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Consequences of duality relations in curved space-time

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Abstract. The duality relations of systems of charges and monopoles are discussed. An analysis of the equations of motion in curved space-time shows that fields measured by means of a charge and fields measured by means of a monopole are not the same physical entity. It is shown that this conclusion is related to the known noncovariant nature of the presently accepted equations of motion of the system. The compatibility of this result with a new monopole theory is pointed out. The interrelations between this conclusion and the experimental search for monopoles is pointed out.

I. Introduction

This work discusses the interrelations between charges, monopoles and the electromagnetic fields associated with them. The work assumes the duality relations between charges and monopoles and between the electromagnetic fields associated with them. An analysis of these systems in a flat space-time and in a curved space-time is carried out. It is found that electromagnetic fields measured by means of charges and the corresponding fields measured by means of monopoles are not the same physical entity. The relation between this conclusion and the known noncovariant nature of the presently accepted equations of motion of the system [1] is discussed. The common trend of these results and of the conclusions of a dynamical analysis carried out in a flat space-time [2] is presented. A comparison of the results with the experimental data is briefly discussed. The whole work is done within the framework of classical field theory.

The term 'known world' denotes systems of charges, photons and the electromagnetic fields associated with them; the term 'dual world' denotes systems of monopoles, photons and the electromagnetic fields associated with them; the term 'extended world' denotes systems of charges, monopoles, photons and the electromagnetic fields associated with them. Subscripts (e), (m) and (w) denote quantities associated with charges, monopoles and photons, respectively. Units where the speed of light $c = 1$ are used. Greek indices run from 0 to 3. $,\mu$ denotes partial differentiation with respect to x^μ.

The first section of this work is the introduction. The duality relations between charges and monopoles are discussed in the second section. The implications of these relations and the nature of the electromagnetic fields are analysed in the third section. The charge-monopole interactions are discussed in the fourth section. A review of the experimental side is presented in the fifth section. Concluding statements are the contents of the last section.

II. Charges, monopoles and duality

Let us examine the equations of motion of a closed system of the known world [3]. The fields satisfy Maxwell equations

$$F_{(e,w)\mu\nu,\lambda}+F_{(e,w)\nu\lambda,\mu}+F_{(e,w)\lambda\mu,\nu}=0 \tag{1}$$

$$F^{\mu\nu}_{(e,w),\nu}=-4\pi J^{\mu}_{(e)} \tag{2}$$

where

$$F_{(e,w)\mu\nu}=A_{(e,w)\nu,\mu}-A_{(e,w)\mu,\nu} \tag{3}$$

is the antisymmetric tensor of the fields and $J_{(e)}$ denotes the 4-current of the charges. The metric of the flat space-time is diagonal and its entries are $(1,-1,-1,-1)$. The contravariant electromagnetic tensor takes the following form

$$F^{\mu\nu}_{(e,w)}=g^{\mu\alpha}g^{\nu\beta}F_{(e,w)\alpha\beta}=\begin{pmatrix}0 & -E_x & -E_y & -E_z\\ E_x & 0 & -B_z & B_y\\ E_y & B_z & 0 & -B_x\\ E_z & -B_y & B_x & 0\end{pmatrix}. \tag{4}$$

Physical quantities are related to measurements. Classical electromagnetic fields of the known world are measured by means of the Lorentz force exerted on charges

$$\frac{dP^{\mu}_{(e)}}{d\tau}=F^{\mu\nu}_{(e,w)}J_{(e)\nu} \tag{5}$$

where $P^{\mu}_{(e)}$ denotes the 4-momentum of the charges and τ denotes the invariant time.

Let us turn to a system of monopoles and fields. In this case, unlike that of charges and fields, there is no experimental data from which the laws of motion of the system can be inferred. The problem is resolved by means of the duality transformations which are introduced into the theory. These transformations are

$$e\to g;\qquad g\to -e \tag{6}$$

$$\tilde{F}^{\mu\nu}_{(m,w)}=\tfrac{1}{2}\varepsilon^{\mu\nu\alpha\beta}g_{\alpha\gamma}g_{\beta\delta}F^{\gamma\delta}_{(e,w)}$$

$$=\begin{pmatrix}0 & -B_x & -B_y & -B_z\\ B_x & 0 & E_z & -E_y\\ B_y & -E_z & 0 & E_x\\ B_z & E_y & -E_x & 0\end{pmatrix} \tag{7}$$

where e, g denote the strength of the electric charges and that of the magnetic monopoles, respectively. $\varepsilon^{\alpha\beta\gamma\delta}$ is the completely antisymmetric unit tensor of four indices. Applying duality to a system of charges, photons and fields, a dual system of monopoles, photons and fields is obtained. The equations of motion of this system are

$$\tilde{F}_{(m,w)\mu\nu,\lambda}+\tilde{F}_{(m,w)\nu\lambda,\mu}+\tilde{F}_{(m,w)\lambda\mu,\nu}=0 \tag{8}$$

$$\tilde{F}^{\mu\nu}_{(m,w),\nu}=-4\pi J^{\mu}_{(m)} \tag{9}$$

$$\frac{dP^{\mu}_{(m)}}{d\tau}=\tilde{F}^{\mu\nu}_{(m,w)}J_{(m)\nu}. \tag{10}$$

The fields $\tilde{F}_{(m,w)\mu\nu}$ of the dual world can be derived from 4-potentials B_{μ}

$$\tilde{F}_{(m,w)\mu\nu}=B_{(m,w)\nu,\mu}-B_{(m,w)\mu,\nu}. \tag{11}$$

The foregoing analysis can be put in the following words. Relying upon experiment, we have the equations of motion of classical electrodynamics of a system of charges and fields (1), (2) and (5). Applying duality, we have the equations of motion of a dual system of monopoles and fields (8), (9) and (10). Hence, both experiment and duality do not provide an answer to the problem of the form of the equations of motion of systems of the extended world which contain charges and monopoles. Before addressing this problem, let us analyse the nature of the electromagnetic fields of the known world and those of the dual world.

III. Measurements of the fields of a photon

It is assumed that photons of the two systems are indistinguishable. Fields of photons of the known world satisfy homogeneous Maxwell equations, where (2) takes the form

$$F^{\mu\nu}_{(w),\nu}=0. \tag{12}$$

It is well known that (12) is just a reformulation of the expression obtained from (8) after the subscript (m) is dropped. Similarly, the homogeneous equation obtained from (9)

$$\tilde{F}^{\mu\nu}_{(w),\nu}=0 \tag{13}$$

and that obtained from (1), after dropping the subscript (e), are identical.

Another aspect of the problem can be found in the examination of the invariant

$$I=\tfrac{1}{2}F^{\mu\nu}F_{\mu\nu}. \tag{14}$$

This invariant is negative for fields of a single charge and vanishes for fields of a single photon. In the special relativistic metric used above, (14) can be put in the form B^2-E^2. This expression vanishes also for fields of a photon of the dual world. Hence, the assumption that photons of the known world are the same as photons of the dual world is compatible with the structure of the equations of motion and with the test carried out by means of (14).

Let us take a photon and measure its fields at a space-time point x^{μ} by means of a charge. Assume that (5) yields

$$F^{\mu\nu}_{(w)} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{15}$$

If these fields are measured by means of a monopole then (10) yields the dual quantity

$$\tilde{F}^{\mu\nu}_{(w)} = \tfrac{1}{2}\varepsilon^{\mu\nu\alpha\beta} g_{\alpha\gamma} g_{\beta\delta} F^{\gamma\delta}_{(w)} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 \end{pmatrix}. \tag{16}$$

It follows that, in this case, the two different measurements yield the same values for the electromagnetic fields.

Let us repeat the measurements in a curved space-time. In this case one is unable to use the diagonal metric $(1, -1, -1, -1)$ in all space. In the following discussion, the metric $g_{\mu\nu}$ denotes a symmetric tensor used in general relativity. The equations of motion undergo some changes. The partial differentiation $,\mu$, is replaced by the covariant differentiation $;\mu$. The completely antisymmetric unit tensor of general relativity is [4, 5]

$$\begin{aligned} \eta^{\alpha\beta\gamma\delta} &= \varepsilon^{\alpha\beta\gamma\delta}/\sqrt{-g} \\ \eta_{\alpha\beta\gamma\delta} &= \sqrt{-g}\,\varepsilon_{\alpha\beta\gamma\delta} \end{aligned} \tag{17}$$

where $g = \det g_{\mu\nu}$. Using these expressions, the dual relations between the fields of a photon take the following form

$$\tilde{F}^{\mu\nu}_{(w)} = \tfrac{1}{2}\eta^{\mu\nu\alpha\beta} g_{\alpha\gamma} g_{\beta\delta} F^{\gamma\delta}_{(w)}. \tag{18}$$

Now assume that these fields are measured by means of a charge and by means of a monopole. The first measurement yields $F^{\mu\nu}$ and the second measurement yields $\tilde{F}^{\mu\nu}$ as written in (18). It is evident that the electric field of $\tilde{F}^{\mu\nu}$ of (18) is, generally, different from that of $F^{\mu\nu}$ which is written on the right hand side of this expression. The same statement holds for the magnetic fields of these tensors.

Let us take, for example, the Schwartzschild metric [6]. This metric, in terms of local Cartesian coordinates at an exterior point on the x-axis, is diagonal and takes the following form

$$g_{00} = \lambda, \qquad g_{11} = -1/\lambda, \qquad g_{22} = g_{33} = -1 \tag{19}$$

where $\lambda \neq 1$. In this case $\det g_{\mu\nu} = -1$ and $\eta^{\alpha\beta\gamma\delta} = \varepsilon^{\alpha\beta\gamma\delta}$.

Assume that, at this point, a measurement of the fields of a photon by means of a charge yields

$$F^{\mu\nu}_{(w)} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & \lambda & 0 \\ -1 & -\lambda & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{20}$$

This is an acceptable result and the fields satisfy the relation $F^{\mu\nu}F_{\mu\nu} = 0$. The analogous measurement, which is carried out by means of a monopole, yields the results obtained from the substitution of (19) and (20) in (18)

$$\tilde{F}^{\mu\nu}_{(w)} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \lambda \\ 0 & 0 & 0 & 0 \\ -1 & -\lambda & 0 & 0 \end{pmatrix}. \tag{21}$$

It is evident that neither the electric field of (21) nor its magnetic field are the same as their counterparts in (20).

It was mentioned above that physical quantities are related to measurements and that electromagnetic fields are measured by means of a charge or by means of a monopole. A comparison of (20) with (21) shows that, for a given photon, a charge and a monopole "see" different fields at the same point of space-time. This cannot happen if fields measured by means of a charge and fields measured by means of a monopole are the same physical entity. Using this conclusion, one can put the results of (15) and (16) in the following words. If the metric is $(1, -1, -1, -1)$ then measurements of the fields of the same photon by means of a charge and by means of a monopole yield the same values for the two *different* entities.

It is interesting to note that a charge and a monopole "see" the same energy-momentum of the photon's fields. This claim can be proved in the following way. Consider the energy-momentum tensor of the photon's fields in the Lorentzian metric $(1, -1, -1, -1)$. It is found that, in this metric, the energy-momentum tensor is invariant under the duality transformations (7). (For example, $T^{00} = (E^2 + B^2)/8\pi$; $T^{0i} = (\vec{E} \times \vec{B}/4\pi)_i$ etc.) The equality of these tensors in one frame guarantees their equality in all frames and the required result is proved. This is the point which is not satisfied by the fields tensors (4) and (7). The duality relations (7) show that, in the Lorentzian metric written above, $F^{\mu\nu} = \tilde{F}^{\alpha\beta}$ where $\mu\nu \neq \alpha\beta$. Hence, the equality of the fields in a Lorentzian metric cannot be extended to all frames.

IV. Approaches to charge-monopole interaction

Having the results of the previous section at hand, let us turn to a system of the extended world which consists of charges and monopoles. It was pointed out above that neither experiment nor duality provide an answer to the question of charge-monopole interaction. At this point two different courses of action can be taken. One may adopt another assumption and derive a theory of the system. Another course is to try to extract theoretical conclusions from the experimental data. Let us examine the theoretical approach.

A well known assumption says that fields associated with charges and fields associated with monopoles play the same role in the Lorentz force exerted on a charge (and on a monopole). The following expressions are obtained [7]

$$\frac{dP^{\mu}_{(e)}}{d\tau} = F^{\mu\nu}_{(e,m,w)} J_{(e)\nu} \tag{22}$$

$$\frac{dP^{\mu}_{(m)}}{d\tau} = \tilde{F}^{\mu\nu}_{(e,m,w)} J_{(m)\nu} \tag{23}$$

where $F^{\mu\nu}_{(e,m,w)} = F^{\mu\nu}_{(e,w)} - \tilde{\tilde{F}}^{\mu\nu}_{(m)}$. The irregular theory [7] derived from this assumption has no counterpart in ordinary classical electrodynamics.

The results of the analysis of electromagnetic fields in a curved space-time cast further doubts on the plausibility of this assumption. It is shown in the previous section that a photon has two different kinds of fields. One kind is measured by means of a charge and the other kind is measured by means of a monopole. This conclusion applies to fields of a photon, i.e.–a particle which belongs to the known world and to the dual world as well. Fields associated with monopoles belong only to the dual world and, like fields of photons of this world, they are not the same physical entity as fields associated with charges. Therefore, it is not clear why these different entities should have the same dynamical properties.

The noncovariant nature of quantum mechanical theories that follow (22) and (23) was already mentioned in the introduction [1]. The classical approach presented in this work shows also this aspect. Indeed, if one claims that fields measured by means of charges and fields measured by means of monopoles are identical then the tensorial relation (18) is violated and covariance fails. If, on the other hand, two kinds of fields are introduced then an incompatibility with the use of a single potential follows, because two different numerical sets cannot be directly derived from a single potential. In special relativity, where the metric $(1, -1, -1, -1)$ can be used in all space, one finds that the two kinds of fields take the same numerical values. However, the use of a single potential within the framework of special relativity violates the ordinary transition to the general theory were the tensorial relations are conserved.

Another point is the opposite parities of the electric fields associated with charges and the electric fields associated with monopoles. The addition of these kinds of fields in (22) and (23) shows that the equations do not conserve parity. An examination of the two kinds of magnetic fields yields the same results. A violation of the time reversal symmetry can be shown in a similar way.

The problem of the extended world can be approached differently. It is well known that the equations of motion of the known world can be derived by means of the variational principle [3]. An extention of the theory can be achieved if it is assumed that the variational principle holds for an electromagnetic system of the extended world [2]. It is shown that the application of this principle yields equations of motion of particles of the extended world which are not the same as (22) and (23). The following results are obtained

$$\frac{dP^{\mu}_{(e)}}{d\tau} = F^{\mu\nu}_{(e,w)} J_{(e)\nu} \tag{24}$$

$$\frac{dP^{\mu}_{(m)}}{d\tau}=\tilde{F}^{\mu\nu}_{(m,w)}J_{(m)\nu}. \tag{25}$$

The consequences of these expressions can be put in the following words. Fields associated with charges do not accelerate monopoles; fields associated with monopoles do not accelerate charges; charges and monopoles interact indirectly through the exchange of real photons.

It is shown that the theory derived in this way is regular and that its structure is an extention of the canonical structure of classical electrodynamics of the known world. A glance at (24) and (25) reveals that they are incompatible with (22) and (23). Hence, it is found that there is no Lagrangian function from which (22) and (23) can be derived. Related discussions can be found in the literature [8, 9].

It has been shown in the previous section that, in a curved space-time, a charge and a monopole "see" different fields of the same photon. The introduction of the variational principle as an element of the theory of the extended world uncovers another aspect of the different nature of electromagnetic fields. Fields associated with charges and fields associated with monopoles have different dynamical properties. Charges and monopoles "see" or do not "see" at all, the appropriate fields as written in (24) and (25).

The Lagrangian function from which (24) and (25) are derived contains no interaction term that is a contraction of the charges' currents with a potential associated with monopoles. Even in the presence of monopoles the charges' interaction term remains in the form $J^{\mu}_{(e)}A_{(e,w)\mu}$. Similarly, the interaction term of monopoles is $J^{\mu}_{(m)}B_{(m,w)\mu}$, where $B_{(m,w)\mu}$ is defined in (11). Therefore, if a quantum mechanical theory follows this Lagrangian and uses the appropriate minimal interactions written above, then it shows that charges do not interact with fields associated with monopoles and monopoles do not interact with fields associated with charges. This conclusion is a quantum mechanical analogue of (24) and (25). These kinds of interactions have direct implications on the experimental search for monopoles.

V. An examination of the experimental data

Let us leave the theoretical approach and examine the experimental data of monopole search. The experiments are carried out by means of instruments which are eventually based upon the interaction of charges with the fields of a monopole. Hence, according to (24) and (25), a monopole cannot trigger the recording apparatus.

A rather extensive search for monopoles, obeying (22) and (23), has been carried out in the last decades [10]. Many new reports of experimental search have been published since then [11–22]. A common conclusion of these last reports is the failure to detect monopoles.

Today there is only one report of a single monopole event which has not been refuted [23]. The statistical significance of this event decreases every day as negative results of other experiments continue to accumulate. Several authors have already suggested that this single event is due to a machine problem or that it is hard to be reconciled with their own results [11, 18, 20, 22]. It can be concluded that the experimental confirmation of (22) and (23) is very far from being established. This conclusion agrees with (24) and (25) which predict the failure of all experimental attempts to detect monopoles by means of the interaction of a charge with the fields of a monopole.

I. Conclusions

This work discusses systems which consist of charges and monopoles, within the framework of ordinary classical electrodynamics. Using metrics of general relativity, it shows that fields measured by means of a charge and fields measured by means of a monopole are not the same physical entity. Classical theories which use a common potential for the two kinds of fields are incompatible with this conclusion. These results are consistent with a new classical theory of systems of charges and monopoles [2]. This new theory is not in disagreement with experiment.

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