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A non relativistic supersymmetric two body equation for scalar and spinor particles

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Abstract. In this paper, we extend the bound-state equations for scalar and spinor particles according to the super-symmetric model of relativistic two-particle systems with Schrodinger's equation in a unified manner.

1. Introduction

In recent years, supersymmetry has been intensively used as a theoretical tool in connection with various models of elementary-particle physics. Supersymmetry ideas have also been widely applied in quantum-mechanics starting with the early work of Witten as an example of the simplest supersymmetric field theory [1]. At the beginning the motivation for studying supersymmetric quantum mechanics was the discussion of the underlying mechanism responsible for spontaneous SUSY (break-up) in arbitrary models. Since then the subject has seen continued interest and many articles constructing new and more realistic models have appeared, so that it is well developed by now [2–5].

In recent years, the possibility of extending super-symmetry to the study of three-dimensional two-particle systems was discussed by several authors. In a paper by Urrutia and Hernandez [6], a three-dimensional quantum-mechanical supersymmetric model which reproduces, as a particular case, the long-range behaviour of the nucleon-nucleon potential (one-pion exchange model) is proposed. A detailed survey of the quantum mechanics of this model, its scattering regime and the spontaneous supersymmetry (break-up) are discussed later by D'Olivio, Urrutia and Zertuche [7].

As is well known, the Beth–Salpeter equation is generally used to treat a relativistic two-particle problem in the framework of quantum field theory. Its supersymmetric extension and the analgoue of Wick-Cutkosky's model in particular are worked out in a paper by Delbourgo and Jarvis [8]. A supersymmetric generalization of the quasi-potential equation is considered by Zaikov [9–10]. Since even for the simplest scalar chiral superfields, fields with spin 0 and 1/2 are

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Editors' note

on the paper

"A non-relativistic supersymmetric two-body equation for scalar and spinor particles"

by Fu-bin Li, published in Helv. Phys. Acta 63, 922 (1990)

Profs. Z. Aydin and U. Yilmazer have informed us that the above-mentioned article is identical practically word by word with their own paper "Euclidean supersymmetry and relativistic two-body systems" which appeared almost three years ago in Nuovo Cimento 99 A, 85 (1988).

This fact escaped both the referee and the editor. We regret this unfortunate circumstance, and we cannot but condemn the inadmissible practice of claiming authorship of duplicated material.

contained in one multiplet, in the supersymmetric case, two-particle bound-state equations for scalar and spinor particles are written in a unified manner.

In a series of papers Crater and Van Alstine used Dirac's constraint mechanics and supersymmetry to obtain consistent descriptions of two interacting particles [11–16], either or both of which may have spin one-half. They made the naive quark model fully relativistic, and obtained a good one-parameter fit to the meson spectrum. By combining a supersymmetric description of a spinning particle in an external field with the Wheeler–Feynman dynamics they also constructed a repeatable relativistic dynamics for spin-1/2 and spinless particles in mutual scalar or vector interactions.

2. Euclidean supersymmetry

In general, supersymmetric quantum mechanics studies any quantum-mechanical model whose Hamiltonian can be written as

$$H = \tfrac{1}{2}\{Q, Q^+\} = \tfrac{1}{2}\{QQ^+ + Q^+Q). \tag{1}$$

where Q and Q^+ are spinorial charges satisfying

$$Q^2 = Q^{+2} = 0, \qquad [Q, H] = [Q^+, H] = 0 \tag{2}$$

SUSY quantum mechanics is thus a $(1+0)$-dimensional field theory, and consequently for the representations of super-fields and generators one uses a superspace consisting of only time t and a Grassman coordinate θ.

An alternative supersymmetric generalization of quantum-mechanical models is recently given in a paper by Sokatchev and Stoyanov [17], using a supersymmetric extension of the three-dimensional Euclidean symmetry. Their starting point is the supersymmetrization of the Schrödinger equation. Although this equation is a nonrelativistic one, the equations of motion for the physical components, after eliminating the auxiliary fields, are Lorentz's invariant, namely free Klein–Gordan and Dirac equations.

In this paper, adopting the principle of minimal coupling to the electromagnetic field, $p_\mu \to p_\mu - ieA_\mu$, in the original supersymmetric Schrödinger equation, we will be able to obtain relativistic wave equations for the particles interacting with external fields, we will extend this model to two-body systems in the last section.

As is pointed out above, the supersymmetry algebra in this alternative approach is not obtained by letting $d = 1$ in the super-Poincare algebra in four space-time dimensions, but it is the supersymmetric extension of the three-dimensional Euclidean group [17]. Let us, therefore, first discuss this extension. In a paper by Rembielinski and Tybor possible superkinematical [18] groups are listed. They extended the classification of the kinematical groups given by Bacry and Levy–Leblond [19] to the supersymmetric case. Let us write down the algebra satisfied by the generators of their super-Galilei group

$$\left.\begin{aligned}
&[J_k, J_l] = i\varepsilon_{klm} Jm, \qquad (\varepsilon_{123} = 1);\\
&[J_k, P_l] = i\varepsilon_{klm} P_m,\\
&[P_k, P_l] = 0,\\
&[J_k, Q_\alpha] = \frac{1}{2}\sigma^k_{\alpha\beta} Q_\beta,\\
&[P_k, Q_\alpha] = 0,\\
&\{Q_\alpha, Q_\beta\} = N(\sigma^l \varepsilon)_{\alpha\beta P_l},
\end{aligned}\right\} \tag{3}$$

where P_k's are three translation generators, J_k's are the 0(3) generators ($k = 1, 2, 3$) and Q_α ($\alpha = 1, 2$) are the supersymmetry generators. The normalization constant N which appears in the last relation is written so that algebra (3) is the same as that of ref. 17.

The generators P_k and Q_α can have the following differential equations:

$$P_k = -i\partial_k, \qquad Q_\alpha = i\frac{\partial}{\partial\theta^\alpha} + \frac{iN}{2}(\sigma^k\theta)_\alpha P_k, \tag{4}$$

and the covariant derivative

$$\mathscr{D}_\alpha = i\frac{\partial}{\partial\theta^\alpha} - \frac{iN}{2}(\sigma^k\theta)_\alpha P_k, \tag{5}$$

anticommutes with Q_α as usual $\{\mathscr{D}_\alpha, Q_\beta\} = 0$. It also satisfies

$$\{\mathscr{D}_\alpha, \mathscr{D}_\beta\} = N(\sigma^k\varepsilon)_{\alpha\beta} P_k. \tag{6}$$

The superfield $\phi(t, x; \theta)$ must be polynomial in θ

$$\phi(t, x; \theta) = A(t, x) + \theta^\alpha\psi_\alpha(t, x) + \theta^\alpha\theta_\alpha B(t, x), \tag{7}$$

and it will play the role of wave function in the model.

Now we write the supersymmetric Schrödinger equation for the particles interacting with external electromagnetic fields by modifying the similar equation of ref. 17 with the minimal substitutions as follows:

$$\left(i\frac{\partial}{\partial t} + e\Phi\right)\phi(t, x; \theta) = \frac{4}{N^2}\mathscr{D}^\alpha\mathscr{D}_\alpha\phi(t, x; \theta) - m\phi^+(t, x; \theta), \tag{8}$$

where P_k in the covariant derivative expressions are replaced by $\mathscr{D}_k = P_k + eA_k$, and m is the mass. Also we have

$$\phi^+(t, x; \theta) = \bar{A} + \theta^\alpha\bar{\psi}_\alpha + \theta^\alpha\theta_\alpha\bar{B}. \tag{9}$$

In order to obtain the equations of motion for the superfield components, one can make use of the following equations:

$$\left.\begin{aligned} &\phi(t, x; \theta)|_{\theta=0} = A(t, x), \\ &\mathscr{D}_\alpha \phi(t, x; \theta)|_{\theta=0} = i\psi_\alpha(t, x), \\ &\mathscr{D}^\alpha \mathscr{D}_\alpha \phi(t, x; \theta)|_{\theta=0} = 4B(t, x). \end{aligned}\right\} \tag{10}$$

Thus eq. (8) implies

$$\left(i\frac{\partial}{\partial t} + e\Phi\right)A = \frac{16}{N^2}B - m\bar{A}. \tag{11}$$

Applying the operator $\mathscr{D}_\alpha$ on both sides of (8), and using relation (6) we find with the help of projections (10)

$$\left(i\frac{\partial}{\partial t} + e\Phi\right)\psi_\alpha = \frac{4}{N}[\sigma^k(P_k + eA_k)\psi]_\alpha - m\bar{\psi}_\alpha. \tag{12}$$

Similarly, the application of $\mathscr{D}^\alpha\mathscr{D}_\alpha$ on (8), and use of the following identity:

$$\mathscr{D}^\alpha\mathscr{D}_\alpha\mathscr{D}^\beta\mathscr{D}_\beta = N^2\mathscr{D}^k\mathscr{D}_k, \tag{13}$$

leads to the equation of motion

$$\left(i\frac{\partial}{\partial t} + e\Phi\right)B = (P^k + eA^k)(P_k + eA_k)A - m\bar{B}. \tag{14}$$

Now we can easily eliminate auxiliary field B (and also $\bar{A}$) from eqs. (11) and (14). The result is (taking $N = 4$ for simplicity)

$$\left[\left(i\frac{\partial}{\partial t} + e\Phi\right)^2 + (P^k + eA^k)(P_k + eA_k) - m^2\right]A = 0, \tag{15}$$

which is a Klein–Gordon equation for a scalar particle interacting with external electromagnetic fields.

Equation of motion (12) for the spinor component of the superfield ϕ can be brought to the relativistic form

$$(i\partial\mu + eA_\mu)\sigma_\mu\psi + m\bar{\psi} = 0, \tag{16}$$

which is a Weyl equation, or a form of Dirac's equation

$$[(i\partial_\mu + eA_\mu)\gamma^\mu + m]\chi = 0, \tag{17}$$

for the spinor $\chi_\alpha = (\psi_\alpha/\bar{\psi}_\alpha)$. Thus, after using the equations of motion for the auxiliary field, the Lorentz invariance appears as a dynamical symmetry of this nonrelativistic quantum-mechanical model.

3. Extension of the model to two-particle systems

In the previous section we essentially followed the method of Ref. 17 in introducing the minimal electromagnetic coupling to the supersymmetric Schrödinger equation. Now we will extend this equation to a two-body case.

Let us first write down the expansion of the corresponding Bethe–Salpeter amplitude $\psi(x_1, x_2, t_1, t_2; \theta_1, \theta_2)$:

$$\begin{aligned}\psi(x_1, x_2, t_1, t_1; \theta_1, \theta_2) = {} & \psi(x_1, x_2, t_1, t_2; 0, 0) + \theta_1\psi(x_1, x_2, t_1, t_2; 1, 0)\\ & + \theta_2\psi(x_1, x_2, t_1, t_2; 0, 1) + \theta_1^2\psi(x_1, x_2, t_1, t_2; 2, 0)\\ & + \theta_2^2\psi(x_1, x_2, t_1, t_2; 0, 2) + \theta_1\theta_2\psi(x_1, x_2, t_1, t_2; 1, 1)\\ & + \theta_1^2\theta_2\psi(x_1, x_2, t_1, t_2; 2, 1) + \theta_1\theta_2^2\psi(x_1, x_2, t_1, t_2; 1, 2)\\ & + \theta_1^2\theta_2^2\psi(x_1, x_2, t_1, t_2; 2, 2),\end{aligned} \tag{18}$$

where $\psi(a, b)$ $(a, b = 0, 1, 2)$ are the components of the wave function and summation over the spinor indices is understood.

For the two-particle supersymmetric wave function we postulate the following equation which is a straightforward extension of the single-particle case:

$$\left(i\frac{\partial}{\partial t_2} + e\Phi_1\right)\left(i\frac{\partial}{\partial t_1} + e_1\Phi_2\right)\psi = \frac{16i}{N^4}\mathscr{D}_1^\alpha\mathscr{D}_{1\alpha}\mathscr{D}_2^\beta\mathscr{D}_{2\beta}\psi - m_1m_2\psi^+, \tag{19}$$

where t_1 and t_2 are the time parameters for each particle and could be taken as equal for instantaneous interaction. $e_1, m_1(e_2, m_2)$ are the charge and the mass of the first (second) particle. Φ_1 is the scalar potential due to the second particle at the place of the first one, $\Phi_1 = \Phi_1(x_2)$ (similarly $\Phi_2 = \Phi_2(x_1)$). The covariant derivatives $\mathscr{D}_1^\alpha$ and $\mathscr{D}_2^\alpha$ act in the subsuperspace of the first and second particle. Again P_{1k} and P_{2k} in the covariant-derivative expressions are replaced by $\mathscr{D}_{1k} = P_{1k} + e_1A_k(x_1)$ and $\mathscr{D}_{2k} = P_{2k} + e_2A_k(x_2)$, respectively.

Since the two-particle amplitude (18) contains nine superfield components we must write nine equations; however, not all $\psi(a, b)$'s are physical, some of them play nondynamical roles so that they have to be eliminated.

As in the single-particle case we can easily write the following projections by covariant-derivative operators:

$$\begin{aligned}&\psi|_{\theta_1=\theta_2=0} = \psi(0, 0), \qquad \mathscr{D}_{1\alpha}\psi|_{\theta_1=\theta_2=0} = i\psi_\alpha(1, 0),\\ &\mathscr{D}_1^\alpha\mathscr{D}_{1\alpha}\psi|_{\theta_1=\theta_2=0} = 4\psi(2, 0), \qquad \mathscr{D}_2^\alpha\psi|_{\theta_1=\theta_2=0} = i\psi_\alpha(0, 1),\\ &\mathscr{D}_2^\alpha\mathscr{D}_{2\alpha}\psi|_{\theta_1=\theta_2=0} = 4\psi(0, 2), \qquad \mathscr{D}_{1\alpha}\mathscr{D}_{2\beta}\psi|_{\theta_1=\theta_2=0} = -\psi_{\alpha\beta}(1, 1),\\ &\mathscr{D}_1^\alpha\mathscr{D}_{1\alpha}\mathscr{D}_{2\beta}\psi|_{\theta_1=\theta_2=0} = 4i\psi_\beta(2, 1), \qquad \mathscr{D}_{1\alpha}\mathscr{D}_2^\beta\mathscr{D}_{2\beta}\psi|_{\theta_1=\theta_2=0} = 4i\psi_\alpha(1, 2),\\ &\mathscr{D}_1^\alpha\mathscr{D}_{1\alpha}\mathscr{D}_2^\beta\mathscr{D}_{2\beta}\psi|_{\theta_1=\theta_2=0} = 16\psi(2, 2),\end{aligned} \tag{20}$$

where space and time variables are suppressed for simplicity. Applying $\mathscr{D}_{i\alpha}(i = 1, 2)$ or products of them on both sides of (19) and making use of relations (20) we finally obtain the coupled equations of motion for the components

$$\left(i\frac{\partial}{\partial t_2} + e_2\Phi_1\right)\left(i\frac{\partial}{\partial t_1} + e_1\Phi_2\right)\psi(0, 0) = \frac{(16)^2i}{N^4}\psi(2, 2) - m_1m_2\bar{\psi}(0, 0), \tag{21a}$$

$$\left(i\frac{\partial}{\partial t_2}+e_2\Phi_1\right)\left(i\frac{\partial}{\partial t_1}+e_1\Phi_2\right)\psi_\alpha(1,0) = -\frac{64i}{N^3}[\sigma_1^k\mathscr{D}_{1k}\psi(1,2)]_\alpha - m_1m_2\bar{\psi}_\alpha(1,0), \tag{21b}$$

$$\left(i\frac{\partial}{\partial t_2}+e_2\Phi_1\right)\left(i\frac{\partial}{\partial t_1}+e_1\Phi_2\right)\psi(2,0) = \frac{16i}{N^2}[\mathscr{D}_1^k\mathscr{D}_{1k}\psi(0,2)] - m_1m_2\bar{\psi}(2,0), \tag{21c}$$

$$\left(i\frac{\partial}{\partial t_2}+e_2\Phi_1\right)\left(i\frac{\partial}{\partial t_1}+e_1\Phi_2\right)\psi_\alpha(0,1) = -\frac{64i}{N^2}[\sigma_1^k\mathscr{D}_{2k}\psi(2,1)]_\alpha - m_1m_2\bar{\psi}_\alpha(0,1), \tag{21d}$$

$$\left(i\frac{\partial}{\partial t_2}+e_2\Phi_1\right)\left(i\frac{\partial}{\partial t_1}+e_1\Phi_2\right)\psi(0,2) = \frac{16i}{N^2}\mathscr{D}_2^k\mathscr{D}_{2k}\psi(2,0) - m_1m_2\bar{\psi}(0,2), \tag{21e}$$

$$\left(i\frac{\partial}{\partial t_2}+e_2\Phi_1\right)\left(i\frac{\partial}{\partial t_1}+e_1\Phi_2\right)\psi_{\alpha\beta}(1,1) = \frac{16i}{N^2}[\sigma_1^k\sigma_2^l\mathscr{D}_{1k}\mathscr{D}_{2l}\psi(1,1)]_{\alpha\beta} - m_1m_2\bar{\psi}_{\alpha\beta}(1,1) \tag{21f}$$

$$\left(i\frac{\partial}{\partial t_2}+e_2\Phi_1\right)\left(i\frac{\partial}{\partial t_1}+e_1\Phi_2\right)\psi_\alpha(1,2) = \frac{4i}{N}[\sigma_1^k\mathscr{D}_{1k}\mathscr{D}_2^l\mathscr{D}_{2l}\psi(1,0)]_\alpha - m_1m_2\bar{\psi}_\alpha(1,2), \tag{21g}$$

$$\left(i\frac{\partial}{\partial t_2}+e_2\Phi_1\right)\left(i\frac{\partial}{\partial t_1}+e_1\Phi_2\right)\psi(2,1) = \frac{4i}{N}[\mathscr{D}_1^k\mathscr{D}_{1k}\sigma_2^l\mathscr{D}_{2l}\psi(0,1)]_\alpha - m_1m_2\bar{\psi}_\alpha(2,1), \tag{21h}$$

$$\left(i\frac{\partial}{\partial t_2}+e_2\Phi_1\right)\left(i\frac{\partial}{\partial t_1}+e_1\Phi_2\right)\psi(2,2) = i\mathscr{D}_1^k\mathscr{D}_{1k}\mathscr{D}_2^l\mathscr{D}_{2l}\psi(0,0) - m_1, m_2\bar{\psi}(2,2), \tag{21i}$$

Obviously from eqs. (21a) and (21i) we can eliminate $\psi(2,2)$ (and also $\bar{\psi}(0,0)$), the result is

$$\left(i\frac{\partial}{\partial t_2}+e_2\Phi_1\right)^2\left(i\frac{\partial}{\partial t_1}+e_1\Phi_2\right)^2 + (p_1+e_1A)^2(p_2+e_2A)^2 - m_1^2m_2^2\Bigg]\psi(x_1,x_2,t_1,t_2;0,0) = 0 \tag{22}$$

which is the two-body equation for two scalar particles interacting with each other minimally, and much resembles the Bethe–Salpeter equation for the same system.

Similarly, couplings among the other equations can be removed easily. Thus from (21b) and (21g) we find

$$\left(i\frac{\partial}{\partial t_2}+e_2\Phi_1\right)^2\left(i\frac{\partial}{\partial t_1}+e_1\Phi_2\right)^2 - \sigma_1^k\mathscr{D}_{1k}\sigma_1^l\mathscr{D}_{1l}\mathscr{D}_2^m\mathscr{D}_{2m} - m_1^2m_2^2]\psi_\alpha(1,0) = 0. \tag{23}$$

Equations (21c)–(21e) and (21d)–(21h) give

$$\left\{\left(i\frac{\partial}{\partial t_2}+e_2\Phi_1\right)^2\left(i\frac{\partial}{\partial t_1}+e_1\Phi_2\right)^2 + [p_1+e_1A(x_1)]^2[p_2+e_2A(x_2)]^2 - m_1^2m_2^2\right\} \times \psi(2,0) = 0, \tag{24a}$$

$$\left[\left(i\frac{\partial}{\partial t_2}+e_2\Phi_1\right)^2\left(i\frac{\partial}{\partial t_1}+e_1\Phi_2\right)^2-\mathscr{D}_1^k\mathscr{D}_{1k}(\sigma_2^l\mathscr{D}_{2l})^2-m_1^2m_2^2\right]\psi_\alpha(0,1)=0, \quad (24b)$$

respectively, and are identical to the corresponding eqs. (22) and (23).

Equation (21f) for the tensorial component $\psi_{\alpha\beta}(1,1)$ may also be written in the form

$$[(i\partial_{1\mu}+e_1A_{2\mu})(i\partial_{2\nu}+e_2A_{1\nu})(\gamma_1^\mu\otimes\gamma_2^\nu)+m_1m_2]\psi(1,1)=0. \quad (25)$$

4. Conclusion and discussion

To summarize, we have proposed a nonrelativistic supersymmetry two-body equation which is essentially in the form of the product of Schrödinger's operators for each particle, and obtained the relativistic "Bethe–Salpeter-like" two-body equations for fermion-fermion, scaler-fermion and scalar-scalar systems. Out of the nine superfield components of the two-body wave function (18) only three of them are physical: the scalar component $\psi(0,0)$ for the spin 0-spin 0 system, the spinorial component $\psi_\alpha(1,0)$ for the spin 1/2-spin 0 system and finally the tensorial component $\psi_{\alpha\beta}(1,1)$ for the case of spin 1/2-spin 1/2 systems. The other two components $\psi(2,0)$ and $\psi(0,1)$ give no new information, and the last four components $\psi(2,2)$, $\psi(0,2)$, $\psi(1,2)$ and $\psi(2,1)$ are nonphysical. The elimination of these unphysical (auxiliary) fields from the nonrelativistic set of equations (21) gives us the relativistic equations for the physical components of the two-body superfield. Thus, one can conclude that the relativistic invariance for the physical component field may be considered as a dynamical symmetry of the nonrelativistic supersymmetric quantum-mechanical system.

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