Renormalization group study of scalar field theories

Helvetica Physica Acta
Vol. 59, 1986 833-834

0018-0238/86/050833-02$1.50+0.20/0

RENORMALIZATION GROUP STUDY OF SCALAR FIELD THEORIES

Anna Hasenfratz
SCRI, Florida State University
Tallahassee, Fl. 32306, USA

Peter Hasenfratz
Institute for Theoretical Physics
University of Bern
Sidlerstrasse 5, CH-3012 Bern, Switzerland

November 1985

An approximate RG equation is derived and studied in scalar quantum field theories in d dimensions. The approximation allows an infinite number of different couplings in the potential, but excludes interactions containing derivatives. The resulting non-linear partial differential equation can be studied by simple means. Both the Gaussian and the non-Gaussian fixed points are described qualitatively correctly by the equation. The RG flows in d = 4 and the problem of defining an "effective" field theory are discussed in detail.

The scalar field theory is regularized in momentum space : The change of the effective potential $V(\varphi)$ under the RG transformation $\Lambda^{cut} \to e^{-t}\Lambda^{cut}$ is descrived by the approximate RG equation:

$$\left(1+\frac{\partial f}{\partial \varphi}\right)\frac{\partial f}{\partial t} = \frac{A_d}{2}\frac{\partial^2 f}{\partial \varphi^2} + \left(1+\frac{\partial f}{\partial \varphi}\right)\left[\left(1-\frac{d}{2}\right)\varphi\frac{\partial f}{\partial \varphi} + \left(1+\frac{d}{2}\right)f\right], \qquad (1)$$

where

$$f(\varphi,t) = \frac{\partial}{\partial \varphi}V(\varphi,t),$$

d is the dimension, A_d is a constant.[1] The equation has a non-trivial fixed-point solution corresponding to the usual fenomagnetic phase transition for $2<d<4$. In d=3 the leading and subleading critical indices ν and ω are predicted to be 0.687 and 0. 595 respectively, while $\eta=0$ is implied by the approximation. In d=4 only the Gaussian fixed-point exists ($f^*(\varphi)\equiv 0$) corresponding to a free field theory. The flow speed of the "almost marginal" operator ("φ^4" coupling) in d=4 can be determined both in the perturbative and in the non- perturbative regime.

The approximation and other aspects of the analysis are related to those of Wilson's recursion relation[2]. Eq (1) is most easily derived from the exact integro- differential functional equation of Wegner and Houghton[3].

[1] A. Hasenfratz, P. Hasenfratz, Bern Univ. Preprint BUTP-85/26

[2] K. Wilson, Phys.Rev. B4 (1971) 3184; Phys.Rev. D6 (1972) 419.

[3] F.J. Wegner and A. Houghton, Phys.Rev. A8 (1972) 401.