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Quarks with Q^2 dependent effective masses as a confinement effect

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Abstract. The non-relativistic nuclear quark model has many qualitative successes. The quarkonium picture is successful to describe $c\bar{c}$ states even though the c and \bar{c} quarks have a high velocity and seems also to be qualitatively valid for the description of hidden strangeness states. The quark-parton description of the nucleon is an 'asymptotic' picture which leads to scaling. The scaling is mildly violated in agreement with *QCD* down to Q^2 values as small as 1 GeV^2 .

The relationship between the nuclear quark model and the *QCD* quark-gluon pictures of a hadron is extensively discussed and a proposal is made to continuously connect them. To that end we introduce valence quarks as hadron-like object with Q^2 varying effective masses. We analyse some consequences of this assumption and find in particular that higher twist effects in leptonproduction are less important than in the conventional picture.

Introduction

The quark model was originally proposed in the form of a non-relativistic picture for the hadron structure [1] quite analogous to the picture of nuclei as made up of protons and neutrons. This picture revealed to be much more successful than the previous $SU(3)$ description. New mass formulae were derived, magnetic moments were successfully derived. Predictions for leptonic and weak decay processes were obtained. Many relations between hadron-hadron interactions (total and differential cross sections) were obtained [2]. In 1969, Feynman proposed the parton idea as a means to predict the behaviour of hadron collisions at extreme energies [3]. His ideas were later extended to leptonproduction at high Q^2 [4] and gave rise to the so-called quark-parton model for deep inelastic scattering. The discovery of asymptotic freedom [5] and the advent of *QCD*¹⁾ led to the introduction of a relativistic and field theoretic view of hadron structure. The nuclear quark structure of a hadron is quite distinct from its quark-gluon structure. Neither of them are ruled out by experiment; on the contrary, both of them do play a significant role in explaining data. The problem is how to go from one picture to the other. It is not only how to go from current quarks to constituent quarks but how to go from a relativistic many (infinite) body system to a non-relativistic two or three body one.

¹⁾ The main references relevant to the discovery of *QCD* can be found in Ref. 71.

The permanent boundedness (or confinement) of quarks and gluons does play a role in that problem but nobody can yet explain realistically its origin and how it manages to connect the two above pictures of hadron structure. There is however a phenomenological way to approach the problem because all physical phenomena under study do depend on confinement forces. Leptoproduction is particularly interesting from that point of view because, with varying Q^2 , we probe distances of the order of the hadron radius down to very small distances.²⁾

In this work we are proposing a simple way to parametrize the effect of confinement forces through the introduction of an effective and Q^2 dependent mass for valence quarks. It should be recognized from the outset that the justifications we can give cannot be deduced from perturbative QCD . Indeed the concept of valence quark cannot be extracted from perturbation theory because it is closely linked to the composite nature of hadrons. If it were the case it would mean that we can calculate bound state from a perturbative calculation. Bound state effects are presumably $O(e^{-1/g})$ contributions (g is the QCD coupling), so it is not possible. However, we shall show that there are many qualitative and phenomenological reasons why the use of Q^2 dependent quark masses is justified and that this concept provides a simple way to bridge the gap between perturbative and confined QCD . In other words, it allows to apply QCD down to Q^2 values much less than one and provides a connection between the nuclear quark and the quark-gluon structure of a hadron. This connection has *observable implications* on the behaviour of structure functions for small Q^2 . With the requirement that valence quarks are softly virtual before emitting gluons we find that *higher twist-effects are much smaller* than in the conventional picture. We do not claim that our proposal takes into account all of the confinement effects. In particular we have no fundamental way to compute the quark wave function in a hadron. We can only tell that the analysis of low Q^2 leptoproduction processes will give rise to experimental indications on its form. We also think that our proposal provides a justification for the use of non-relativistic models.

Our work is developed as follows.

In Section 1 we remind the main properties of perturbative QCD and introduce the notion of effective mass.

In Section 2 we analyse the reasons one can find in perturbative QCD for the existence of hadrons.

In Section 3 the hadronization of quarks and gluons in jets is considered. The possibility that perturbative QCD remains valid even if confinement forces are acting is discussed and adopted.

In Section 4 the partonic structure of hadrons is reviewed and critically considered. Experimental evidences for each component are considered.

In Section 5 we explain why a valence quark is different from a QCD quark and review some experimental evidences that valence quarks are hadron-like objects.

In Section 6 we examine the reasons which allow us to think that QCD quarks can become valence quarks and vice-versa if suitable conditions are fulfilled. We introduce our concept of effective mass and a boundary condition for its value which expresses the fact that a *stable* quark in a hadron must be a constituent quark.

²⁾ Data at $Q^2 = 200 \text{ GeV}^2$ correspond to a distance of the order of 0.01 fm.

In Section 7 we analyse the expected behaviour of the effective mass as a function of Q^2 .

In Sections 8 and 9 we explain the new partonic description of lepton production processes. The meaning of the impulse approximation in a confining environment is discussed.

In Section 10 we examine the consequences on the description of higher twist effects.

In Section 11 quark mass effects in QCD corrections are considered and seen to be *non leading*.

In Section 12 we sum up briefly the most important implications of our work.

1. Properties of perturbative QCD

(i) Quarks and gluons are the quanta of the theory. Both carry a colour charge. Their masses are given without ambiguity by the Lagrangian (\mathcal{L}_{QCD}). Gluon masses are zero because of gauge invariance. Quark masses are introduced as parameters. They depend on flavour. They (partly) originate from weak and electromagnetic interactions.

(ii) The coupling between quarks (q) and gluons (G) is introduced through a unique dimensionless coupling constant g . It gives rise to the virtual processes

$$\begin{aligned} q &\rightarrow qG & q\bar{q} &\rightarrow G \\ G &\rightarrow GG & \text{and } GG &\rightarrow GG. \end{aligned} \tag{1}$$

(iii) It is a *renormalisable* theory. It is a necessary condition for perturbation theory to make sense.

(iv) Because *non* abelian gauge invariance is at work, the field theory so defined has the well-known property of asymptotic freedom.

Quarks and gluons behave as (almost) free particles at small distances (compared to hadronic ones). Properties (iii) and (iv) imply that the renormalisation procedure can be carried out at sufficiently high energy. The tree approximation is modified taking into account all virtual effects, vertices and propagator loop corrections. The renormalization group equations lead

- to replace the coupling g by an effective coupling $g(Q^2)$ which *tends to zero* when $Q^2 \rightarrow \infty$;

- to make quark masses Q^2 dependent according to the equation [6]

$$Q \frac{dm(Q)}{dQ} = \gamma\left(g, \frac{m}{Q}\right) m(Q); \tag{2}$$

$m(Q)$ or $m(Q^2)$ is called the running quark mass and $\gamma(g, m/Q)$ is the anomalous dimension.

(v) The vacuum is the usual perturbative vacuum i.e. it fulfills the equations

$$\langle 0 | q\bar{q} | 0 \rangle = 0 = \langle 0 | G_a^{\mu\nu} G_{\mu\nu}^a | 0 \rangle. \tag{3}$$

Comments

(a) The coupling and masses present in \mathcal{L}^{QCD} are unrenormalised quantities. They do not depend on the renormalisation point. Through regularisation they

are replaced by renormalised quantities which depend on the subtraction point (say μ). For instance in the dimensional regularisation scheme we have [7]

$$g = \mu^{\varepsilon/2} g_R Z_g, \quad \varepsilon = 4 - d \quad (4)$$

and

$$m = Z_m m_R \quad (5)$$

where Z_g and Z_m are the renormalisation constants which also depend on μ . It follows that g_R and m_R also depend on μ . g_R is related to the effective coupling $g(Q^2)$ setting

$$g(\mu^2) = g_R \quad (6)$$

while m is related to the running mass appearing in the R.G. (renormalization group) equation (2) setting

$$m(Q = \mu) = m_R. \quad (7)$$

(b) One can always introduce a value $Q^2 = \Lambda^2$ such that to the LLA [8] (leading logarithmic approximation)

$$\frac{g^2(Q^2)}{4\pi^2} = \frac{\alpha_S(Q^2)}{\pi} = \frac{1}{b_0 \ln \frac{Q^2}{\Lambda^2}} \quad (8)$$

with

$$b_0 = \frac{11 - \frac{2}{3}f}{4}, \quad (9)$$

f being the number of flavours. Λ is determined experimentally. The renormalisation group improved perturbation theory in *QCD* is an asymptotic expansion in terms of the parameter $\alpha_S(Q^2)/\pi$. Therefore 'asymptotic' *QCD* can only be applied when $Q^2 > Q_0^2$ where

$$\frac{\alpha_S(Q_0^2)}{\pi} < 1. \quad (10)$$

QCD does not tell us what this value Q_0^2 is. This is equivalent to say that we have no a priori knowledge of the value of Λ^2 .

We shall come back to this important point later on.

(c) If i and j are two different quark flavours, the running masses $m_i(Q^2)$ and $m_j(Q^2)$ satisfy the relation

$$\lim_{Q^2 \rightarrow \infty} \frac{m_i(Q^2)}{m_j(Q^2)} = \frac{m_i}{m_j} \quad (11)$$

where m_i and m_j are the bare masses previously introduced in \mathcal{L}^{QCD} . Bare quark masses can in principle be observed in leptoproduction. The meaning of quark masses in the context of perturbative *QCD* is intensively discussed in Ref. 9.

Conclusion

Asymptotic *QCD* is a field theory very analogous to *QED*. As long as we are working inside distances shorter than hadronic ones we can deal with quarks and

gluons in much the same way as with the electron and the photon. If we could do our experiments within 0.1 or 0.2 fm we could see quarks radiating gluons and gluons interacting with themselves. However our observations always involve macroscopic distances i.e. are always under the influence of confining forces. A priori these very strong forces could have completely washed out the short range behaviour. As we have learned from the numerous observations and calculations done in the past eight years, it is not at all the case. This very fact is important to be studied in details. Confining forces do not 'shuffle the cards'. We have to understand how in the hope to find a simple mechanism to generate them.

Our analysis does not depend whether quarks and gluons are *absolutely* or *partially* confined because it relies only on our present phenomenological understanding of data. Indeed, in such a context we know that if quarks exist they must be heavier than 10 GeV [10]. This requires at least the breakdown of asymptotic QCD i.e. the appearance of a strong force around 1 fm.

2. Does perturbative QCD suggests existence of hadrons?

This question is obviously to be asked as soon as one tries to probe confinement forces from a phenomenological point of view. As we shall try to explain its answer is far from being trivial.

(i) The mere existence of the Landau singularity in the effective QCD coupling has been taken as 'evidence' for the existence of hadrons through the appearance of a critical length Λ^{-1} which determines the breakdown of perturbation theory. Many experimental determinations of Λ from detailed analysis of deep inelastic lepton-hadron scattering have been made [11]. They range from 0.2 to 0.5 and even up to 0.9 GeV [12]. Some recent experiments indicate that Λ could be as small as 100 MeV and even less [13]. The very different values of Λ one gets from one experiment to the other means we essentially do not know its value. Two attitudes can then be taken. Either we 'feel' Λ is bigger than 200 MeV and we are allowed to relate the appearance of the Landau singularity to the existence of hadrons. Or Λ is much less than 200 MeV and then we are tempted to say that perturbative QCD *does not know* the existence of confining forces. So consequently no definite answer can be given yet at this level.

(ii) When $Q^2 \sim \Lambda^2$ we are in the I.R. (infrared region) of QCD. In this region the 3 and 4 gluon contributions play an important role. For instance, the GG interaction graph (Fig. 1) comes from the term $gf_{abc}(\partial_\mu A_\nu^a)A_\mu^b A_\nu^c$ in \mathcal{L}^{QCD} . When we replace g by the effective coupling the interaction strength becomes very strong. Moreover when the two gluons are in a colour singlet the above interaction is *attractive*. Therefore we expect the formation of a gluon condensate which

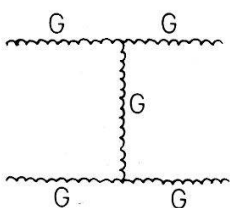


Figure 1
The $GG \rightarrow GG$ interaction.

screens colour³⁾ [14]. Valence quarks are dressed with gluons. The gluon cloud delocalises the valence quark colour and strengthens it. Each pointlike valence quark of perturbative *QCD* becomes an extended particle. We expect this extension to be of the order of the hadronic radius. We also expect that the colour clouds will *superimpose* as much as possible. The concept of dressed valence quark is the field theory picture for the phenomenological *constituent* valence quarks of the nuclear quark model. It is also called valon [15]. Each valence quark is a coloured object and keeps its individuality inside a hadron. The necessity to saturate colour appears experimentally a necessary condition to form a hadron⁴⁾. However since the existence of confining forces between coloured objects is not really proved⁵⁾ it is not clear why we need it yet.

(iii) Even if confining forces do indeed exist, it is not clear how to produce the Yukawa force between two hadrons as a Van der Waals force. For instance, a linearly confining force generates a residual interaction between two non coloured objects, which decreases as an inverse power of their distance⁶⁾ [16]. It is only in the bag model that we can avoid such a residual force. In a manifestly covariant theory with positive definite metric the existence of such a force would mean the existence of a zero mass hadron. However here the metric is necessarily not positive definite since we are dealing with a non abelian gauge theory. Therefore the conclusion is that perturbative *QCD* does not tell us anything serious about the existence and the properties of hadrons or, in other words, the numerous predictions concerning the hadronic spectrum and their static properties do all depend on assumptions which look *completely outside perturbative QCD*. However this does not mean that the use of perturbative *QCD* is not helpful in studying hadronic properties. Indeed there is quite a number (if not all) of physical phenomena which clearly depend on both perturbative *QCD* and confinement forces because they depend on both short and long range forces. Thanks to this occurrence we can hope, on a phenomenological basis, to probe the properties of confinement forces since we can extract the dynamical features which reflect the action of short range forces. The separation of the short and long range features of each physical phenomena is not necessarily trivial. There is no a priori rule for doing this since we have no previous experience with *confined* quanta.

3. The hadronization of quarks and gluons as seen in jets

In high $Q^2 e^+ e^-$ and lepton-hadron scattering, final states come from the hadronization of quarks and gluons leading to jet formation. Observations have shown that, during hadronization processes, the underlying information concerning the scattering of *QCD* quanta is not completely lost; quite the contrary, jet formation can almost be described by perturbative *QCD* processes. We want to discuss briefly and intuitively the process of jet formation.

³⁾ This force is responsible for the possible existence of glueballs.

⁴⁾ In a simple potential model, it is easy to show that the forces between two quarks is most attractive when they form a colour singlet but we cannot rule out the existence of coloured hadrons.

⁵⁾ Of course numerous arguments have been proposed which suggest the existence of such forces.

⁶⁾ A residual force of the type $|r|^{-7}$ is not yet ruled out by experiments.

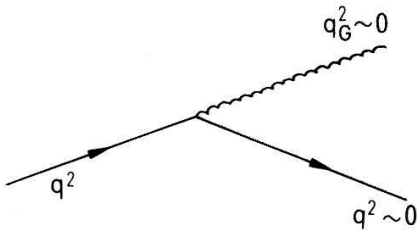


Figure 2
A virtual quark transforms to an almost real quark through gluon emission.

Jet formation always follows colour separation. This colour separation is a *short distance* process. To be specific we consider the production of a $q\bar{q}$ pair in e^+e^- annihilation. $q\bar{q}$ production is a short distance process. We expect that each quark will begin to evolve according to perturbative QCD. Its evolution is governed by the process of gluon emission (Fig. 2).

For simplicity we take the final particles on shell. If the emitted gluon takes a finite fraction x of the initial quark momentum, we find

$$q^2 = \frac{|\vec{q}_{\perp G}|^2}{x(1-x)}, \quad x \in]0, 1[. \tag{12}$$

The proper life-time of the quark is

$$\tau = \frac{1}{\sqrt{q^2}} \tag{13}$$

and its 4-momentum is

$$q = (\sqrt{q^2}, \vec{0}). \tag{14}$$

In the CMS (center of mass system) of the $q\bar{q}$ pair, its lifetime becomes⁷⁾

$$\tau_{\text{CMS}} = \frac{s + q^2}{2\sqrt{q^2}s} \frac{1}{\sqrt{q^2}} \sim \frac{\sqrt{s}}{q^2}. \tag{15}$$

It is also an estimate of the distance it travels ($c = 1$) before gluon emission and hadronization occur. For perturbative evolution to make sense we need to compare this distance with Λ^{-1} . One can easily see that Λ^{-1} is not invariant with respect to the chosen reference frame.⁸⁾ To be compatible with the scale breaking properties of QCD, it is necessary to *assume* that Λ^{-1} measures the *instantaneous* distance in *the rest frame of the parton*. Then, in the CMS, it becomes

$$\frac{1}{\Lambda_{\text{CMS}}} = \gamma \left(\frac{1}{\Lambda} \right) \sim \sqrt{\frac{s}{q^2}} \frac{1}{\Lambda} \tag{16}$$

and since s is assumed very big, it is also the time during which perturbative QCD

⁷⁾ s is the square of the photon energy.

⁸⁾ If Λ was invariant one would readily enter in conflict with the scale breaking property of QCD. See Ref. 72.

is valid. We get that q^2 must be bigger than q_c^2 given by

$$q_c^2 \sim \Lambda^2. \quad (17)$$

After this time, partons have travelled a distance

$$d \sim \frac{\sqrt{s}}{\Lambda^2}. \quad (18)$$

Next we compare this distance with the *confinement* distance. In the rest frame of a hadron it is given by its radius R_h . In the CMS of the quark, this radius becomes

$$R_{\text{CMS}} \sim \frac{\sqrt{s}}{\Lambda} R_h. \quad (19)$$

Confinement forces *do not act instantaneously* in this system. Two possibilities occur:

(a) $d \leq R_{\text{CMS}}$ i.e. $R_h > 1/\Lambda$; then quarks and gluons evolve first according to perturbative *QCD* but, since *QCD* becomes strong before confining forces begin to act, the distribution of hadrons will depend essentially on the *last stage* of the *QCD* evolution; this last stage is not easily calculable;

(b) $d > R_{\text{CMS}}$ i.e. $R_h \leq 1/\Lambda$; then confining forces do begin to act before perturbative *QCD* breaks down; their only role should be to freeze quarks and gluons into hadrons; a stage of preconfinement⁹⁾ exists which is completely amenable to perturbative *QCD*.

Data seem to support case (b). The small values of Λ found in some recent experiments [17] do also favor it. It remains to clarify how confining forces do manage to arrange quarks and gluons into hadrons.

In this work we shall adopt the view that

$$\frac{1}{\Lambda} \gg R_h, \quad (20)$$

Λ^{-1} being referred to an instantaneous distance in the quark rest frame while R is an instantaneous distance in the hadron rest frame.

A consequence of this assumption is that the perturbative regime of the hadronization process should be more and more clearly seen in the CMS since, in that system, both distances are dilated by \sqrt{s} . This is quite compatible with observations both in D.I.S. (deep inelastic scattering) and in e^+e^- annihilation.

The above picture can be generalized taking into account that the emitted gluon can also radiate. The processes are seen in Figs. 3a and 3b. Here the initial gluon is offshell by an amount given by

$$q_G^2 = q^2 x - \frac{|\vec{q}_{\perp G}|^2}{1-x} < q^2 x < q^2; \quad (21)$$

q_G^2 is always smaller than the initial quark virtuality.

⁹⁾ It is a delicate matter to estimate down to what values of Q^2 is the fragmentation described by perturbative *QCD*. See for instance Ref. 73.

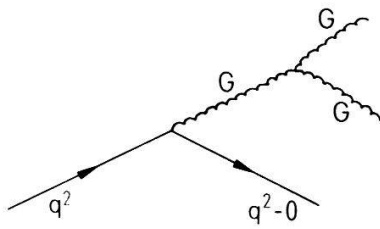


Figure 3a
Before hadronization a hard gluon can decay into two gluons.

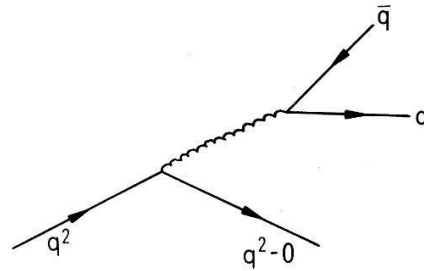


Figure 3b
A hard gluon becomes a $q\bar{q}$ pair.

For gluons too, confinement forces will begin to act when q_G^2 is sufficiently small. Here we take again the view that, in the gluon rest frame, it happens as soon as

$$\sqrt{q_G^2} = R_h^{-1} \sim \Lambda. \tag{22}$$

We finally get a state composed of a soup of quarks and gluons whose offshellness is less or equal than R_h^{-1} . From that moment on confinement forces do act strongly on the subsequent evolution of this soup. We cannot fully describe what happens yet but we can at least put forward on a qualitative level some characteristics of this evolution.

First of all, there is a constraint which expresses the only way to avoid the Landau singularity of the QCD coupling α_s : it is to form *colour singlet* objects. This implies a colour shuffling which however should not alter strongly the original kinematical configuration. In other words, colour permutations must occur through very soft quarks and gluons only.

We believe this is made possible because the true vacuum is *not* the ‘empty’ Fock space vacuum used in perturbative QCD. There are indeed several evidences that¹⁰⁾

$$\begin{aligned} \langle 0 | q\bar{q} | 0 \rangle &\neq 0 \\ \text{and} \\ \langle 0 | G^2 | 0 \rangle &\neq 0. \end{aligned} \tag{23}$$

The first relation leads to the spontaneous breaking of chiral symmetry in QCD and is ultimately responsible for quark masses [18]. Both vacuum expectation values are also intimately connected to resonance properties through the OPE (operator product expansion) and QCD sum rules and required to be $\neq 0$ [19]. Though the vacuum is in a colour zero state, it is a sea of $q\bar{q}$ pairs and of gluons of any colour.

We assume that colour rearrangement does occur through them.

For instance, if, before hadronization, a red quark should become a green one, then it interacts with the vacuum according to Fig. 4 and the $R\bar{G}$ colour produced locally in the vacuum is promptly delocalized. It propagates somewhere else and finally neutralizes the colour of another cluster of quarks and gluons which needs it to hadronize.

¹⁰⁾ $G^2 \equiv G_{\mu\nu}^a G^{a\mu\nu}$; $q\bar{q} \equiv \sum_a q^a \bar{q}^a$.

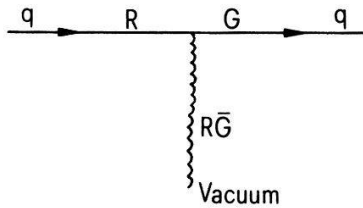


Figure 4

The vacuum is a sink of gluons and $q\bar{q}$ pairs. Thanks to that a quark can easily change its colour.

We confess this picture is presently completely qualitative but it looks quite reasonable.

4. The partonic structure of hadrons

The description of hadronization we have given up to now does not tell us much about the quark-gluon structure of a hadron. Indeed, perturbative *QCD* only tells us that hadrons are composed of quarks and gluons and that the *total flavour of the quarks* is equal to the hadronic flavour.

Even the distinction between valence quark and sea quark is outside it. Questions like

- what is a valence quark?
- what is the amount of sea quarks?
- what is the amount of gluons?
- how are these particles distributed?

are questions which cannot be answered correctly if due care of confinement forces is not taken.

In principle, simple experiments can be done to answer them. But the answers they give depend strongly on our prejudices. Since we lack a basic description of the hadronic wave function we have to use a trial and error procedure, deriving at each step a model which is able to interpret all data.

The point is that there is as yet no model which is compatible with the successful features of the old nuclear quark model.

In this section we shall consider successively all questions raised above.

What is a valence quark?

This is the most difficult and basic question to answer. Perturbative *QCD* cannot answer it. For instance a u quark in a proton can be a valence quark but it can also be a sea quark. An s quark cannot be a valence quark. An antiquark cannot be a valence quark either. So we can say for sure when a quark is *not* a valence quark but we *cannot say with certainty* when a quark is a valence quark. Such a situation is not surprising because the concept of valence quark comes from the old nuclear quark model which was created outside *QCD*.

In other words the above question leads to another deeper question: is a quark in a hadron the same as a 'perturbative' quark and ultimately what is it

like? We do not want to answer this question now but we intend to come back to it later.

What is the amount of sea quarks?

The existence of sea quarks inside the hadrons is a necessity because gluons can produce virtual quark-antiquark pairs. Since they are confined their momentum is

$$p \geq \frac{1}{R_h} \tag{24}$$

so that the virtual process (Fig. 5) is governed by

$$\alpha_s \left(\frac{1}{R_h^2} \right). \tag{25}$$

From the relation (8) and $R_h < \Lambda^{-1}$, we see that it can be less than one i.e. the amount of $q\bar{q}$ pairs inside a hadron is governed by *perturbation theory*. If $1/R_h \approx \Lambda$ the coupling would be very large and we would expect a big amount of such pairs inside any hadron. A better estimate of the average gluon momentum is given by the average value of its transverse momentum or by the intrinsic transverse momentum of the quarks. This *last* quantity has also the advantage to be measurable. Putting

$$p \sim \sqrt{\langle p_{\perp}^2 \rangle} \tag{26}$$

implies that the production of $q\bar{q}$ pairs is governed by

$$\alpha(\langle p_{\perp}^2 \rangle). \tag{27}$$

There are indications that $\langle p_{\perp}^2 \rangle$ is rather high (see also Section 10) and of the order of a fraction of GeV^2 . Therefore

$$\langle p_{\perp}^2 \rangle \gg \Lambda^2 \tag{28}$$

as is required for this process to be describable by perturbative QCD. Experimental data do indeed indicate that the amount of $q\bar{q}$ pairs is relatively small.

Let us quote two examples.

(a) Comparison between cross sections for the processes

$$\begin{aligned} \pi^- N &\rightarrow \mu^+ \mu^- X, \\ \pi^+ N &\rightarrow \mu^+ \mu^- X \end{aligned} \tag{29}$$

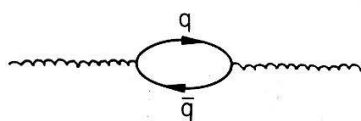


Figure 5
At sufficiently high energy, more and more gluons are seen as $q\bar{q}$ pairs.

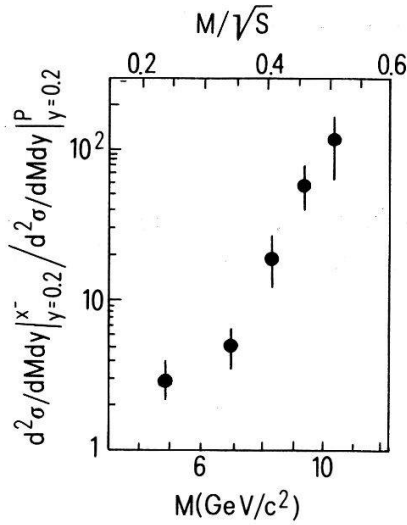


Figure 6

The ratio of $\pi^-p \rightarrow \mu^+\mu^-x$ and $pp \rightarrow \mu^+\mu^-x$ cross-sections is always bigger than one and is an increasing function of the effective mass of the $\mu^+\mu^-$ pair.

shows clearly that they are dominated by the subprocesses [10]

$$\begin{aligned} \bar{u}u &\rightarrow \mu^+\mu^-, \\ \bar{d}d &\rightarrow \mu^+\mu^-. \end{aligned} \quad (30)$$

If sea quarks are abundant both of them do contribute to the two processes and their cross sections should be about equal. One observes on the contrary that

$$\sigma_{\text{tot}}(\pi^-N \rightarrow \mu^+\mu^-X) \approx 4\sigma(\pi^+N \rightarrow \mu^+\mu^-X) \quad (31)$$

i.e. that the process $\bar{u}u \rightarrow \mu^+\mu^-$ is highly dominant for π^-N scattering while it is $\bar{d}d \rightarrow \mu^+\mu^-$ which dominates π^+N scattering. One has also compared the processes $\pi^-p \rightarrow \mu^+\mu^-X$ to $pp \rightarrow \mu^+\mu^-X$. In this last process the only antiquarks which can be found in the initial state are necessarily *sea* quarks. The result is given in Fig. 6 [21]. For any value of the $\mu^+\mu^-$ effective mass M , the ratio¹¹⁾

$$\frac{\frac{d^2\sigma^{\pi^-}}{dM dy}}{\frac{d^2\sigma^p}{dM dy}} > 2 \quad (32)$$

(here y is the rapidity of the $\mu^+\mu^-$ pair) and increases to 100 when

$$M \sim 10 (\text{GeV}/c)^2. \quad (33)$$

Therefore sea quarks are concentrated on small values of the fraction x of the initial momenta of the hadrons.

These two experiments show also that the quarks which build up the flavour of the hadron do play a very important role in these reactions and in particular

¹¹⁾ The energy is equal to 225 GeV and $y = 0.2$.

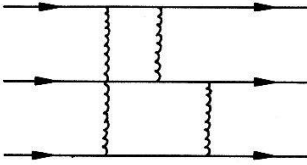


Figure 7
The simplest view of an undisturbed baryon in QCD.

bring with them an important fraction of the hadron momentum. This strong kinematical difference is not understandable within perturbative QCD.¹²⁾

(b) The D.I.S. lepton-hadron experiments allow us to probe further the sea quarks distribution. They all agree to the following:

(i) the momentum fraction of antiquark is of the order of 10% and it increases with increasing incident energy or with increasing Q^2 ;¹³⁾

(ii) the sea distribution is concentrated towards low x values obeying a $(1-x)^n$ law with [22]

$$n \geq 5 \tag{34}$$

which means that there are no more sea quarks once $x \geq 0.6$.¹⁴⁾

(c) As a last phenomenon which shows that sea quarks are confined to small x values, we want to consider the behaviour of fragmentation functions when $x \rightarrow 1$. This behaviour has been thoroughly studied [23]. Two mechanisms are proposed. The first assumes that there are *no* sea quarks inside the wave functions ψ_H (Fig. 7), the fragmentation function $F_{q_s/H}(x)$ for a sea quark is given by the graph of Fig. 8. Its behaviour when $x \approx 1$ is then given by

$$F_{q_s/H}(x) \sim (1-x)^{2n_H+n_s-1} \sim (1-x)^5 \tag{35}$$

where n_H is the number of spectator quarks and n_s is the number of sea quarks.

¹²⁾ In perturbative QCD quark masses for the u and d quarks are respectively of the order of 5 and 10 MeV. We must assume that in a hadron

$$\langle v \rangle_{\text{quark}} = V_{\text{hadron}};$$

this implies

$$p(\bar{u} \text{ or } d) = \frac{m_{u(d)} V}{\sqrt{1-V^2}} = \frac{m_{u(d)}}{M} P$$

i.e. that the mean fraction of any quark is

$$\langle x \rangle \sim \frac{m_{u(d)}}{M} \sim 0.05 \text{ to } 0.1.$$

It is indeed true that sea quarks are seen with about this momentum fraction. It is not true for the non singlet [i.e. the so-called valence] quark distribution. The argument suggests an effective mass for these, which is much bigger and of the order of $M/3$. This observation *must be* explained. See also Ref. 74.

¹³⁾ Most data points have $Q^2 \geq 3 \text{ GeV}^2$.

¹⁴⁾ There are still considerable uncertainties concerning this number. We give here an order of magnitude. See Ref. 21.

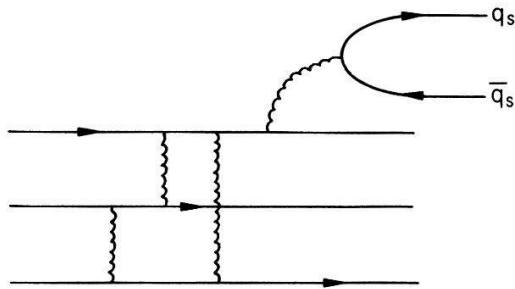


Figure 8
The perturbation acting on the baryon excites the gluon field which produces a $q\bar{q}$ pair.

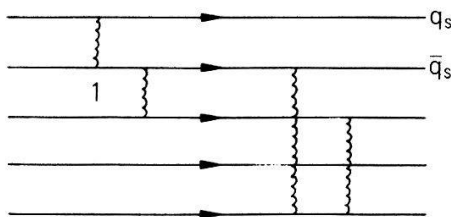


Figure 9
The view of a baryon if $q\bar{q}$ pairs are important for all x .

The second assumes that ψ_H does contain sea quarks at all x (Fig. 9); then the counting rule gives

$$F_{q_s/H}(x) \sim (1-x)^{2n_H-1} \sim (1-x)^7. \tag{36}$$

The first behaviour is favoured by data. For instance for the fragmentation

$$p \rightarrow \pi^+ + x \tag{37}$$

the diagram as given by Fig. 10 implies

$$G_{\pi/p}(x) \sim (1-x)^{2+2-1} \sim (1-x)^3. \tag{38}$$

It qualitatively agrees with experiment [24].

In conclusion,

- (i) in a hadron, sea quarks are produced through perturbative QCD; their masses are the current masses;
- (ii) they are confined to small momenta;
- (iii) when a hadron is moderately disturbed they are seen to contribute to less than 10% of its momentum.

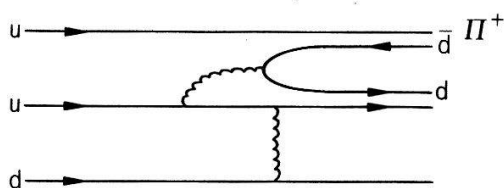


Figure 10
Quark-gluon diagram which describes the fragmentation of a proton in a $\pi^+ + X$.

What is the amount of gluons?

We have many arguments both theoretical and experimental to tell that gluons are an important component of a hadron. We shall consider here some of them.

(a) It has been demonstrated that in an undisturbed nucleon the gluon field energy is responsible for almost all its mass m_N . It is the meaning of the approximate relation [25]

$$\langle n | -\frac{9\alpha_s}{8\pi} G^2 | N \rangle \sim m_N \bar{N}N. \tag{39}$$

Moreover for any heavy pair $q\bar{q}$ it has been shown that the dominant non perturbative effect comes from a term

$$\langle B^2 \rangle = \frac{1}{4} \langle 0 | G^2 | 0 \rangle. \tag{39'}$$

(b) In deep inelastic lepton-hadron scattering gluons are only observed through the sea quarks they generate. However we get an estimate of their importance through the momentum sum rules

$$\int_0^1 F_2^{ep}(x) dx = \sum_i \int_0^1 Q_i^2 x (q_i(x) + \bar{q}_i(x)) dx \tag{40}$$

and

$$\int_0^1 F_2^{\nu N}(x) dx = \sum_i \int_0^1 x (q_i(x) + \bar{q}_i(x)) dx. \tag{41}$$

where $q_i(x)(\bar{q}_i(x))$ is the flavour i quark (antiquark) distribution. Assuming that only u and d quarks contribute (this is a good approximation) and, knowing that $u(x) = 2d(x)$ for the proton, we should have

$$\int_0^1 F_2^{ep}(x) dx = \frac{1}{3}, \tag{42}$$

$$\int_0^1 F_2^{\nu N}(x) dx = 1 \tag{43}$$

if¹⁵⁾

$$\int_0^1 xu(x) dx = \frac{2}{3} \quad \text{and} \quad \int_0^1 xd(x) dx = \frac{1}{3}. \tag{44}$$

The experimentally measured values are respectively about 0.15 and 0.5.¹⁶⁾ At $Q^2 = 5 \text{ GeV}^2$ one finds that 45% of the total momentum is taken up by gluons.

(c) In hadron-hadron scattering, gluons are of course directly seen because gluon-gluon interaction occurs. The hadronic matter behaves as a continuum. It is therefore much more difficult to make quantitative statement. However, in some special kinematical region it turns out to be possible. Such is the case of the large transverse momentum processes. We know that they can be described correctly in

¹⁵⁾ We are neglecting the sea quark contribution.
¹⁶⁾ This is already an old result. See for instance Ref. 75.

terms of the hard subprocesses between quarks and gluons [26]. In particular the contributions of the processes $qG \rightarrow qG$ and $GG \rightarrow GG$ are *important*.

(d) In deep inelastic scattering the gluons we see are only those which radiate $q\bar{q}$ pairs. The distribution of these gluons can be studied through a determination of the evolution of the singlet structure functions F_2^{ep} and $F_2^{\nu N}$ at least if data are accurate enough. This has recently been done by Duke, Owens and Roberts [26] who find their mean momentum fraction

$$x_G = \frac{\int_0^1 xG(x, Q^2) dx}{\int_0^1 G(x, Q^2) dx} \quad (45)$$

to be respectively 0.13 at $Q^2 = 5 \text{ GeV}^2$ and 0.09 at $Q^2 = 90 \text{ GeV}^2$.

So, like sea quarks, *radiative* gluons are concentrated at small x .

5. Why a valence quark can be different from a QCD quark

In this section we shall pursue our investigation of the valence quark nature.

We have already explained that perturbative QCD *does not make any distinction* between valence quarks and sea quarks. In this section we want to explain why we believe that valence quarks are in fact quite different from sea quarks and try to describe some of their characteristics.

(a) An a priori definition of a valence quark

We define a valence quark as a *permanent* quark or in other words a quark *whose existence is not linked* to the fluctuations of the gluonic surrounding field. It is independent of the gluonic field but it is obliged to move inside it so that its effective mass has nothing to do with its QCD lagrangian mass. In other branches of physics too, one has many examples of objects with effective masses. We consider here three examples.

(1) If a sphere of radius R is moving with constant velocity v in a liquid of density ρ , its total kinetic energy is given by

$$E = \frac{1}{2}(M + \frac{1}{2}M')v^2 \quad (46)$$

where M is the true mass of the object while

$$M' = \frac{4}{3}\pi\rho R^3 \quad (47)$$

is the mass of the liquid which occupies the same volume. If such a sphere is accelerated the total thrust on it is entirely given through M' only i.e. by

$$T = \frac{1}{2}M' \frac{dv}{dt}. \quad (48)$$

So, when accelerating the sphere its own true mass does not count anymore [28].

(2) The classical Hamiltonian for a relativistic scalar particle in an elec-

tromagnetic field is

$$H = [m^2 + (\vec{p} - e\vec{A})^2]^{1/2} + eA_0. \quad (49)$$

Therefore in a field such that

$$\langle \vec{A} \rangle = 0 = \langle A_0 \rangle, \quad (50)$$

its effective mass is given by

$$m_{\text{eff}}^2 = m^2 + e^2 \langle \vec{A}^2 \rangle. \quad (51)$$

It can again be much different from m^2 if the field fluctuations are big [29].

(3) If a Dirac particle is embedded in an external scalar potential V , it obeys the equation

$$(\not{p} - m - V)\psi = 0. \quad (52)$$

If this potential has a mean value $\langle V \rangle$ different from zero, the particle gets an effective mass

$$m_{\text{eff}} = m + \langle V \rangle \quad (53)$$

when $\langle V \rangle \gg 0$ then¹⁷⁾

$$m_{\text{eff}} \gg m. \quad (54)$$

Such an equation is also obtained from a quark in a bag. $\langle V \rangle$ is generated by the gluon field.

So we shall define a valence quark as a permanent *QCD* quark dressed with gluons and moving inside gluons. This dressing delocalises its colour but its flavour remains strongly localised.¹⁸⁾ The effective mass that it acquires in that way is much bigger than its *QCD* mass. Moreover we shall argue that such a dressed quark can never be liberated because its effective mass would increase to infinity.

(b) *Experimental evidences that valence quarks¹⁹⁾ are hadron-like objects*

(1) The successes of the nuclear quark model in describing the meson and baryon spectroscopy are astonishing [30]. However they depend in an essential way on the fact that the assumed effective masses of the u and d quarks obey qualitatively

$$\begin{aligned} m_u \simeq m_d &\simeq \frac{M_{\text{proton}}}{3}, \\ m_s &\simeq \frac{M_{\text{proton}}}{2}. \end{aligned} \quad (55)$$

¹⁷⁾ This example has been put forward by many people with different aims. Some physicists have seen in it a way to explain confinement by requiring $\langle V \rangle < 0$. See for instance Ref. 76. Here we take the same point of view as in Ref. 77.

¹⁸⁾ Such a dressed quark has been called valon by Hwa (Ref. 78). For the same idea, see Ref. 79.

¹⁹⁾ We do not use the expression constituent quarks because constituent quarks do always contain a valence quark.

Once these big masses are assumed, mesons can be described as two-body objects and baryons as three-body objects interacting through a *one* gluon exchange and possibly also through a scalar confining potential [31]. This last potential apart from being not yet unambiguously proven²⁰⁾ is of *long range* i.e. only effective for

$$r \geq 0.5 \text{ fm.} \quad (56)$$

Therefore, if we look only at distances less than 0.5 fm, it looks as if the effect of confinement forces is *entirely taken into account by the chosen effective masses*.

(2) The differential cross sections for hadron-hadron scattering at fixed $\cos \theta$ or equivalently in the kinematical region where

$$\frac{s}{t} \xrightarrow[t \rightarrow \infty]{s \rightarrow \infty} \text{constant} \quad (57)$$

behave in a very simple way as a function of energy. It is well known that they have a simple power law behaviour which can be deduced assuming that each hadron is composed of two or three constituents [32]. For example, the formula

$$\frac{d\sigma}{dt} (pp \rightarrow pp) \xrightarrow{s \rightarrow \infty} \frac{1}{s^{2(n_p+n_p-1)}} f_{pp} \left(\frac{t}{s} \right) \quad (58)$$

agrees with experiment for $n_p = 3$. Such a formula is deduced using a scale transformation under the hypotheses

- the constituents behave as almost free
- the coupling which governs their interaction is dimensionless.

This last condition is fulfilled by *QCD*. What is more surprising is that the number of constituents we need to assume to get agreement with experiment is equal to the *minimum* number of quarks. Moreover we can completely forget the existence of gluons in spite of the fact that they can interact *directly* with the same strength through the three gluon coupling. So, we have to recognize that the mesons and baryons wave functions are dominated by their *valence* quark Fock states $|q\bar{q}\rangle$ and $|qqq\rangle$. These valence quarks are hadron-like objects which are distinct from each other by their colour. Once we take these objects as our basic units the corrections to it are calculable from perturbative *QCD* [33]. All this does not tell us anything more *on how to describe* a valence quark.

(3) Magnetic moments of hadrons can be qualitatively described by the *nuclear* quark model based on the $SU(6)_F \times O(3)$ wave functions. One must stress that these wave functions do not take into account the interactions between the quarks.

In unit of nuclear magneton the quark magnetic moment is expressed by

$$\vec{M}_q = \left(\frac{g_q}{2} \frac{m_p}{m_q} \frac{e_q}{e} \right) \vec{\sigma} \quad (59)$$

where m_p , e , and m_q , e_q are the masses and charges of the proton and the quark respectively. g_q is the gyromagnetic ratio of the quark.

²⁰⁾ We expect from such a potential an inversion of the 3P_J multiplets for unsymmetrical $Q\bar{q}$ objects where Q is a heavy quark.

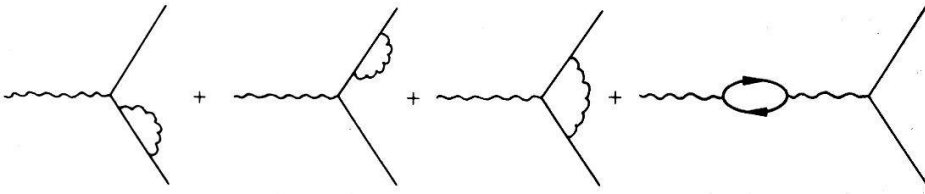


Figure 11
 QED graphs which contribute in $\mathcal{O}(\alpha)$ to the magnetic moment of a charged lepton.

For a particle like a lepton, g is almost equal to 2. The dominant correction to this value comes from the diagrams of Fig. 11 and it is given by

$$\frac{1}{2} \left(\frac{\alpha}{\pi} \right) \approx \frac{1}{860} \tag{60}$$

However, in the case of quarks, the contribution of the diagram of Fig. 12 is much bigger. Indeed it is proportional to²¹⁾

$$\frac{1}{\pi} \alpha_s \left(\left(\frac{1}{R_H} \right)^2 \right) \approx \frac{1}{4} \tag{61}$$

if we take

$$\Lambda \equiv \Lambda_{\overline{MS}} \approx 100 \text{ MeV.}^{22)} \tag{62}$$

This means that g_q has no reason to be exactly equal to 2. Nevertheless the nuclear quark model does assume this value and it replaces the almost massless QCD quarks by ‘heavy’ constituent quarks. Let us present briefly the actual status of its predictions. From the experimental values of M_p and M_n one finds²³⁾

$$\begin{aligned} M_u &= 1.85, \\ M_d &= -0.97. \end{aligned} \tag{63}$$

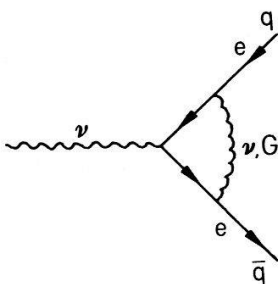


Figure 12
 A graph which can contribute to the quark magnetic moment.

²¹⁾ We assume (see above) that perturbative QCD is still applicable for distances as large as R_h .
²²⁾ A precise determination of $\Lambda_{\overline{MS}}$ has been done by T. B. Mackenzie and C. P. Lepage (Ref. 80); they find

$$\Lambda_{\overline{MS}} = 100 \begin{matrix} +34 \text{ MeV} \\ -25 \text{ MeV} \end{matrix}$$

²³⁾ Here and in the following we use the $SU(6) \times O(3)$ wave functions for the baryons.

These values correspond to

$$\begin{aligned} m_u &= 338 \text{ MeV}, \\ m_d &= 322 \text{ MeV}. \end{aligned} \quad (64)$$

These masses are about equal to $\frac{1}{3}$ of the nucleon mass and satisfy isospin symmetry at the level of 5%.

To find M_s we use the most recent data on M_{Λ^0} [34]. It gives

$$M_s = -0.614 \quad (65)$$

and corresponds to

$$m_s = 509 \text{ MeV}. \quad (66)$$

From the $m_{\Lambda} - m_p$ mass difference²⁴⁾ one gets

$$m_s = 497 \text{ MeV}. \quad (67)$$

Another determination of m_s has been done by Martin [35] from a simultaneous fit of the $b\bar{b}$, $c\bar{c}$ and $s\bar{s}$ spectra. He finds

$$m_s = 518 \text{ MeV}. \quad (68)$$

Our value sits in between and can be considered as satisfactory.

There is some difficulty in explaining the experimental values of the other hyperon magnetic moments. A first way to illustrate it is to write the sum rule

$$M_s = 4(M_p - M_{\Sigma^+} + \frac{1}{4}M_{\Sigma^-}). \quad (69)$$

Since [36]

$$\begin{aligned} M_{\Sigma^+} &= 2.30 \pm 0.14, \\ M_{\Sigma^-} &= -1.38 \pm 0.37, \end{aligned} \quad (70)$$

we find

$$M_s = 0.37 \pm 0.93 \quad (71)$$

which is hardly compatible with the previous value.

A second way is to use the measurements of Ξ^0 and Ξ^- magnetic moments [37]. They give

$$(M_{\Xi^-} - M_{\Xi^0})_{\text{EXP}} = 0.487 \pm 0.076 \quad (72)$$

while the nuclear quark model gives

$$M_{\Xi^-} - M_{\Xi^0} = \frac{1}{3}(M_u - M_d) = 0.94. \quad (73)$$

Here, there is a clear discrepancy. We think it questions as much the experimental result as the model itself. Keeping this in mind we still consider that the model is *qualitatively* correct [38].

(4) Small t elastic pp scattering can be described at high energy considering

²⁴⁾ The validity of the relation $m_{\Lambda} - m_p = m_s - m_u$ is extensively studied by H. J. Lipkin (see Ref. 81).

the amplitude as a *sum* over quark-quark scattering amplitudes [39]. Though such a model depends on a number of assumptions which are rather arbitrary (i.e. the form of $q-q$ scattering amplitude and the form of the proton wave function) it suggests

- (a) that the number of quarks is finite; it can be equal to three [39];
- (b) that the $q-q$ elastic scattering amplitude has the same form as *the small t pp scattering amplitude* i.e. it is 'Pomeron like'.²⁵⁾

6. From QCD quarks to valence quarks

As we have seen QCD quarks are point-like particles analogous to leptons and prompt to radiate gluons, they are almost massless and behave as relativistic particles. They are 'seen' when probing distances much shorter than R_h .

At the opposite, valence quarks are extended particles analogous to hadrons which look almost free. They are heavy with a mass about equal to $m_p/3$ and behave as non relativistic particles. They are seen when distances bigger or of the order of R_h are probed.

We here pursue the idea that understanding these two quite different pictures of the quarks will help to improve our understanding of the nature of confinement forces.

The case of deep inelastic lepton-hadron scattering processes

In D.I.S. processes, we can vary at will the distance over which the proton is probed. Our present understanding of the behaviour of the structure functions do show us how QCD quarks transform progressively in valence quarks when larger and larger distances are involved.

This transformation becomes almost obvious once we adopt the hypothesis repeatedly discussed by F. Martin [40] i.e. there exists a $Q^2 = Q_0^2$ such that

$$xG(x, Q_0^2) = 0 = xq_s(x, Q_0^2) \quad (74)$$

where $G(x, q^2)$ and $q_s(x, Q^2)$ are respectively the gluon and sea quark distributions.

Martin has tested his hypothesis numerically using the Altarelli-Parisi evolution equations [41]. For instance starting from²⁶⁾

$$\langle G \rangle_2 = 0.48 \quad (75)$$

at 2 GeV^2 , he finds

$$\langle G \rangle_2 = 0,$$

²⁵⁾ Quarks behave like hadrons. This is one of the basic principles of the massive quark model of Preparata, see for instance Ref. 82.

²⁶⁾ If $D(x, Q^2)$ is any structure function we define

$$\langle D \rangle_2 = \int_0^1 xD(x, Q^2) dx.$$

at (76)

$$Q_0^2 = 0.09 \text{ GeV}^2$$

when second order corrections in α_s are included. This calculation suggests the following *boundary condition*²⁷⁾

$$\boxed{\langle G \rangle_2 = 0 = \langle q_s \rangle_2 \text{ at } Q_0^2.} \quad (77)$$

It implies

$$\langle q_v \rangle_2 = 1 \quad (78)$$

at $Q^2 = Q_0^2$ i.e. that valence quarks take all the hadron momentum.

It is clear that we cannot *deduce* this boundary condition from *QCD*. However it is strongly supported by the evolution equation of *QCD* and by the non relativistic quark model.

Since at rather low values of Q^2 ($< 5 \text{ GeV}^2$) we have

$$\begin{aligned} \langle G \rangle_2 &\sim 45\% \\ \langle q_s \rangle_2 &\sim 7\% \\ \langle q_v \rangle_2 &\sim 48\% \end{aligned} \quad (79)$$

it means that, when Q^2 decreases,

- (i) the momentum carried by the valence quarks increases rapidly;
- (ii) the momentum carried by the sea quarks and gluons decreases rapidly.

We must interpret this behaviour in the following way. At high Q^2 we see gluons through the process $G \rightarrow q\bar{q}$ only i.e. we see (Fig. 13) the *radiating* gluons through the interaction of the incident photon or vector boson with the virtual quarks they produce. These quarks are necessarily *sea* quarks. The sea quarks and gluon distribution are tied together. When Q^2 decreases, the probe can no more distinguish the quark from the antiquark in virtual pairs. Therefore it can no more interact with them and it does not see gluons either because they are neutral particles. We shall attach a meaning to the valence distribution when $Q^2 = 0$ and say that it represents the momentum carried by the *constituent* quarks. In other words we assume that the *QCD* valence (and point-like) quarks are always seen as point-like by the electromagnetic or weak probes but get an effective mass

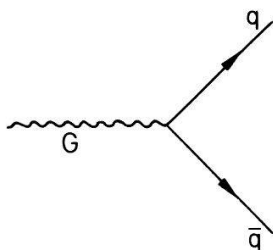


Figure 13
In D.I.S. some gluons are seen as $q\bar{q}$ pairs.

²⁷⁾ In D.I.S., $1/Q$ measures the distance seen by the incoming virtual photon or weak boson.

$m(Q^2)^{28}$ which is such that

$$\boxed{\begin{array}{l} m(Q^2) \rightarrow m_{\text{constituent}} \\ \text{when } Q^2 \rightarrow \left(\frac{1}{R_h}\right)^2. \end{array}} \quad (80)$$

This boundary condition on the valence quark masses amounts to assume that the only effect of confining forces is to give them a mass which is of the same order as the proton mass. Within it, we can easily interpret the condition

$$\langle q_v \rangle_2 = 1 \quad \text{when } Q^2 \sim 0; \quad (81)$$

indeed, when $Q^2 \rightarrow 0$, the probe does only see a *finite number* of (valence) quarks. Therefore, if $\langle p \rangle$ is their mean momentum value and P the proton momentum, then

$$\frac{\langle p \rangle}{P} \sim \frac{1}{3} \quad (82)$$

and

$$\frac{m_q}{M} \sim \frac{1}{3} \quad (83)$$

since we know there are only 3 valence quarks.

What happens when we consider higher and higher Q^2 ? We know the valence quark distribution will evolve according to [41]

$$\langle q_v \rangle_2 = \langle q_v(Q_0^2) \rangle_2 e^{-[64/3(33-2f)]\bar{s}} \quad (84)$$

where

$$\bar{s} = \ln \frac{\ln \frac{Q^2}{\Lambda^2}}{\ln \frac{Q_0^2}{\Lambda^2}} \quad (85)$$

as soon as $Q^2 > Q_0^2 > \Lambda^2$. Therefore when $Q^2 \rightarrow \infty$

$$\langle q_v(Q^2) \rangle_2 \rightarrow 0. \quad (86)$$

This condition means

- (i) there is no more distinction between valence and sea quarks;
- (ii) the valence quark masses are the same as the sea quark masses;
- (iii) the number of quarks is indefinite.

From (86) we find it is necessary that, for any quark seen, the quantity

$$\frac{\langle p \rangle}{P} \sim 0 \quad (87)$$

i.e. that

$$\frac{m_q}{M} \sim 0; \quad (88)$$

²⁸⁾ The notion of effective quark mass has been proposed as early as 1972 in an unpublished paper by H. Faissner et al (Ref. 83).

this is the case when m_q is equal to the current quark mass. For instance

$$\frac{m_q}{M} \sim 0.01 \quad (89)$$

when $m_q = 10$ MeV.

This is of the order of the *smallest* observable values of x (~ 0.015).
QCD tells us also that when $Q^2 \rightarrow \infty$

$$\begin{aligned} \langle q_s \rangle_2 &\rightarrow \frac{3f}{16+3f}, \\ \langle G \rangle_2 &\rightarrow \frac{16}{16+3f} \end{aligned} \quad (90)$$

where f is the number of flavours [42]. We get asymptotically Table I. Table I shows that whatever the number of flavours sea quarks and gluons take each about half of the hadron momentum when $Q^2 \rightarrow \infty$. The fact that we still get an

Table I
 Evolution of the momentum fractions of sea quarks and gluons for different numbers of flavours.

f	$\langle q_s \rangle_2$	$\langle G \rangle_2$
4	0.43	0.57
6	0.53	0.47

important gluon contribution is related to the asymptotic freedom property of the interaction.

Though we have introduced the concept of effective mass of valence quarks through deep inelastic scattering we expect this concept to play a role in all other processes as e.g. lepton-pair production and fragmentation in hadron-hadron interactions.

7. The Q^2 behaviour of $m(Q^2)$

(a) *The usual definition of $m(Q^2)$ in QCD*

In any renormalisable theory the mass of a particle depends on the momentum at which it is defined. In *QED*, since we can define the mass of an electron at rest and 'almost' free,²⁹⁾ we can also talk of the electron mass as an intrinsic physical quantity.

In *QCD* we can only see quarks through interaction processes which involve a sufficiently large squared momentum transfer Q^2 . Therefore it is a *necessity* to introduce the concept of effective mass $m(Q^2)$. The way to introduce it is the

²⁹⁾ We can see an electron through the Compton effect with a very soft photon.

following [43]. One takes the unrenormalized propagator

$$S_0^{-1}(p) = \not{p}A^0(p^2) - m^0B^0(p^2) \quad (91)$$

Then the effective mass $m(p^2)$ is defined by

$$m(p^2) = m^0 \frac{B^0(p^2)}{A^0(p^2)} \quad (92)$$

which in terms of renormalized quantities is written

$$m(p^2) = m \frac{B(p^2)}{A(p^2)}. \quad (93)$$

$m = m_0 - \delta m$. While A and B are renormalization scheme dependent, $m(p^2)$ is independent of it.

However this definition does not take properly into account the existence of confining forces since the chosen propagator has a pole and therefore, there exists an *asymptotic* one particle quark state. This amounts to deny the existence of confining forces. We have no indication in the above equations that Q^2 cannot be taken equal to zero and that the constituent mass is *not* a pole of the propagator $S(p)$. If we consider that $m_{\text{constituent}}$ is a pole of $S(p)$ it was shown by F. Martin [44] that this leads to predict a quark distribution in the pion which disagrees with the experimental one.

(b) A more general definition of $m(Q^2)$

A way out of the difficulty with the interpretation of the quark structure function of the pion [44] is to introduce a quark propagator which has a *cut* in the p^2 plane. Such a proposal has been made several times in the past [45].

In a model independent way the authors of Ref. 46 have shown why, in a coloured channel, Green functions should have a cut which has nothing to do with the usual threshold cuts.

The renormalized propagator should be written

$$S^{-1}(p) = \not{p}A(p^2) + B(p^2), \quad (94)$$

with

$$\text{disc } A(p^2) \neq 0,$$

and obeys a dispersion relation

$$S(p) = \frac{1}{\pi} \int_{m_0^2}^{+\infty} \frac{\text{disc } S(m')}{m'^2 - p^2} d(m'^2) \quad (95)$$

without subtraction.³⁰⁾

However nobody knows what m_0 should be and what is the behaviour of the

³⁰⁾ The asymptotic freedom property of QCD guarantees the vanishing of the propagator when $p^2 \rightarrow \infty$.

discontinuity. It is natural however to make the assumptions:

$$(i) \quad m_0 = m_{\text{current}}; \quad (96)$$

(ii) the discontinuity is important only when

$$m_{\text{current}} < m' < m_{\text{constituent}}. \quad (97)$$

If this is true, it means that inside a hadron a quark has no well defined mass. It gets an apparent mass which depends on the nature of our observations and which is always inside the interval (97). In deep inelastic scattering, it is Q^2 which parametrizes its variation. We are left with the problem of giving an estimate of the Q^2 dependence of this effective mass not only for high Q^2 but also for Q^2 down to $(1/R_h)^2$.

It is necessary to mention here that there exists a possibility to construct a propagator which is an entire function of p^2 [47]. For instance, a zero mass particle inside a constant self-dual field A_μ has a Lorentz invariant propagator of the form

$$S(p) = \frac{1}{p^2} [1 - e^{-p^2/B}]. \quad (95')$$

It has no singularity in $p^2 = 0$. Therefore, the particle is *confined*. Here the concept of mass has completely lost its meaning and, in particular, the inequalities (97) are no longer suggested. Though it is a very interesting possibility we shall not pursue it here as it amounts to deny any physical reality to the effective mass concept.

(c) *The Q^2 dependence of the quark mass*

The effective mass $m(Q^2)$ gets two contributions:

(1) the *perturbative* contribution ruled by *QCD* which is written

$$m_{\text{QCD}}(Q^2) = m_c \left(\frac{1}{\ln \frac{Q}{\Lambda}} \right)^d \quad (98)$$

with

$$m_c = m_{\text{current}}, \quad d = \frac{12}{33 - 2f}; \quad (99)$$

this Q^2 evolution of m ignores *completely* confining forces and is compatible with the propagator being a meromorphic function;

(2) the *non perturbative* or confining contribution is related to the chiral symmetry breaking contribution [43]. It is present only because³¹⁾

$$\langle 0 | q\bar{q} | 0 \rangle \neq 0 \quad (100)$$

i.e. the vacuum is a non *perturbative* one.

This contribution has the form

$$\frac{64\pi \alpha_s(Q^2)}{9 Q^2} (|\langle 0 | q\bar{q} | 0 \rangle|). \quad (101)$$

³¹⁾ This operator comes in the *OPE* of the quark field propagator.

One can easily understand the presence of the $1/Q^2$ contribution noticing $\langle 0 | q\bar{q} | 0 \rangle$ has dimension M^3 .

Adding together the two contributions one finds

$$m(Q^2) = m_c \left(\frac{1}{\ln \frac{Q}{\Lambda_{MS}}} \right)^d + 4\pi \frac{\alpha_s(Q^2)}{Q^2} v(Q^2) \tag{102}$$

where

$$v(Q^2) = |\langle 0 | \bar{q}q(M) | 0 \rangle| \left(\frac{\ln \left(\frac{Q}{\Lambda_{MS}} \right)}{\ln \left(\frac{M}{\Lambda_{MS}} \right)} \right)^{-d} \tag{103}$$

is written

$$v(Q^2) = \hat{v} \left(\ln \frac{Q}{\Lambda_{MS}} \right)^{-d} \tag{104}$$

Since we are interested in low Q^2 we take $f = 3$ and consequently

$$d = \frac{4}{9} \tag{105}$$

We can easily compute that with³²⁾

$$\left(v \left(\left(\frac{1}{R_h} \right)^2 \right) \right)^{1/3} = 135 \text{ MeV} \tag{106}$$

and

$$\Lambda \equiv \Lambda_{\overline{MS}} = 100 \text{ MeV}, \tag{107}$$

the second term of (102) is equal to 300 MeV.

It is interesting to notice that the value (106) of v falls well below the upper bound found in Ref. 48. The above expression is therefore very important on qualitative ground for our purpose. Indeed it indicates that at low Q^2 the effective mass begins to increase much faster than what is predicted by QCD. A last remark should be made concerning the behaviour of $m(Q^2)$ when $Q^2 \rightarrow 0$. Does the $1/Q^2$ behaviour subsists at that limit? If this is so, our considerations suggest that *free* quarks do not exist simply because they would have an INFINITE mass. It is an appealing possibility which would 'explain' confinement and would mean that the gluonic energy associated with a free quark is necessary infinite.³³⁾

³²⁾ We are aware that the use of the above formula of $m(Q^2)$ for $Q^2 \sim (1/R_h)^2$ looks unreasonable because in the OPE many other terms do contribute. However these terms will not modify qualitatively the fact that for low Q^2 the effective mass begins to increase at least as fast as $1/Q^2$. We are taking

$$\alpha_s(Q^2) = \frac{12\pi}{27 \ln \frac{Q^2}{\Lambda_{MS}^2}}$$

³³⁾ Recently Politzer [see Ref. 88] has suggested that spontaneous chiral symmetry breaking accounts for the success of the non-relativistic model. The reason is that loop contributions are suppressed by a $Q^2/m^2(Q^2)$ term for $Q < m(Q)$. In our case $m(Q^2)$ is not limited to 300 MeV but can become very big.

We want to end this section with several remarks relevant to our understanding of valence quarks.

(i) The OZI rule [49] cannot be understood unless a fundamental difference is made between valence quark and sea quarks. For instance, in the case of the ϕ , the s and \bar{s} quarks are not of the same kind as all $q\bar{q}$ pairs which surround them. They cannot annihilate easily while every other $q\bar{q}$ pair has an ephemeral existence. As is well known this is suggested by the fact that the $\phi \rightarrow 3\pi$ decay is suppressed by a factor 50. This factor becomes 5×10^3 for the $\psi \rightarrow 3\pi$ decay [50].

(ii) It may well be that the increase of the effective quark mass at small Q^2 does compensate the increase of the effective coupling $\alpha_s(Q^2)$ and therefore regularizes the theory at low Q^2 [51].

(iii) We have made no particular assumption about the behaviour of $\alpha_s(Q^2)$ when $Q^2 \rightarrow 0$. It is possible that its Q^2 dependence is also of the $1/Q^2$ type when Q^2 is small enough.

The potential we use successfully in quarkonium studies is of the form

$$V = -\frac{4}{3} \frac{\alpha_s}{r} + \frac{r}{a^2}. \quad (108)$$

It is given by the one gluon exchange contribution if the effective coupling behaves as

$$\alpha_s(Q^2) = \alpha_s + \frac{3}{2a^2} \frac{1}{Q^2}. \quad (109)$$

The use of the Richardson potential [52] leads also to the same behaviour. In it, the effective coupling takes the form

$$\bar{\alpha}_s(Q^2) = \frac{12\pi}{27} \frac{1}{\ln\left(1 + \frac{Q^2}{\Lambda^2}\right)} \quad (110)$$

so that

$$\bar{\alpha}_s(Q^2) \xrightarrow{(Q^2/\Lambda^2) \ll 1} \frac{12\pi}{27} \frac{\Lambda^2}{Q^2}. \quad (111)$$

A more basic argument in favor of this behaviour comes from the work of Ref. 53. The infrared behaviour of $\alpha_s(Q^2)$ is determined from Dyson's equations and Slavnov-Taylor's identities. It is shown that

$$\alpha_s(Q^2) \xrightarrow{Q^2 \rightarrow 0} 0.07 \frac{M^2}{Q^2} \quad (112)$$

with M^2 fixed by the condition

$$3\alpha_s(M^2) = 1. \quad (113)$$

(iv) The non relativistic behaviour of valence quarks has been noticed since the first successful predictions of $SU(6)$ [54]. The present successes of the non relativistic quark model for the $s\bar{s}$, $c\bar{c}$ and $b\bar{b}$ systems and for hadrons [55] is astonishing since we expect important relativistic corrections to it [56]. It seems that confining forces transform relativistic quanta into non relativistic ones. This possibility is suggested by the bag model.

A massless free quark in a sphere of radius R has, for its most stable solution, the wave function

$$q(\vec{r}) \div \begin{pmatrix} ij_0\left(\frac{rx_0}{R}\right)\psi \\ -ij_1\left(\frac{rx_0}{R}\right)\vec{\sigma} \cdot \hat{r}\psi \end{pmatrix} \tag{114}$$

where

$$x_0 = 2.04. \tag{115}$$

When $r = R$ its small and large components are equal since

$$j_0(x_0) = j_1(x_0). \tag{116}$$

However, the ratio j_1/j_0 decreases rapidly when $r < R$. Table II shows its values as a function of r/R .

Table II

The ratio between the big and small components of the quark wave-function.

$\frac{j_1\left(2.04 \frac{r}{R}\right)}{j_0\left(2.04 \frac{r}{R}\right)}$	$\frac{r}{R}$
1	1
0.37	0.5
0.17	0.25

Therefore though its mass is zero, because of confining forces, its wave function is only weakly relativistic. If $m \neq 0$ such effect is more pronounced.

8. The partonic description of deep inelastic leptonproduction processes

The usual partonic description of leptonproduction is based on two assumptions which are:

(i) the use of the impulse approximation which allows to describe the scattering of the photon with the nucleon as an *incoherent* superposition of photon-quark scattering processes;

(ii) quark masses are negligible and the offshellness of virtual quarks is neglected. Scaling violation is entirely described by perturbative *QCD* and, as far as transverse momenta are neglected, every virtual particle remains onshell and massless.

In particular no distinction is made between valence quarks and sea quarks on the level of their mass.

We shall try to make clear in this section that

- the way the impulse approximation is used
- the absence of clear distinction between valence and sea quarks are both indicative of the neglect of *confinement forces*.

(a) *The impulse approximation*

The use of the impulse approximation is justified on the basis of two observations.

First, the wave length of the virtual photon behaves as $1/\sqrt{Q^2}$ (where $q^2 = -Q^2$ is the square of the 4-momentum transferred to the nucleon by the incident lepton), so that it is *small* when Q^2 is *large*.

Second, the photon energy ($Q_0 = \nu$ in the Lab system) determines the interaction time $\tau = 1/Q_0$.

When Q_0 is large, τ is very small and the nucleon structure appears to be *frozen*, the photon interacts with each charged constituent separately. Each constituent appears *to be free* during the interaction time.

Therefore when Q^2 and ν are 'large enough' the photon-nucleon interaction is replaced by the photon-quark interaction of Fig. 14 with

$$p^2 = m^2 = (p + q)^2 \quad (117)$$

i.e.

$$2pq + q^2 = 0. \quad (118)$$

The fuzziness of the expression 'large enough' is related to our ignorance of the effective interaction strength between the quarks but the possibility to use the impulse approximation does not really depend on it. If the nucleon is composed of n quarks tightly held together we can write

$$m = \frac{M}{n} \quad (119)$$

and

$$\frac{Q^2}{2M\nu} = \frac{1}{n} \quad (120)$$

which shows that the value of n is restricted to be equal to the inverse of the number of constituents. However the nucleon is not a rigid body and the above situation which implies action at a distance or a $T = 0^\circ\text{K}$ situation is not realistic. Correspondingly, $Q^2/2M\nu$ is known to take any value between 0 and 1.

Next we want to show that the real situation is far from being as idyllic as described above because it is not always possible to consider that the scattered quark is on its mass-shell.

We consider the vertex nucleon -quark-nucleon residue (Fig. 15). P , p and p_R are respectively the 4-momenta of the nucleon, the quark and the nucleon residue.

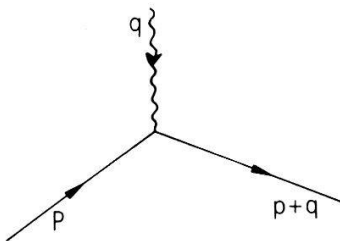


Figure 14
The quark- $\gamma(W)$ vertex in the impulse approximation.

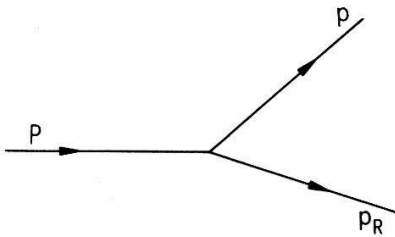


Figure 15

The fragmentation of one particle into two particles or clusters.

If M is the nuclear mass and if

$$x = \frac{p_0 + p_L}{p_0 + p_R} \tag{121}$$

we find the *kinematical* relation

$$M^2 = \frac{p^2}{x} + \frac{p_R^2}{1-x} + \frac{|\vec{p}_\perp|^2}{x(1-x)}; \tag{122}$$

this relation is valid for all x such that

$$\epsilon \leq x \leq 1 - \epsilon \quad (\epsilon > 0). \tag{123}$$

Relation (122) *constrains* the quark and nucleon residue masses. We extract from it three particular interesting situations.

$$(1) \quad p^2 = 0 = |\vec{p}_\perp|^2; \tag{124}$$

it leads to

$$M^2 = \frac{p_R^2}{1-x}. \tag{125}$$

However, in quantum mechanics, $|\vec{p}_\perp| = 0$ means that the nucleon radius is infinite i.e. that the quarks do not interact with each other in the nucleon. Therefore we must also have $M^2 = 0$ and $p_R^2 = 0$. Once we neglect the quark mass we *must* neglect all other masses in order to be consistent. The only scales in the problem are Q^2 and $2p \cdot q$ and before QCD corrections are included, all physical quantities can only depend on $Q^2/2pq$. This situation corresponds to the naïve parton model.

$$(2) \quad p_R^2 = M_R^2 = \mathcal{O}(M^2) \tag{126}$$

then

$$p^2 = -\frac{xM_R^2 + |\vec{p}_\perp|^2}{1-x} + M^2x. \tag{127}$$

The mass of the virtual quark (p^2) varies continuously with $|\vec{p}_\perp|^2$ and x . When $x \rightarrow 1$, it becomes

$$p^2 \xrightarrow{x \rightarrow 1} -\frac{M_R^2 + |\vec{p}_\perp|^2}{1-x} + M^2 \tag{128}$$

i.e. the virtuality of the quark becomes high unless M_R^2 and $|\vec{p}_\perp|^2$ are proportional to $1-x$. These last conditions seem quite unnatural and shall not be kept here.

In the present situation we expect corrections in the dimensionless structure functions of the order of

$$\begin{aligned}\frac{\langle p^2 \rangle}{Q^2} &= \pm \frac{\langle M_R^2 \rangle + \langle |\vec{p}_\perp|^2 \rangle}{Q^2} \frac{1}{1-x} \\ &= \pm \frac{\bar{m}^2}{Q^2} \cdot \frac{1}{1-x}\end{aligned}$$

which becomes important when $x \rightarrow 1$.

In a usual description of a bound state, a highly virtual component is associated with the *short distance* behaviour of the wave function. Moreover the above condition is quite foreign to the much discussed target mass corrections [57].

We shall call the present possibility, the *highly virtual quark* (H.V.Q.) case, its underlying assumption being that the nucleon residue is *for any x softly virtual*.

(3) $p^2 = m^2$, is finite and >0 then

$$p_R^2 = M^2(1-x) - \frac{m^2(1-x) + |\vec{p}_\perp|^2}{x}. \quad (130)$$

(i) When $x \rightarrow 1$ we observe that

$$p_R^2 \rightarrow -|\vec{p}_\perp|^2; \quad (131)$$

the nucleon residue is on the average *softly virtual* since

$$\langle p_R^2 \rangle \sim \left(\frac{1}{R_h} \right)^2 \sim 0.06 \text{ GeV}^2. \quad (132)$$

The nucleon residue virtuality changes continuously with x . It is minimum when

$$|\vec{p}_\perp|^2 = 0 \quad (133)$$

and

$$x = \frac{m}{M} \quad (134)$$

and is given by

$$p_R^2 = (M-m)^2; \quad (135)$$

it corresponds to the situation of a rigid nucleon from which a fraction m of its total mass M is removed [see above].

(ii) When $x \rightarrow 0$,

$$p_R^2 \rightarrow -\frac{m^2 + |\vec{p}_\perp|^2}{x}, \quad (136)$$

so that the nucleon residue becomes highly virtual.

We shall call the present possibility the *softly virtual quark* (S.V.Q.) case because for all values of x the quark remains on (or close to) its mass-shell.

The problem which is raised by this discussion is the following. When we use the impulse approximation, should we use a H.V.Q. or a S.V.Q.? The answer to this question is clear in atomic or nuclear physics but we believe it is still open in particle physics.

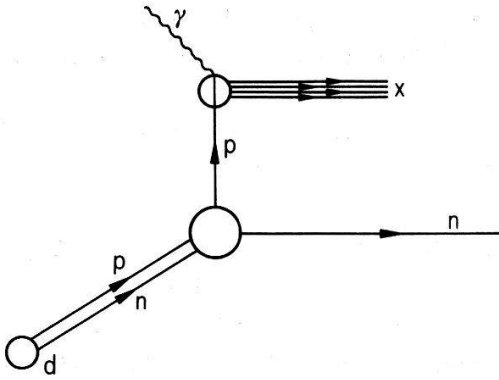


Figure 16

The first order Fermi correction is computed through the impulse approximation. Here the neutron n and the cluster X are *real* objects.

Indeed in atomic and nuclear physics we are dealing with composite systems which are governed by a force which is strong at short distances and weak at large distances. A simple illustration of the impulse approximation in such a situation is given when we calculate Fermi corrections. We are dealing with the reaction



which, in the impulse approximation, gets the contribution of the diagram of Fig. 16.

Since the neutron is a *free* particle in the final state it is necessary to ascribe the necessary offshellness to the scattered proton and we encounter without ambiguity the H.V.Q. case, the quark being here the proton.

In particle physics however the situation is reversed. We are dealing with a composite system which is governed by a weak force when distances are small compared to its radius and strong when distances are of the same order of magnitude as its radius.

Physically this implies

- (i) that the nucleon residue *can never be* a free particle (like the neutron);
- (ii) that the fragmentation of the quark *cannot be independent* of the fragmentation of the nucleon residue while in the above example the proton fragmentation is quite independent of the neutron evolution;
- (iii) that as soon as the photon is hard enough it sees a quark which is *almost free* while for the above nuclear reaction the proton is seen as much strongly bound in the deuteron.

These remarks lead us to propose a new kind of impulse approximation which takes explicitly into account the asymptotic freedom and *confinement properties* of colour forces.

(b) Towards a new form of the impulse approximation

This new form is based on the following assumptions:

- (i) it is valid down to

$$Q^2 > \left(\frac{1}{R_h}\right)^2 \sim 0.06 \text{ GeV}^2; \quad (138)$$

- (ii) the quarks seen by the probe (γ or W 's) are *softly* virtual;
- (iii) – sea quarks are seen as perturbative *QCD* quarks i.e. with a mass equal to their current mass.
 - valence quarks are seen with a Q^2 varying effective mass $m(Q^2)$ where $m(Q^2)$ obeys a relation of the type of (102).

Let us discuss them.

Assumption (i) is compatible with the general spirit of the impulse approximation. It is also compatible with the observed *early* scaling. It means that the parton model can be used down to Q^2 values much smaller than 1 GeV^2 .

Assumption (ii) is compatible with the successful counting rule for form factors [58] and structure functions. Indeed the validity of the latter does not depend on the degree of softness of the quarks [59].

Assumption (iii) needs more comments.

First, a strong difference is made between valence quarks and sea quarks. They are distinguished by their *effective* masses. Sea quarks are seen as perturbative *QCD* quarks because they are virtually produced by gluons. These gluons must be *highly virtual* otherwise the production of sea quarks would not be describable perturbatively. These quarks should also live a very short time in such a way that they cannot dress themselves before being annihilated. All this is compatible with the fact that the sea quark pair production process must be highly localised inside the nucleon. Once a sea quark is scattered, it receives enough energy to allow its hadronization to occur. This hadronization process is a *long time* scale process.

Valence quarks are long-lived particles. They are dressed with gluons³⁴⁾ with which they interact strongly. When they are scattered, the probe not only accelerates them but it must accelerate the surrounding gluons so that their mass can be much different from their current mass and can of course vary with Q^2 .

The distinction made between valence and sea quarks is, in our view, a consequence of confinement forces and the notion of effective mass is a simple way to parameterize their effect.

We must add that effective quark mass corrections are not the only corrections to the naïve parton model. Notice that in addition to the effective mass correction we have also

- target (or here nucleon) mass corrections
- quark transverse momentum corrections.

We postulate that those corrections taken together give a good description of the effect of confinement forces in deep inelastic scattering. This new form of the impulse approximation should then be used as the zero order approximation when

³⁴⁾ Our concept of valence quark corresponds to the concept of valon as introduced by R. C. Hwa (see above). It is however entirely different in so far as we do not consider that one quark dressed with gluons forms a well defined particle. Therefore in the description of deep inelastic scattering we do not need to introduce a two level structure scheme in which a nucleon is seen as composed of valons, each valon being itself a composite system made up of a quark dressed with gluons. A structure function is therefore written as

$$F_h(x, Q^2) = \sum_v \int_0^1 G_{v/h}(y) F_v\left(\frac{x}{y}, Q^2\right) dy$$

when $(G_{v/h})(y)$ is the probability to find a valon in a hadron h .

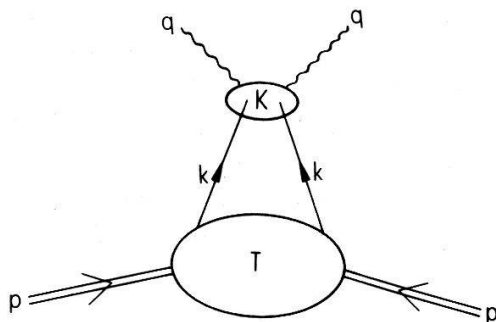


Figure 17

The covariant parton model graph. K describes the quark-vector boson interaction. T describes the proton-quark scattering amplitude.

applying the QCD corrections. The way these corrections are affected by the introduction of a quark effective mass will be discussed in Section 11.

As a final but important remark we point out that our proposal is quite different from the usual improvement of the impulse approximation provided by the so-called covariant parton model [60]. In this covariant formulation the inclusive deep inelastic amplitude is given by the handbag diagram of Fig. 17 and it allows to take both the quark virtuality and its transverse momentum into account. T does contain the bound state effects. However the consequences we deduce from this graph (i.e. the scale breaking effects) are strongly dependent on the assumptions and approximations made. In particular writing a spectral representation for T seems to us, if not in conflict, at least questionable in view of the fact that *no physical state* exist because of confinement forces [61].³⁵⁾

9. An alternative parton description of deep inelastic scattering

In this section we want to discuss several properties and consequences of a parton description which includes

- the softly virtual quark property inside the impulse approximation
- the notion of Q^2 dependent effective valence quark mass.

A few obvious properties of this model are:

- (i) it is valid for all $Q^2 > (1/R_h)^2$;
- (ii) it should be applicable outside the region $x = 0$; in particular, it is valid near $x = 1$;
- (iii) target mass and transverse momentum corrections are intimately connected and tend to *compensate each other*;
- (iv) it does not lead to $R = (\sigma_L/\sigma_T) = 0$ [62].

A few comments are in order. Property (i) is obvious from what we have already said. Property (ii) can be understood from the fact that, near $x = 0$, the nucleon residue becomes highly virtual unless the quark gets a squared *negative* mass. The latter is only possible if it is no longer a quark but a collective quark

³⁵⁾ The authors of ref. (61) find a M^2/Q^2 correction not included in Nachtmann's moments from such a special representation.

gluon excitation which is exchanged. It is a 'Regge trajectory'.³⁶⁾ It can also be said that, in this limit, the incident photon behaves essentially like a ρ particle i.e. there is a region where a vector meson dominance principle applies.³⁷⁾ When x is near 1 the correction of the type p^2/Q^2 is now much less important than in the previous formulation since it does not contain the $(1-x)^{-1}$ factor. As we shall further discuss, this fact has an important implication on the higher twist contributions. Property (iii) is an immediate consequence of the new kinematics of the process. As shown in Ref. 62 the basic kinematical variable is no longer x but

$$x_N = x \frac{1 + \sqrt{A_m}}{1 + \sqrt{A_M}} \quad (139)$$

where

$$A_m = 1 + \frac{4m^2}{Q^2}, \quad A_M = 1 + \frac{4M^2x^2}{Q^2}. \quad (140)$$

To recover x it is necessary to set $M^2 = 0$ and $m^2 = 0$. On the other hand x_N is closer to x than the Nachtmann variable

$$N = \frac{2}{1 + \sqrt{A_M}}. \quad (141)$$

We shall come back to this point soon. Before that we want to examine the Q^2 behaviour of the valence quark mass.

A critical analysis of the $m(Q^2)$ function

The $m(Q^2)$ function we have written in equation (102) is the best we presently know. However it is somewhat ambiguous and probably grossly approximate.

First, it is not quite fair to identify Λ to $\Lambda_{\overline{MS}}$ because it contains only the leading order correction in α_s which strictly implies that Λ is undetermined. Therefore our identification has only an experimental basis.³⁸⁾ We have done it because it leads to

$$\alpha_s(Q^2) = 0.76 \quad (142)$$

for $Q = 1/R_h$ i.e. a value less than one.

Second the $1/Q^2$ term comes from using the O.P.E. For very small values of Q^2 , it is not necessarily reliable. Therefore we cannot take for granted a *strict* $1/Q^2$ behaviour. For instance, a

$$\frac{1}{Q^2 + a^2} \quad (143)$$

³⁶⁾ It is wellknown that when $x \rightarrow 0$ structure functions have a behaviour close to $1/\sqrt{x}$. See for instance the results obtained by the EMC group (Ref. 84).

³⁷⁾ The virtual photon is close to a real photon. For a real photon we can no longer associate $1/\sqrt{Q^2}$ with its wave length since a frame in which the photon energy is zero does no longer exist.

³⁸⁾ Notice from the Y decay one infers that $\alpha_s(m_b) \approx 0.16$. This value is obtained if $\Lambda_{\overline{MS}} \approx 25$ MeV. See for instance Ref. 85.

behaviour could, as well, be correct. However the difference between a $1/Q^2$ and a $1/Q^2 + a^2$ may be enormous and is crucial in our case.

The two following examples illustrate clearly the situation.

(1) From formula (102) of Section 7 and taking $R_h = 0.8$ fm, we get the results of Table III for a u quark. We observe that already at 1 GeV^2 the valence

Table III

The evolution of the Q^2 -dependent effective mass according to formula (102)

$Q^2(\text{MeV}^2)$	$m(Q^2)$ (MeV)
$\left(\frac{1}{R_h}\right)^2$	304
0.1	202
0.5	21
1	11

quark mass is almost equal to the current quark mass. If, however, we take³⁹⁾

$$Q^2 = 1 \text{ GeV}^2 \quad (144)$$

and adjust the coefficient of $1/Q^2 + a^2$ in such a way that the effective mass equals 300 MeV when $Q = 1/R_h$, we find (see Table IV) that, for $Q^2 = 1 \text{ GeV}^2$, it is still significantly different from the current quark mass. However in both cases when $Q^2 > 5 \text{ GeV}^2$ we can say that the valence quark mass is *close to the current quark mass*.

Table IV

The evolution of the Q^2 -dependent effective mass for $a = 1 \text{ GeV}^2$

$Q^2(\text{MeV}^2)$	$m(Q^2)$ (MeV)
$\left(\frac{1}{R_h}\right)^2$	300
0.1	231
0.5	102
1	66
2	39

The difficulty to observe $m(Q^2)$

Is there a way to observe $m(Q^2)$? We think the answer is positive at least if we are willing to make precise measurements of deep inelastic scattering processes at values of Q^2 less than 1 GeV^2 .

Indeed we have shown [62] in a previous work that the valence quark distribution has a maximum near

$$x_N = \frac{m(Q^2)}{M} \quad (145)$$

³⁹⁾ This number is almost equal to the expected gluonium mass.

In fact, this statement is exact only when Q^2 is much higher than $4M^2$. When it is of the same order of magnitude or less one can show that the maximum sits *above* this value.⁴⁰⁾ It is impossible to tell by how much in a model independent way because this depends on the assumed quark 'wave function'. However experimental data do indicate that all non-singlet structure functions have a maximum when x is close to 0.2 for the different values of Q^2 [63]. Our model implies that as Q^2 increases from very low values one must see a displacement towards the small values of x . The rate of displacement is also predicted to be of the $1/Q^2 + a^2$ type for $Q^2 \leq 2 \text{ GeV}^2$ and $1/\ln(Q^2/\Lambda^2)$ type for bigger values of Q^2 . Unfortunately, it is a very difficult experimental task to measure correctly the position of the maximum on non singlet distributions and a fortiori to measure its displacement with Q^2 . Moreover, there are very few data for Q^2 values less than 2 GeV^2 . Therefore our description is not presently testable at this level. We think that low Q^2 data are urgently needed.⁴¹⁾

10. Consequences on the description of higher twist effects

We have already said in the preceding section that, in our formulation, it appears that target mass and transverse momentum corrections should be applied at the same time and tend to compensate each other. We want now to make a full analysis of this statement.

The presently estimated value of the intrinsic transverse momentum of the scattered quarks is much higher than $(1/R_h)^2$. In general, the intrinsic transverse momentum distribution is assumed to be Gaussian in the analysis of lepton production and of Drell-Yan type processes [64]. $\langle p_{\perp}^2 \rangle$ is found in the range 0.2 GeV^2 to 0.8 GeV^2 .

These big values of $\langle p_{\perp}^2 \rangle$ imply that our variable x_N is substantially closer to x than the usual Nachtmann variable. As an illustration, if we take

$$\begin{aligned} Q^2 &= 1 \text{ GeV}^2, \\ p_{\perp}^2 \sim \langle p_{\perp}^2 \rangle &= 0.5 \text{ GeV}^2, \end{aligned} \tag{146}$$

⁴⁰⁾ Formula (75) of Ref. 62 can also be written

$$W(\xi, Q^2) = \left(\frac{2}{Q^2 + 4M^2\xi^2} \right) \int_{X_0(\xi, \mu, M)}^{+\infty} \left\{ 1 + \frac{4M^2\xi}{Q^2} X \right\} \frac{dX}{X} F(X) \langle \bar{\omega}(Q^2) \rangle.$$

The functions

$$\frac{\xi^2}{Q^2 + 4M^2\xi^2} \quad \text{and} \quad \left\{ 1 + \frac{4M^2\xi}{Q^2} X \right\}$$

are both *monotonously* increasing with ξ . Therefore the experimental maximum can sit above. How much depends on the details of the model and especially on the quark 'wave-function' $F(X)$. Consistently with the parton model philosophy $F(X)$ does not depend on Q^2 in our model. This is unlike the assumption of Ref. 86.

⁴¹⁾ Using the fit of Buras and Gaemers [87] one finds for $Q^2 = Q_0^2 = 20 \text{ GeV}^2$ that the distribution xF has a maximum for $x = 0.09$. The experimental error made is as big as this last value. To be able to localize the position of the maximum reliably it is necessary to collect data within very small $\Delta x \approx 0.01$.

we find that, around $x \approx 0.8$,⁴²⁾

$$\frac{N}{x} = 0.71 \tag{147}$$

while

$$\frac{x_N}{x} \approx 0.97. \tag{148}$$

Since almost all data points are for $x \leq 0.8$, we arrive at the conclusion that Cornwall–Norton moments of structure functions are a better approximation than the Nachtmann moments.

More arguments can be brought in favour of this statement. We discuss them successively.

1. The use of the Nachtmann variable together with the assumption of massless quarks is badly inconsistent.⁴³⁾ Indeed this variable is found from the relations (Fig. 18)

$$\begin{aligned} (NP + q)^2 &= 0, \\ P^2 &= M^2. \end{aligned} \tag{149}$$

However, these two relations imply that the initial quark mass is equal to

$$m = NM = \frac{2x}{1 + \sqrt{1 + 4 \frac{M^2 x^2}{Q^2}}} M \tag{150}$$

i.e. is of the same order of magnitude as the target mass. It is incompatible with the first of the equations (149) and also with gauge invariance. Moreover it implies that the target behaves as a rigid body.

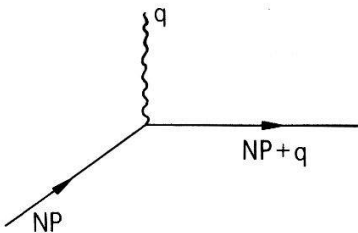


Figure 18

The quark- $\gamma(W)$ vertex when the proton mass is not neglected. N is the Nachtmann variable.

⁴²⁾
$$N = \frac{2x}{1 + \left(1 + \frac{4M^2 x^2}{Q^2}\right)^{1/2}}$$

is the usual Nachtmann variable.

⁴³⁾ It is a consequence of the kinematical considerations of Ref. (62) but the argument given here is much simpler.

2. We start from the Nachtman moment expression of a non singlet structure function⁴⁴⁾

$$M(n, Q^2) = \int_0^1 \frac{dx}{x^2} N^{n+1} W(x, Q^2). \quad (151)$$

When n is large, the integral is dominated by values of x close to 1. On the other hand, the counting rule allows us to state that

$$W(x, Q^2) \xrightarrow{x \rightarrow 1} (1-x)^\alpha \quad (152)$$

with $\alpha \simeq 3$. Therefore, for large n , we can estimate the target mass effect in $M(n, Q^2)$.⁴⁵⁾ We first express x in terms of N

$$x = \frac{N}{1 - \frac{M^2}{Q^2} \cdot N^2} \quad (153)$$

so that, when $x = 1$, we have the equality

$$N_0 = 1 - \frac{M^2}{Q^2} \cdot N_0^2 \quad (154)$$

with $N_0 = N(x = 1)$. Using (153), we get

$$M(n, Q^2) = \int_0^{N_0} N^{n-1} \left(1 + \frac{M^2}{Q^2} N^2\right) dN W(x(N), Q^2). \quad (155)$$

We renormalize N setting

$$N = \rho N_0. \quad (156)$$

The integral becomes

$$M(n, Q^2) = N_0^n \int_0^1 \rho^{n-1} \left(1 + \frac{M^2 N_0^2}{Q^2} \rho^2\right) d\rho W[x(N(\rho)), Q^2]. \quad (157)$$

We replace W by its boundary value as given by relation (152) so that

$$W \simeq \frac{1}{\left(1 - \frac{M^2}{Q^2} N_0^2 \rho^2\right)^\alpha} \left(1 - \rho N_0 - \frac{M^2}{Q^2} N_0^2 \rho^2\right)^\alpha. \quad (158)$$

It can be transformed into the expression

$$W \simeq (1-\rho)^\alpha \left(\frac{1 + \frac{M^2}{Q^2} N_0(1+\rho)}{1 + \frac{M^2}{Q^2} N_0(1-\rho^2)} \right)^\alpha. \quad (159)$$

⁴⁴⁾ We consider the case of scalar particles.

⁴⁵⁾ An analogous calculation has been done in Ref. 65. Since we disagree with the result of these authors, we think it is worth to explain it.

The integral becomes

$$M(n, Q^2) \approx N_0^n \int_0^1 \rho^{n-1} (1-\rho)^\alpha \left(1 + \frac{M^2 N_0^2}{Q^2} \rho^2\right) \left(\frac{1 + \frac{M^2}{Q^2} N_0(1-\rho)}{1 + \frac{M^2}{Q^2} N_0(1-\rho^2)}\right)^\alpha d\rho \quad (160)$$

i.e.

$$M(n, Q^2) \approx N_0^n \int_0^1 \rho^{n-1} (1-\rho)^\alpha \left[1 + \mathcal{O}\left(\frac{M^2}{Q^2} \rho\right)\right] d\rho. \quad (161)$$

We see that there exists no N_0^α term contrary to what is claimed by the authors of Ref. 65.

It is useful to go a little further calculating the integral when M^2/Q^2 are supposed to be small. Replacing N_0 by

$$1 - \frac{M^2}{Q^2} \quad (162)$$

we find

$$M(n, Q^2) \approx N_0^n \int_0^1 \rho^{n-1} (1-\rho)^\alpha \left(1 + \frac{M^2}{Q^2} \rho(\rho + \alpha)\right) d\rho. \quad (163)$$

If $n(M^2/Q^2)$ is also supposed to be small, we find

$$M(n, Q^2) \approx \left(1 - \frac{nM^2}{Q^2}\right) B(n, \alpha + 1) \quad (164)$$

$$\approx \left(1 - \frac{nM^2}{Q^2}\right) e^{-(\alpha+1) \ln n} \Gamma(\alpha + 1) \quad (165)$$

Since N_0 is always less than one, target mass corrections tend to *decrease* the moments. As pointed by Duke and Roberts [66] there is no evidence that a factor N_0^n is at work since its introduction does not improve the quality of the fits. This is not surprising in our view since the neglect of valence quark mass as well as transverse momentum effect in the usual picture induces a too important modification of the scaling variable x into N . Therefore, if one takes into account those corrections too, in first approximation, we expect that $M(m, Q^2)$ will be multiplied by a term

$$1 + \frac{na}{Q^2} \quad (166)$$

where a is a coefficient which reflects binding effects i.e. which can be considered to be of the order of $\langle m_\perp^2(Q^2) \rangle$.⁴⁶⁾ The whole higher twist effect on $M_n(Q^2)$ is then equal to

$$\begin{aligned} M(n, Q^2) &\approx (QCD) \left(1 - n \frac{M^2}{Q^2}\right) \left(1 + \frac{n \langle m_\perp^2(Q^2) \rangle}{Q^2}\right) \\ &\approx (QCD) \left(1 + \frac{n}{Q^2} (\langle m_\perp^2(Q^2) \rangle - M^2)\right) \end{aligned}$$

⁴⁶⁾ We remind the reader that, in our framework, N becomes

$$x_N = \frac{N}{2} (1 + \sqrt{1 + 4m_\perp^2/Q^2}).$$

where (QCD) contains the QCD correction term as well as all proportionality factors.

Since $\langle m_{\perp}^2(Q^2) \rangle$ is of the same order as M^2 , we observe an important compensation.

Therefore our framework gives a hint for the explanation of the fact that there are no real evidence for important higher twist effects.

11. Quark mass effects on QCD corrections

The fact that in our formulation u and d quark masses cannot be neglected at low Q^2 implies some modifications of QCD corrections. They have already been studied in connection with the existence of thresholds due to the massive quarks s , c and b [67]. Mass dependences manifest themselves first in the expression of the effective coupling and second in the anomalous dimensions. For instance, β_0 becomes

$$\beta_0 = 11 - \frac{2}{3} \sum_{\text{flavours}} K\left(\frac{m_j^2}{Q^2}\right). \quad (168)$$

$K(x^2)$ is equal to

$$K(x^2) = 1 - x^2 + \frac{12x^4}{\sqrt{1+4x^2}} \ln \left(\frac{1 + \sqrt{1+4x^2}}{\sqrt{1+4x^2} - 1} \right). \quad (169)$$

Since

$$\alpha_s = \frac{12}{\beta_0 \ln \frac{Q^2}{\Lambda^2}}, \quad (170)$$

the meaning of (168) is clear:

- (i) $\beta_0(m_j^2/Q^2) > \beta_0(0)$ and the effective coupling is smaller for massive quarks than for massless ones;
- (ii) when a threshold for excitation of a new quark is crossed, β_0 decreases and therefore the scaling violation weakens.

In the above formula the m_j 's are current quark masses which means, in particular, that the u and d quark masses are neglected. In our case, because of confinement forces, u and d quarks stay massive when Q^2 is small. It is not immediately clear what to do with the relation (168) but we propose to set

$$\beta_0 = 11 - \frac{2}{3} \sum_{j=u,d,s} K\left(\frac{m_j^2(Q^2)}{Q^2}\right) \quad (171)$$

where $m_j^2(Q^2)$ are the confined effective masses we have introduced.

As stated by De Rujula and Georgi [68] the function K can be well approximated by

$$\frac{Q^2}{5m_j^2(Q^2) + Q^2} < 1. \quad (172)$$

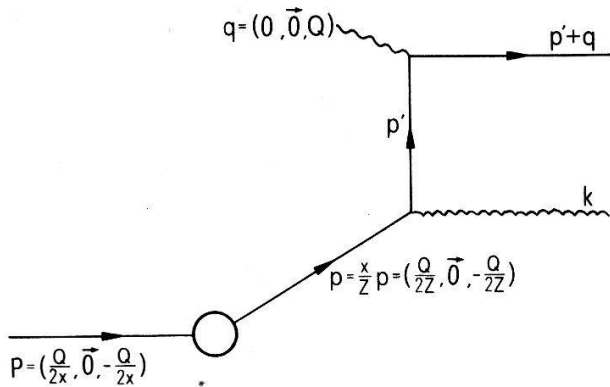


Figure 19
Kinematics of gluon Bremsstrahlung when all particles are massless.

Therefore, the effective coupling stays *smaller* at small Q^2 than in the massless case. Our prescription either *weakens* scaling violations for non singlet moments or leads to an increase of the Λ parameter.

As a second aspect of these effects, we want to explain how the quark mass does modify the Altarelli–Parisi kinematics used to calculate QCD corrections.

We are working in the Breit frame and consider first the Altarelli–Parisi kinematics when all masses and the intrinsic transverse momentum of the quark are neglected. Our notations are fixed according to Fig. 19. It is necessary that

$$P'_L = -\frac{Q}{2}; \tag{173}$$

this means that

$$k_L = p_L - p'_L = \frac{Q}{2Z} (Z - 1). \tag{174}$$

The gluon 4-momentum is

$$k = \left(k_0, \vec{k}_\perp, \frac{Z-1}{2Z} Q \right). \tag{175}$$

Since $k^2 = 0$ (physical gluon), we get

$$k_0 = \sqrt{|\vec{k}_\perp|^2 + \left(\frac{1-Z}{2Z}\right)^2 Q^2}. \tag{176}$$

The 4-momentum of the scattered quark is

$$p' = p - k = \left(\frac{Q}{2Z} - \sqrt{|\vec{k}_\perp|^2 + \left(\frac{1-Z}{2Z}\right)^2 Q^2}, -\vec{k}_\perp, -\frac{Q}{2} \right). \tag{177}$$

Its off-shellness is given by

$$p'^2 = \frac{Q^2}{2Z^2} (1-Z) - \frac{Q}{Z} \sqrt{|\vec{k}_\perp|^2 + \left(\frac{1-Z}{2Z}\right)^2 Q^2}. \tag{178}$$

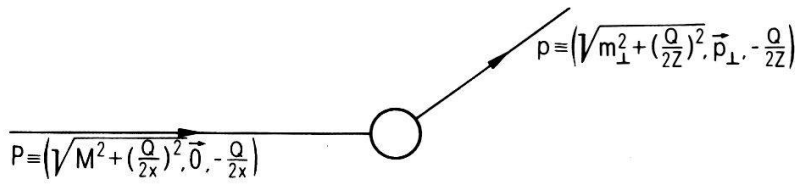


Figure 20

The general expression of the quark momentum in the hypothesis it is softly virtual.

When $Q^2 \gg |\vec{k}_\perp|^2$, it reduces to the wellknown result

$$p'^2 = -\frac{|\vec{k}_\perp|^2}{1-Z}. \tag{179}$$

Moreover we get

$$p'^2 = (p' + q)^2. \tag{180}$$

QCD corrections are calculated from integration of $1/p'^2$ over d^3k .⁴⁷⁾ It is singular when $|\vec{k}_\perp| = 0$. Therefore it is necessary to introduce a *fictitious* mass μ to get a finite result. The outcome of the calculation is the appearance of a $\ln(Q^2/\mu^2)$ term [69].

Next we consider the general case when we include all mass contributions and the intrinsic transverse momentum of the quarks \vec{p}_\perp .

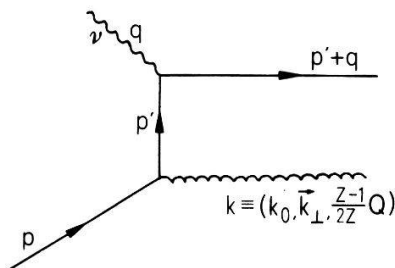


Figure 21

The general expression of the gluon momentum.

The new kinematics is displayed in Fig. 20 and Fig. 21. p' is now given by

$$p' = \left(\sqrt{m_\perp^2 + \left(\frac{Q}{2Z}\right)^2} - \sqrt{|\vec{k}_\perp|^2 + \left(\frac{Z-1}{2Z}\right)^2 Q^2}, \vec{p}_\perp - \vec{k}_\perp, -\frac{Q}{2} \right) \tag{181}$$

and

$$p'^2 = m^2 + 2\vec{p}_\perp \cdot \vec{k}_\perp + \frac{Q^2}{2^2} (1-Z) - 2\sqrt{m_\perp^2 + \left(\frac{Q}{2Z}\right)^2} \sqrt{|\vec{k}_\perp|^2 + \left(\frac{Z-1}{2Z}\right)^2 Q^2}. \tag{182}$$

⁴⁷⁾ We neglect all spin effects here for simplicity.

The calculation of QCD corrections involves the integral

$$\int \frac{d^3k}{p'^2 - m^2} = \int \frac{d^3k}{(p-k)^2 - m^2} = \int \frac{d^2k_{\perp} dk_L}{-2pk} \tag{183}$$

Because of the appearance of the scalar product $\vec{p}_{\perp} \cdot \vec{k}_{\perp}$ the \vec{k}_{\perp} integration over the angle $(\hat{p}_{\perp} \cdot \hat{k}_{\perp})$ ⁴⁸⁾ is crucial. We find

$$\int_{-1}^{+1} \frac{d(\cos \theta)}{p'^2 - m^2} = -\frac{p_0}{|\vec{p}|} \ln \left(\frac{1 - \frac{|\vec{p}|}{p_0}}{1 + \frac{|\vec{p}|}{p_0}} \right) = \frac{p_0}{|\vec{p}|} \ln \frac{p_0^2 \left(1 + \frac{|\vec{p}|}{p_0}\right)^2}{m^2} \tag{184}$$

With

$$p_0^2 = \frac{Q^2}{4Z^2} \left(1 + 4Z^2 \frac{m_{\perp}^2}{Q^2}\right),$$

we get

$$\int_{-1}^{+1} \frac{d(\cos \theta)}{p'^2 - m^2} = \frac{p_0}{|\vec{p}|} \left\{ \ln \frac{Q^2}{4Z^2 m^2} + \ln \left(1 + \left(\frac{1 + \frac{4Z^2}{Q^2} |\vec{p}_{\perp}|^2}{1 + \frac{4Z^2}{Q^2} m_{\perp}^2} \right)^{1/2} \right)^2 \left(1 + \frac{4Z^2}{Q^2} m_{\perp}^2 \right) \right\} \tag{185}$$

When $Q^2 > 4\langle m_{\perp}^2 \rangle = 4(m^2 + \langle p_{\perp}^2 \rangle) > 2 \text{ GeV}^2$, the second term does not contribute and m^2 is almost equal to the current quark mass as we have shown. Since $p_0 = |\vec{p}|$ we recover the old result when we make $\mu = Zm$.⁴⁹⁾

However if

$$Q^2 < 4m_{\perp}^2, \tag{186}$$

there is a Q^2 independent term which gives a significant contribution. This contribution is entirely determined by $m^2(Q^2)$ and $|\vec{p}_{\perp}|^2$ and must be calculated as soon as we go beyond the L.L.A.

⁴⁸⁾ \hat{p}_{\perp} and \hat{k}_{\perp} are unit vectors.

⁴⁹⁾ We see that the soft virtual quark assumption does provide a natural cut-off.

12. Concluding remarks

We hope we have made clear that the notion of soft virtual quark with Q^2 varying effective mass has a number of attractive features which makes it worthwhile to be further studied.

- (a) It looks as an economical way to partly include confinement forces.
- (b) It allows to introduce a close connection between the non relativistic quark model and its relativistic field theoretical version.
- (c) It implies that Cornwall–Norton moments are a better approximation than Nachtmann moments for low and intermediate Q^2 values.
- (d) It suggests that higher twist contributions are much less important than expected in the usual picture.
- (e) It does not change *QCD* corrections at the L.L.A. level.
- (f) Our proposal can be tested in polarized leptonproduction. Indeed, in the conventional model the smallness of the scattered quark mass implies that its helicity is conserved in its scattering with the photon. Here helicity has no reason to be conserved.

One can consider, for instance, the virtual photoproduction process of a transverse photon with a polarized proton. To be specific let us take the photon helicity in the same direction as the proton spin. Then the probability that the forward jet is generated by a u or d quark of effective mass $m(Q^2)$ polarized in the same direction is in first approximation proportional to

$$\left(1 - \frac{m(Q^2)}{2} \frac{p + p'}{pp'}\right)^2$$

where p and p' are the initial and final momenta of the scattered quark. It is clear that this term can be quite small for $Q^2 \sim pp'$ less than a few GeV^2 .

Finally we would like to point out that our assumption leads to a better understanding of the similarity between fragmentation in hadron-hadron collision and fragmentation in leptonproduction noticed in Ref. 70. Indeed in hadron-hadron collisions, the valence quark which fragments is a ball of sea quarks and gluons which behave collectively as one particle. This is precisely our valence quark picture in leptonproduction and therefore we understand why the fragmentation functions are the same.

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