{C}_i (\widehat{v}_i^N)^{-1/2} \cos (\widehat{k}_i^N r), \tag{35}$$

$$\begin{aligned} \widehat{B}_i &= \sum_\alpha A_\alpha \widehat{t}_{i\alpha}^N \cos \delta_\alpha^N, \\ \widehat{C}_i &= \sum_\alpha A_\alpha \widehat{t}_{i\alpha}^N \sin \delta_\alpha^N. \end{aligned} \tag{36}$$

From (18) we know that

$$\widehat{C}_i = \sum_j (i|K^N|j)\, \widehat{B}_j \tag{37}$$

and so, because the A_α are any constants, we have

$$\hat{t}^N_{i\alpha} \tan \delta^N_\alpha = \sum_j (i|K^N|j)\, \hat{t}^N_{j\alpha} \tag{38}$$

or

$$(i|\bar{K}^N|j) = \sum_\alpha \hat{t}^N_{i\alpha} \tan \delta^N_\alpha (t^N)^+_{\alpha j}. \tag{39}$$

As in the case of t^N, $\bar{K}^N$ decomposes into three submatrices. Furthermore, $\bar{K}^N$ is symmetric, which we see at once from (39) remembering that t^N is real. Therefore there are only ten non-vanishing matrix elements of $\bar{K}^N$. These are chosen as the ten parameters describing the scattering. From these parameters one finds the values for δ^N_α and $t^N_{i\alpha}$ by diagonalizing the matrix $\bar{K}^N$.

4. Inner Corrections for $l = 0$

We obtain the inner corrections by introducing the exact masses and the Coulomb potential in the Schrödinger equation for $r \leqslant r_0$. r_0 is chosen as a further parameter. The Coulomb potential is taken as that of a point-like charge and a homogeneous charged sphere with radius r_0. The corrections are only weakly dependent on this special choice of the Coulomb potential.

$$\langle i|V|j\rangle = \begin{cases} \delta_{ij}\ \eta_i\ k_i (m_i\ r)^{-1} & r \geqslant r_0 \\ \delta_{ij}\, \eta_i\, k_i\, m_i^{-1} (1.5 r_0^{-1} - 0.5 r_0^{-3} r^2) & r \leqslant r_0 \end{cases} \tag{40}$$

$$\eta_i = \begin{cases} -e^2 v_i^{-1} & i = 1, 3, 5 \\ 0 & i = 2, 4, 6. \end{cases} \tag{41}$$

The s-wave Schrödinger equation now has the following form:

$$D^N |R^{IN}\} = \begin{cases} 2MU|R^{IN}\} + \Delta |R^{IN}\} & r \leqslant r_0 \\ 0 & r \geqslant r_0 \end{cases} \tag{42}$$

where

$$\Delta = 2M\,V + 2(M - M^N)\, U - (P^2 - P^{2N}). \tag{43}$$

As in the preceding section a general regular solution outside r_0 can be written as

$$R^{IN}_i = \sum_\alpha A_\alpha R^{IN}_{i\alpha} = \sum_\alpha A_\alpha t^{IN}_{i\alpha} (v^N_i)^{-1/2} \sin (k^N_i r + \delta^{IN}_\alpha). \tag{44}$$

The matrix t^{IN} can be chosen to be real and unitary [1].

$$t^{IN} (t^{IN})^+ = 1. \tag{45}$$

Now t^{IN} no longer decomposes into submatrices because of the isospin mixing caused by the Coulomb potential and the mass differences.

From Ref. [1] we have formulae to calculate δ^{IN}_α and $t^{IN}_{i\alpha}$ by perturbation theory to first order in Δ.

$$\delta^{IN}_\alpha = \delta^N_\alpha - X_{\alpha\alpha}, \tag{46}$$

$$\hat{t}^{IN}_{i\alpha} = \hat{t}^{N}_{i\alpha} + \sum_{\beta \neq \alpha} \hat{t}^{N}_{i\beta} \sin^{-1} (\delta^{N}_{\beta} - \delta^{N}_{\alpha}) X_{\beta\alpha}, \tag{47}$$

$$X_{\beta\alpha} = \int_0^{r_0} dr \sum_{i,j} (\hat{m}^{N}_{i})^{-1} (i|\Delta|j) \hat{R}^{N}_{i\beta} \hat{R}^{N}_{j\alpha}. \tag{48}$$

In the numerical calculations $R^{N}_{i\alpha}$ is approximated by the simplest polynomial that vanishes at $r = 0$ and that joins continuously onto the solution (33) at $r = r_0$; i.e.

$$R^{N}_{i\alpha}(r) = r(A_{i\alpha} + r B_{i\alpha}) \quad r \leqslant r_0, \tag{49}$$

$$A_{i\alpha} = \hat{t}^{N}_{i\alpha} r_0^{-1} (\hat{v}^{N}_{i})^{-1/2} (2 \sin \omega_{i\alpha} - \hat{k}^{N}_{i} r_0 \cos \omega_{i\alpha}), \tag{50}$$

$$B_{i\alpha} = \hat{t}^{N}_{i\alpha} r_0^{-2} (\hat{v}^{N}_{i})^{-1/2} (\hat{k}^{N}_{i} r_0 \cos \omega_{i\alpha} - \sin \omega_{i\alpha}), \tag{51}$$

$$\omega_{i\alpha} = \hat{k}^{N}_{i} r_0 + \delta^{N}_{\alpha}. \tag{52}$$

Proceeding in exactly the same way as in the previous section one now obtains the K-matrix with inner corrections

$$(i|\bar{K}^{IN}|j) = \sum_{\alpha} \hat{t}^{IN}_{i\alpha} \tan \delta^{IN}_{\alpha} (\hat{t}^{IN})^{+}_{\alpha j}. \tag{53}$$

The symmetry of $\bar{K}^{IN}$ also follows here from the orthogonality relation (45).

5. Inner and Outer Corrections for $l = 0$

Now we solve the exact radial Schrödinger equation (1). We can express a general solution for $r \geqslant r_0$ in terms of the regular and irregular Coulomb wave functions for $l = 0$.

$$\hat{R}_{i} = \sum_{\alpha} A_{\alpha} \hat{R}_{i\alpha} = \sum_{\alpha} A_{\alpha} \hat{t}_{i\alpha} (\hat{v}_{i})^{-1/2} [\cos \delta_{i\alpha} F_{0i}(r) + \sin \delta_{i\alpha} G_{0i}(r)]. \tag{54}$$

A_{α} are again six arbitrary constants. Because equations (1) and (42) are identical for $r \leqslant r_0$, one chooses $t_{i\alpha}$ and $\delta_{i\alpha}$ such that $\hat{R}_{i\alpha}$ and $\hat{R}^{IN}_{i\alpha}$ join smoothly at $r = r_0$. With the definitions

$$\hat{Q}1_{i\alpha} = \hat{t}_{i\alpha} \sin \delta_{i\alpha}, \tag{55}$$

$$\hat{Q}2_{i\alpha} = \hat{t}_{i\alpha} \cos \delta_{i\alpha}, \tag{56}$$

we get from this condition

$$\hat{Q}1_{i\alpha} = (\hat{v}_{i})^{1/2} \hat{k}_{i}^{-1} [\hat{R}^{IN}_{i\alpha} (F_{0i})' - (\hat{R}^{IN}_{i\alpha})' F_{0i}]_{r=r_0}, \tag{57}$$

$$\hat{Q}2_{i\alpha} = (\hat{v}_{i})^{1/2} \hat{k}_{i}^{-1} [(\hat{R}^{IN}_{i\alpha})' G_{0i} - \hat{R}^{IN}_{i\alpha} (G_{0i})']_{r=r_0}, \tag{58}$$

where the dash denotes the derivative with respect to r. The exact K-matrix follows by comparing (25) with the expansion of the solution (54) in terms of F_{0i} and G_{0i}.

$$\bar{K} = Q1 \cdot Q2^{-1}. \tag{59}$$

Calculating the Wronski determinant with two different solutions of equation (1) one obtains the following condition

$$Q1^T \cdot Q2 = Q2^T \cdot Q1. \tag{60}$$

This ensures that we have a symmetric K-matrix.

6. Results

In order to see the effect of the full corrections Table 1 gives some results for the total cross-sections with a pK^- input channel. The values of K^N for $I = 0$ and 1 are those given by Martin and Ross [6]. The K-matrix value for $I = 2$ and the radius r_0 is given in Table 1.

Table 1
Differential cross-sections times 4π at 100 MeV/c for pK^- scattering in the absence of the pure Coulomb amplitude (in mbarn). The $I = 0$ and 1 K^N matrix values are taken from Martin and Ross [6].

K^N $(I = 2)$ →	0	0	0	0	0.1	−0.1
r_0 →		0.4	0.4	1.0	0.4	0.4
Corrections →	No	DT	Full	Full	Full	Full
$pK^- \to pK^-$	101.8	115.1	111.8	110.0	111.8	111.8
$pK^- \to n\bar{K}^0$	18.8	19.3	18.3	18.6	18.3	18.3
$pK^- \to \Sigma^+\pi^-$	29.9	30.8	31.6	31.4	31.7	31.5
$pK^- \to \Sigma^0\pi^0$	26.5	27.3	25.4	24.7	25.0	25.8
$pK^- \to \Sigma^-\pi^+$	72.0	74.1	74.9	76.4	75.2	74.5
$pK^- \to \Lambda\pi^0$	26.9	27.7	28.5	29.4	28.5	28.5
Σ^+/Σ^-	0.416	0.416	0.422	0.411	0.421	0.422
$\Lambda/(\Sigma^0 + \Lambda)$	0.504	0.504	0.529	0.544	0.533	0.525

The first three columns show the results with inclusion of various corrections. No corrections means no Coulomb and mass difference corrections but with the exact momenta in the kinematical factors. DT corrections means with those corrections described by Dalitz and Tuan [3]. The only difference to the former case is the inclusion of the outer Coulomb correction in the pK^- channel. Full corrections means with all corrections as described in this paper. As one can see, the differences between DT corrections and the full corrections are as large as the DT corrections themselves. The ratios Σ^+/Σ^- and $\Lambda/(\Lambda + \Sigma^0)$ are especially changed by the full corrections, the DT corrections cancelling for these ratios.

Columns 3 and 4 show the results for two different radii r_0. Again the two ratios Σ^+/Σ^- and $\Lambda(\Lambda + \Sigma^0)$ are most affected by r_0.

The last two columns contain results with two different $I = 2$ K-matrix values. The difference in the cross-section values is so small that it would not be possible to determine this K-matrix element from a fit to the available data. Conversely it means that this unknown matrix element will not introduce an important uncertainty on the $I = 1$ and $I = 0$ matrix elements.

In Figure 1 the predictions for the branching ratio $\Lambda/(\Sigma^0 + \Lambda)$ in the three cases with no, DT and full corrections are compared in the energy range from 0 to 200 MeV/c K^- laboratory momentum.

These results show that the corrections due to the Coulomb and mass difference effects are appreciably changed from the conventional Dalitz and Tuan values. For this reason it is important to refit the low energy K^-p data using these new corrections. The results of such a fit will be published later [8].

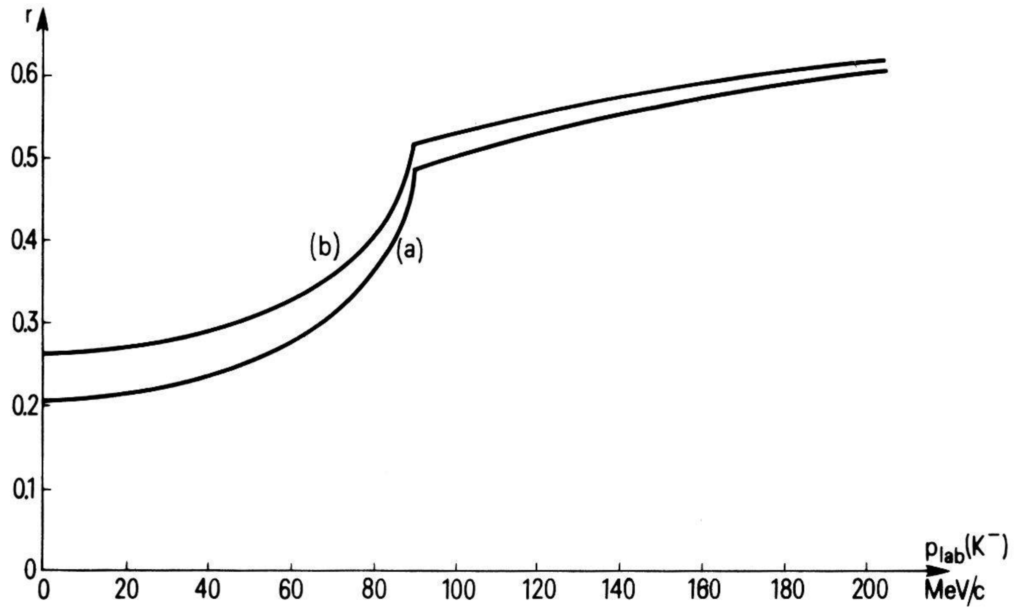

Figure 1
The branching ratio $r = \Lambda/(\Sigma^0 + \Lambda)$ for K^-p scattering in the cases (a) with no or DT corrections and (b) with full corrections. The values for K^N are taken from Martin and Ross [6]; $r_0 = 0.4$ fm.

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