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Bremsstrahlung and Čerenkov radiation of high energy particles in an excited medium

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Abstract. Bremsstrahlung and Čerenkov radiation of a high energy particle in a homogeneous medium excited by a resonant electromagnetic field are investigated. It is shown that the field greatly influences the radiation by the particle moving in the medium. The field leads to a suppression of Čerenkov radiation. In some cases the presence of the field leads to an increase of Bremsstrahlung. The intensities of both Bremsstrahlung and Čerenkov radiation are shown to depend non-linearly on the intensity of the field.

In this article we have investigated Čerenkov radiation and Bremsstrahlung of high energy (E) particles ($E \gg m$)[2]) in a medium consisting of identical independent atoms, excited by an external resonant electromagnetic field varying as $\boldsymbol{\varepsilon}_{\mathrm{ex}} \cos \omega_0 t$. This problem is specially interesting because of the recently discussed possibility of detecting high energy particles with the help of an excited medium [1, 2].

We shall assume that the distance between the atomic energy levels ($E_2 - E_1$) is close to the energy of the quantum of the field, i.e.

$$\omega_0 - (E_2 - E_1) \equiv \Delta\omega_0 \ll \omega_0. \tag{1}$$

In the resonant situation, equation (1), the dielectric constant of the medium depends strongly on the exciting field and oscillates with a frequency [3]:

$$\Omega_0 = (\Delta\omega_0^2 + \tfrac{1}{3}|d_{12}|^2 \varepsilon_{\mathrm{ex}}^2)^{1/2} \tag{2}$$

(d_{12} the matrix element of the modulus of the dipole moment, expressed in terms of the radial wave-functions of the atom).

These oscillations change the conditions of propagation of radiation in the medium, or in other words spectrum of radiation.

To study this effect we shall suppose that the particle enters the medium at time t_x. We shall calculate the dielectric constant $\varepsilon(\omega, t_x)$ for this time and we shall average the intensity of radiation over all possible times t_x.

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[2]) We use the system of units: $h = c = 1$ (c – the velocity of light). m = mass of the particle.

In the resonant situation, equation (1), the wave-function of the atomic electron can be written as a combination [3]:

$$\psi = c_1\, e^{i(\Delta\omega_0 t/2)}\, \psi_1 + \sum_m c_2^m\, e^{-(i\Delta\omega_0 t/2)}\, \psi_2^m \tag{3}$$

in which ψ_i is a wave-function with energy level $E_i (i = 1, 2)$, m = the magnetic quantum number of the excited level, equation (2).

It is simple to obtain a system of equations for the coefficients c_i from the Schrödinger equation (see [3]). The solution of the system is:

$$c_1 = \cos\frac{\Omega_0}{2}t - i\frac{\Delta\omega_0}{\Omega_0}\sin\frac{\Omega_0}{2}t; \qquad c_2^m = i\frac{\mathbf{d}_{12}^{m*}\,\boldsymbol{\varepsilon}_{\mathrm{ex}}}{\Omega_0}\sin\frac{\Omega_0}{2}t; \tag{4}$$

where $\mathbf{d}_{12}^m = \langle\psi_1^*|\hat{\mathbf{d}}_x|\psi_2^m\rangle$;
$\hat{\mathbf{d}}_x$ = dipole moment operator of the atomic electron.

The method to calculate the dielectric constant of the excited medium with the help of the wave function of an atom in the resonant field, equations (3) and (4), is known [4]. The result is:

$$\begin{aligned}\varepsilon_{ij}(\omega; \Delta; t_x) = 1 + (\varepsilon_0(\omega) - 1)\frac{1}{2}\Bigg\{&\Big[(1 + \Delta) + (1 - \Delta)\cos\Omega_0 t_x\Big]\delta_{ij} \\ &- (1 - \Delta)(1 - \cos\Omega_0 t_x)\frac{\varepsilon_{\mathrm{ex}\,i}\,\varepsilon_{\mathrm{ex}\,j}}{\varepsilon_{\mathrm{ex}}^2}\Bigg\}\end{aligned} \tag{5}$$

$\varepsilon_0(\omega)$ = dielectric constant of the non-excited medium, which we suppose to be homogenous.

$\Delta \equiv \Delta\omega_0^2/\Omega_0^2$.

It is evident from equation (5) that:

(a) in the presence of the field the dielectric constant is a tensor, and
(b) the components ε_{ij} vary as a function of the different moments of t_x.

The intensity of Čerenkov radiation for the case $\mathbf{v}_0 \| \boldsymbol{\varepsilon}_{\mathrm{ex}}$ ($\mathbf{v}_0$ = the velocity of the particle) will be:

$$dI_{v(\mathrm{cer})}^{\mathrm{ex}} = \frac{e^2}{c^2}\left[1 - \frac{c^2}{v_0^2\varepsilon_0}\cdot\frac{1}{\sqrt{1 - \gamma\dfrac{(\varepsilon_0 - 1)^1}{\varepsilon_0}}}\right]\omega\, d\omega,$$

$$\gamma \equiv 1 - (\Delta\omega_0^2/\Omega_0^2), \quad 0 \leqslant \gamma \leqslant \tfrac{1}{2}. \tag{6}$$

For a sufficiently strong field $\varepsilon_{\mathrm{ex}}$, one has:

$$\frac{\varepsilon_0}{\varepsilon_0 - 1}\left(1 - \frac{c^2}{v_0^2\varepsilon_0}\right) \leqslant \gamma. \tag{7}$$

Thus Čerenkov radiation will be absent, i.e. it is possible that even in the weakly excited medium, the presence of excitations leads to the disappearance of radiation, if:

$$\frac{\varepsilon_0}{\varepsilon_0 - 1}\left(1 - \frac{c^2}{v_0^2\varepsilon_0}\right) \ll 1. \tag{8}$$

The expression for the intensity of Bremsstrahlung for a general dielectric constant, equation (5), has a complicated form. To estimate the influence of excitation on the Bremsstrahlung spectrum we note that the exciting field changes most strongly the Z-component ($\boldsymbol{\varepsilon}_{ex} \parallel Z$) of the dielectric constant tensor. For this reason we take the dielectric constant to be of the form:

$$\varepsilon_{ij} = \varepsilon_{ZZ}\delta_{ij} = [\varepsilon_0(\omega) - \gamma(\varepsilon_0(\omega) - 1)(1 - \cos \Omega_0 t_x)]\delta_{ij}. \tag{9}$$

The Bremsstrahlung spectrum in this case will be [5]:

$$dI_B^{ex} = \frac{e^2 E_s^2}{6\pi E^2 I} d\omega \int_0^\infty dx \sin\left[x\left(1 - v_0\sqrt{\varepsilon_0} + \frac{1}{2}\gamma\frac{\varepsilon_0 - 1}{\sqrt{\varepsilon_0}}\right) + \frac{E_s^2 x^2}{12\omega E^2 L}\right] J_0\left[\frac{x\cdot\gamma(\varepsilon_0 - 1)}{2\sqrt{\varepsilon_0}}\right]. \tag{10}$$

Here J_0 = Bessel function! $E_s = \sqrt{1700}\cdot m_e$, m_e = mass of the electron, L – the cascade length unit [5]; $\gamma(\varepsilon_0 - 1)/\sqrt{\varepsilon_0} \ll 1$.

From equation (10) in the case of high frequencies:

$$\frac{E_s^2}{12E^2 L}\cdot(1 - v_0\sqrt{\varepsilon_0})^{-1} \ll \omega \tag{11}$$

after the integration, we have:

$$dI_b^{ex} = \frac{e^2 E_s^2}{6\pi E^2 L} d\omega \cdot \left[(1 - v_0\sqrt{\varepsilon_0})^2 + (1 - v_0\sqrt{\varepsilon_0})\gamma\frac{\varepsilon_0 - 1}{\sqrt{\varepsilon_0}}\right]^{-1/2} \tag{12}$$

If one has the inequality

$$1 - v_0\sqrt{\varepsilon_0} \ll \gamma\frac{\varepsilon_0 - 1}{\sqrt{\varepsilon_0}} \tag{13}$$

the spectrum of Bremsstrahlung has the form:

$$dI_B^{ex} = \frac{e^2 E_s^2}{6\pi E^2 L} d\omega \cdot \left[(1 - v_0\sqrt{\varepsilon_0(\omega)})\cdot\gamma\frac{\varepsilon_0(\omega) - 1}{\sqrt{\varepsilon_0(\omega)}}\right]^{-1/2}. \tag{14}$$

If the frequency ω is higher than the characteristic atomic frequency ω_{at}:

$$\omega \gg \omega_{at}$$

we can put:

$$\varepsilon_0(\omega) = 1 - \omega_p^2/\omega^2 \tag{15}$$

where $\omega_p^2 = 4\pi n e^2/m_e$ – the medium plasma frequency.

In this case equation (12) has the form:

$$dI_B^{ex}(\omega \gg \omega_{at}) = \frac{e^2 E_s^2}{3\pi E^2 L}\left[\left(\frac{m^2}{E^2} + \frac{\omega_p^2}{\omega^2}\right)^2 - \left(\frac{m^2}{E^2} + \frac{\omega_p^2}{\omega^2}\right)\frac{\omega_p^2}{\omega^2}\gamma\right]^{-1/2}\cdot d\omega \tag{16}$$

From equation (16) we can conclude that for high frequencies the presence of the exciting field leads to an increase of Bremsstrahlung.

The numerical estimation of the above effects indicates that these should be observed for fields satisfying $\varepsilon_{ex} \sim 10^{-6}\varepsilon_{at}$ ($\varepsilon_{at} \sim 0.5 \times 10^{10}$ v/cm) for $\Delta\omega_0/\omega_{at} \sim$

10^{-6} and $m/E \ll \omega_p/\omega \ll \omega_{at}/\omega$. Such values of ε_{ex} are now available for variable-frequency laser beams [6].

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