QCD and Jets

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A review of jet physics at LEP.

1 Foreword

Happy birthday Henri, happy birthday Raoul. It is a great pleasure for me to participate in this happy and prestigious gathering.

I have known Henri for close to thirty years. We first met in Trieste, for a full-summer workshop, which once took place as a forerunner of what eventually became the ICTP under the leadership of Abdus Salam. This was in 1962. I think that everyone who participated in that workshop must, as I do, consider it as a very worthwhile and pleasant experience and also as the begining of several friendships, as mine with Henri. Henri's research work, which has successfully touched most of the topical aspects of the so prominent role of group theory in particle physics, is well known. I would like here to pay tribute to all that he also did for the success of the Département de Physique Théorique, here in Geneva, and for its development in full harmony with its very big neighbour at CERN. Henri deserves much praise for that. My own association with the University of Geneva, twice for one full academic year, in the framework of the Troisième Cycle de Suisse Romande, has been a very pleasant and interesting experience. It was the ocassion to better know Henri. It has also been a great pleasure for me to hear the brilliant talk of my former "assistant" in that capacity, Jean-Pierre Derendinger.

I met Raoul for the first time several years after I had met Henry but I had of course at that time a knowledge of part of his work. He was already famous. Many former students of Raoul have praised his prominent qualities as a researcher and as a teacher. I shall therefore here focus on something else, praising him as a fellow editor. Almost twenty years ago, in 1971, I handed Raoul something which could have been a poisonous gift, namely the editorship of Physics Letters B, for theoretical particle physics. I had then to look for a job and the editorship of Physics Reports, which was just starting as a new journal, was more than enough. I had been an editor for Physics Letters B for three years. This had been a big job. I could not guess that Raoul was picking it up for (already!) 19 years, during which the journal has become the success which it is today in our field of particle physics. It is well known that during the eighties, Europe has taken the lead worldwide in research in particle physics and now CERN serves by itself half of the world community. It is less known that, during the same period, Europe has also affirmed its leadership in publishing in particle physics. As a recent survey in the CERN Courier indicates, among the 30 most quoted papers of the eighties, those having already collected over 400 citations, one finds 23 of them published in the North Holland journals, including 12 american papers. In the

sixties Europeans were chafing under the desire to be published in the Physical Review or PRL. At present, Americans find it highly worthwhile to publish in European journals.

2 QCD and Jets

The subject of my talk is "QCD and jets". I shall talk about QCD almost only to the extent that it refers to jets. I shall discuss jets almost only in connection with the recent LEP results. Close to 800.000 Z decays have been observed during the first year of experimentation at LEP. This is a wonderful sample for jet analysis. QCD works as expected. This was far from obvious at the time of the " LEP summer study ", in 1978. The precision level now reached is of the order of 10%. This rapid survey is a summary of part of the rapporteur talk on " Jet physics and QCD", which I gave at the Singapore Conference, and which can be consulted for a more detailed and extensive status report on that subject [1]. It also includes in particular a discussion of jets in hadronic collisions where important new results have also been recently obtained and which I shall not touch here. As the early announcement has indicated, I thought initially of talking about "the vacuum in physics", but, when I realized the level of expertise of the other talks, I quickly switched to a more restrictive and technical title. Yet, this talk remains to a great extent a talk about the vacuum. We do not observe quarks and gluons because our too cold vacuum is opaque to the colour which they carry. Instead of the coloured quarks, antiquarks and gluons, shot into the vacuum in Z decay, we see jets of colourless hadrons, mainly π mesons. It takes about 1 GeV per fermi to penetrate the vacuum with a coloured object. The vacuum behaves with respect to the colour field much as a superconductor does with respect to a magnetic field. The penetration work involved is however such that nothing prevents the "stored" energy to "evaporate" as π mesons, which, as a jet of hadrons, altogether take away the energy and momentum of the primordial parton (quark, antiquark or gluon). This may be rather spectacular. Fig. 1 shows a three-jet event associated with the decay of a Z, as seen in the ALEPH detector at LEP. The q-qbar decay of the Z here involves the radiation of an energetic gluon. Quark, antiquark and gluon are seen clearly and almost separately as jets. The clarity of the event structure is such that jets offer an almost direct approach to the study of quark and gluon interactions at short distances. This is where QCD applies in a perturbative way. QCD thus provides the theoretical framework for jet analysis and conversely jet studies provide good tests of QCD.

Evidence for a jet structure in high energy collisions manifested itself about 15 years ago [2]. Whilst jets had been expected to occur in the framework of the parton model and were first analysed that way, QCD, with its gluon radiation, its scaling violations and the rationale provided by assymptotic freedom, soon appeared as offering the proper theoretical ground base [3]. With the huge energy liberated by Z decay, and the impressive statistics already available with LEP results only one year after the machine started, much progress could take place [4]. This is what I shall survey. However, whilst partons (quarks and gluons) can be almost identified with jets when they originate at high energy from an interaction at very short distance, this connection is neither perfect nor one to one. It may appear in a spectacular way as in Fig. 1. It remains nevertheless somewhat ambiguous. There are basic difficulties which cannot be fully overcome. Indeed most of this talk will be about
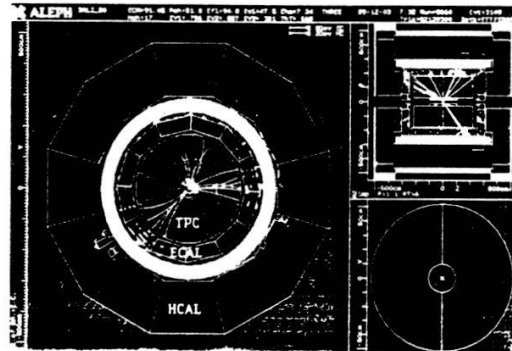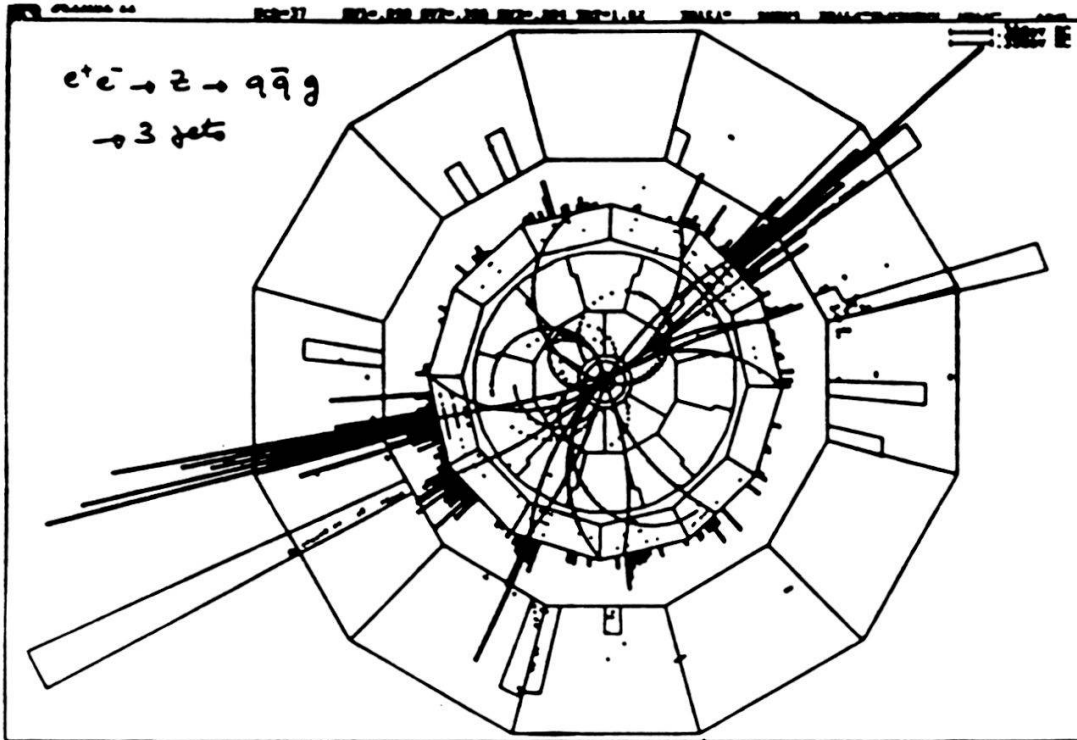
Figure 1: A three-jet event in Z decay (LEP-ALEPH).

what we mean by "almost" when we say that partons can be almost identified with jets. Some of these difficulties have long been known in QED, where radiative corrections and infrared problems have to be dealt with properly. The situation is just far more complicated with QCD. We also do not observe partons but hadrons which result from an hadronization procedure which we can only parametrize.

A few words about QCD are in order to set the stage.

QCD is a gauge theory based on the unbroken SU(3) colour gauge group. The definition of the renormalized coupling constant (α_s)

(i) Introduces a scale μ, which is arbitrary.

(ii) Depends on the renormalization scheme which is followed at that scale.

As is well known, having defined the coupling constant at a particular scale μ, one can obtain it at any scale Q, according to [3]:

$$\frac{\partial}{\partial \ln(Q^2/\mu^2)} \alpha_s(Q) = \beta(\alpha_s(Q)) . \tag{1}$$

Whilst the calculation of any quantity should not depend upon the scale μ at which one starts, a calculation at a fixed order in perturbation theory does. The dependence on the arbitrary scale μ (1) can, to a large extent, be traded for a depencence upon a physical scale Λ, which can be measured, and which sets the rate for the variation of α_s. One then writes [3]:

$$\alpha_s(Q) = \left(b_f \ln \frac{Q^2}{\Lambda^2}\right)^{-1} \left(1 - b'_f \frac{\ln \ln \frac{Q^2}{\Lambda^2}}{b_f \ln \frac{Q^2}{\Lambda^2}} + \cdots\right) , \tag{2}$$

where all the coefficients, which depend upon the number of active flavours f, are known. The value of Λ depends upon the renormalization scheme followed. To be precise, all values mentionned here refer to that known as "modified minimal subtraction with dimensional regularization", with Λ written more precisely as $\Lambda_{\overline{MS}}$. This being said, we shall merely write simply Λ from now on. From (2), it is clear than we can trade a change of scale introduced in the calculation of a particular order in perturbation theory for a modification of the contribution of the next order term. Accordingly, an experimental determination of Λ requires at least two terms in the perturbation expansion in α_s, the leading one and the next to leading one.

QCD has two prominent properties.

(i) It has asymptotic freedom, with, asymptotically:

$$\alpha_s(Q) \sim (\ln Q)^{-1} . \tag{3}$$

The coupling constant "runs" (1), and becomes vanishingly small asymptotically.

(ii) It should confine coloured quarks and gluons into colourless hadrons. At present we have only good indications that the theory implies confinement. We can merely aknowledge the fact replacing a system of partons by a system of hadrons whenever the relevant centre of mass energy becomes small (a fraction of a GeV say). This is done according to models developed from empirical rules which borrow as much as possible from a solidly based phenomenology [5]. When it comes to jet studies, the corresponding effects become weaker with increasing energy. At LEP energy most measurable parameters change but by a few percent
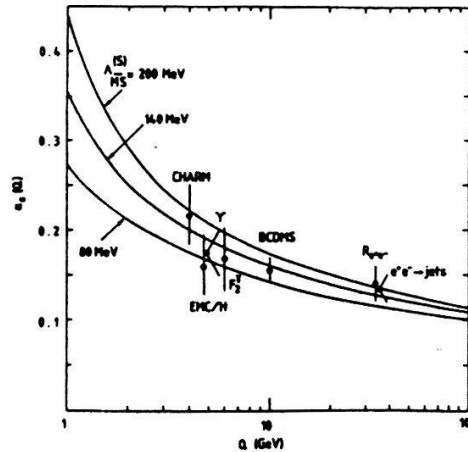
Figure 2: The QCD coupling constant as a function of the scale Q.

when going from a parton description to the related hadron description. This follows from the heavy Z mass. Indeed confinement is a soft process involving, in particular, only small transverse momenta with respect to that of the leading partons.

This is good news with respect to point (ii), which could have lead to big difficulties. However, with respect to point (i), where there are a priori no difficulty of principle, one has to aknowledge the fact that calculations become forbidingly difficult technically as one goes to higher orders in perturbation theory. All calculated quantities are so far limited to leading and next to leading orders in α_s. For most of them, going beyond that should imply new methods of calculation which are not yet available.

Is QCD the right theory for strong interactions?

(i) It is a theory with which we can, in principle, calculate everything. We have no alternative!

(ii) When data exist and calculations can be done, there is good agreement. We shall see many examples of that.

(iii) When data exist and calculation cannot be done yet, there is no clear conflict.

We are in a situation which recalls that of QED in the "pre-Lamb shift" era, when there was a good array of satisfactory results but not yet any single very precise one.

The pre-LEP legacy is summarized by Fig. 2, which combines the information on $\alpha_s(Q)$ in the 3 GeV $< Q <$ 50 GeV range [6]. Information comes from the measurement of the hadronic cross section in electron-positron annihilation, from jet studies at electron-positron colliders, from the analysis of scaling violations in deep inelastic scattering, from the study of quarkonium decay and of the photon structure function. The best value of Λ extracted from this experimental information is [6]:

$$\Lambda = 140 \pm 60 \text{ MeV} . \tag{4}$$

From which one predicts [6]:

$$\alpha_s(M_Z) = 0.11 \pm 0.01 \tag{5}$$

The logarithmic dependence of α on Q is such that the relatively large uncertainty on Λ results in only a 10% uncertainty on $\alpha_s(M_Z)$. Whilst no single experimental determination could be used to claim a running coupling constant, combining them all, one could build quite a good case for it.

3   Testing QCD on the Z

There are three types of tests.The goal is to verify the jet structure imposed by QCD and to measure the value of α_s at the Z mass, $\alpha_s(M_Z)$. One can study:

(i) The Z hadronic width.

In that case, one assumes that long-time physics (parton cascade evolution and hadronization) does not modify the global rate determined by short-time physics,which is given directly by the standard model. This is safe.

(ii) Jet structure.

One then assumes that the parton energy flow,which one can calculate in QCD,is not modified in any overwhelming way by the hadronization processes which can be described only according to phenomenological models. This is quite safe.

(iii) Hadronic yields and particle spectra.

One has then a priori to fully rely on models [5]. However one is helped by the fact that the parton-hadron transition is relatively local in phase space. Model dependence is therefore not too strong. One is lucky but this is deserved luck since it is based on solid phenomenology.

It is clear that theoretical reliability goes down as we move down that list. On the other hand the precision required from data in order to extract new information relevant for each separate entry varies the other way around! We have to make compromises.

3.1   The Z hadronic width

This is in principle the most direct test of QCD.The value of the width is equal to that given by the standard model to zeroth order in α_s times a correction factor which reads:

$$R = 1 + \frac{\alpha_s}{\pi} + 1.411 \left(\frac{\alpha_s}{\pi}\right)^2 + r_3 \left(\frac{\alpha_s}{\pi}\right)^3 + \cdots . \tag{6}$$

Its value (to second order) is now taken as:

$$R = 1.040 \pm 0.004 \tag{7}$$

This is a sizeable effect.The third order term was once calculated and found embarassly big. This now turns out to be a false alarm. Its value should be known soon and one may hope that it will be reasonably small [1]. Present information on the correction factor R at LEP is summarized in Fig. 3. These results are compatible with the expected value (5), but they do not provided a precise test. One can only conclude that $\alpha_s = 0.18 + 0.08$ [7]. An accurate determination of $\alpha_s(M_Z)$ through this most direct measurement is still too demanding for the experimental precision achieved so far. At present a more precise value can be obtained from jet analysis. Yet a significant reduction of the experimental error on R should be expected.

3.2   Jet structure

The jet structure is not always as clear as on Fig. 1. One thus gain at describing the global jet structure through "shape parameters" which do not imply a precise definition of what a jet is. One can parametrize the particle-energy flow with the values of various parameters
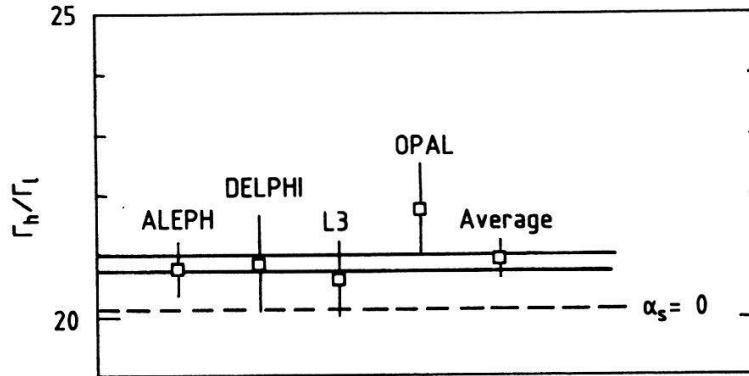
Figure 3: The ratio between the hadronic width and the leptonic width of the Z. The band corresponds to present expectations.

which vary according to the "jettyness" and "planarity" of the events. These parameters can be measured experimentally from the particle distribution observed. They can also be calculated from modelized event generations. They may then refer either to the parton distribution or to the hadron distribution, obtained from the former with the help of a particular hadronization model. Wether considering actual or generated events, one should choose shape parametrers which are "infrared stable" and " collinear stable". They should not change under the radiation of a soft or collinear gluon. In the latter case (generated events), these are required properties so that the calculation makes sense in perturbative QCD, before hadronization is implemented. In the former case (actual data), this eliminates experimental ambiguities separating out nearby tracks or treating soft particles. There are many such parameters. They have names Thrust (T), Major (M), Minor (m), Oblateness (O), C, D, EEC, $AEEC$... [1]. The one before last refers for instance to the two correlated energy flows measured at an angle v of each other, averaged over all events and normalized to the centre of mass energy square. When one takes the difference (asymmetry) between the values at $\pi - v$ and at v, one obtains the last one, AEEC. All parameters depend on α_s, though at various orders.

Let us assume that all these quantities have been measured experimentally. How can they be determined theoretically?

The most direct approach consists in calculating partonic decay amplitudes to increasing order in perturbation theory. This is the well defined "matrix element" approach. The calculation directly involves α_s which can therefore be measured through a fit to the experimental values. However, one is presently limited to a second order calculation in perturbation theory. The maximum complexity corresponds therefore to four partons and it is then calculated to the tree-approximation level only. The full complexity of the events is likely not to be well reproduced. One can in any case modelize the parton-hadron transition. To that end, one can transform low invariant mass sets of partons into clusters of hadrons (HERWIG) or associate sets of hadrons to the colour strings extending between partons (LUND), just to mention two of the different and often followed approaches [5]. At LEP energy this does not change very much the numerical values obtained for the shape parameters (a few percents). One can thus obtain the differential distributions associated with some of the parameters and an array of mean values and distributions.

Such a four-parton maximum complexity falls however short of what is often found at
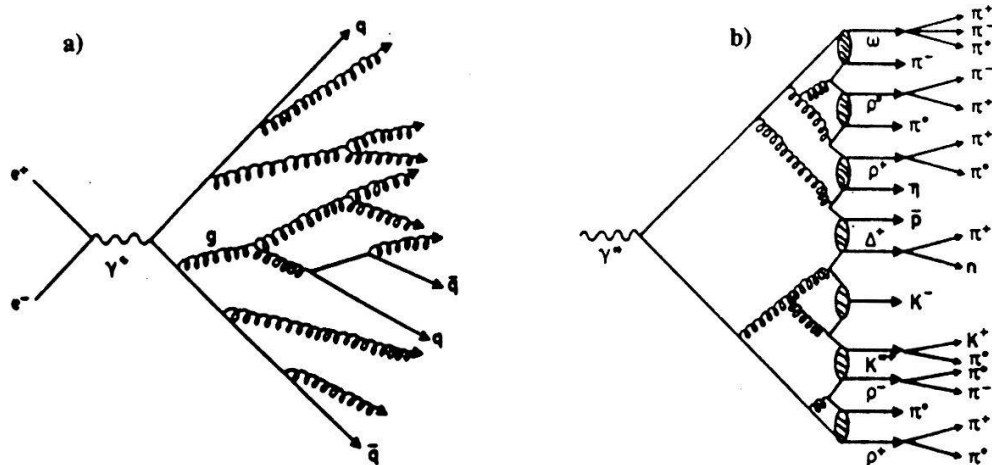
Figure 4: Partonic cascade. (a) Before hadronization, (b) with hadronization.

LEP energy. One is therefore pushed to go further in order to better determine the shape parameters. One can then calculate the partonic cascade using QCD in the double and single leading log approximations, summing all terms where α_s is associated with a large logarithm. This is done through Monte Carlo models which combine the probabilities for a parton not to have branched (Sudakov),with that for it to branch (Gribov-Lipatov, Altarelli-Parisi). This is illustrated by Fig. 4 -a. One can thus reach a rather high parton complexity (up to 8 say at LEP energy), which is a much better match to the actual complexity of the events, as described by the values of a full array of shape parameters. However the cascade (Fig. 4 -a) is then calculated in term of an effective Λ. It is not calculated as an amplitude but as a combination of probabilities. One therefore cannot any longer use a fit to the experimental values to determine $\alpha_s(M_Z)$. Partons can again be transformed into hadrons according to clusters (HERWIG), as illustrated by Fig. 4 -b, or according to strings (JETSET). This is included in the Monte Carlo programmes [5].

One can be tempted to combine the matrix element approach with a partonic Monte Carlo cascade in order to get the best of both worlds. One may improve the fits but one should not fool oneself. When it comes to a determination of $\alpha_s(M_Z)$, one cannot get more than there is in a second order calculation!

A choice has to be made. If one wishes to reproduce very well the event structure, a cascade calculation is necessary. One then cannot actually determine the coupling at the Z. If one wish to measure α_s at the Z, a matrix element calculation has to be followed. One cannot then hope to reproduce the details of the event structure.

The LEP experiments have been very quick and efficient at collecting and assessing information. They were of course helped by an extensive theoretical preparation and by the experience collected at PEP and PETRA [8].

Fig. 5 shows how events become more jetty and more planar, going from PEP energy (29 GeV) to LEP energy (91 GeV) and how a parton shower calculation provides a much better fit than a matrix element one. Fig. 6 illustates with the thrust distribution at LEP how parton shower models have converge as they have developed, adapted and improved. They indeed now all include the QCD evolution parton shower of Fig. 4 -a, which is the key ingredient in reproducing the energy flow. Models have of course adjustable parameters which can be best tuned to a particular set of data. Their predictive power should however not be undermined.
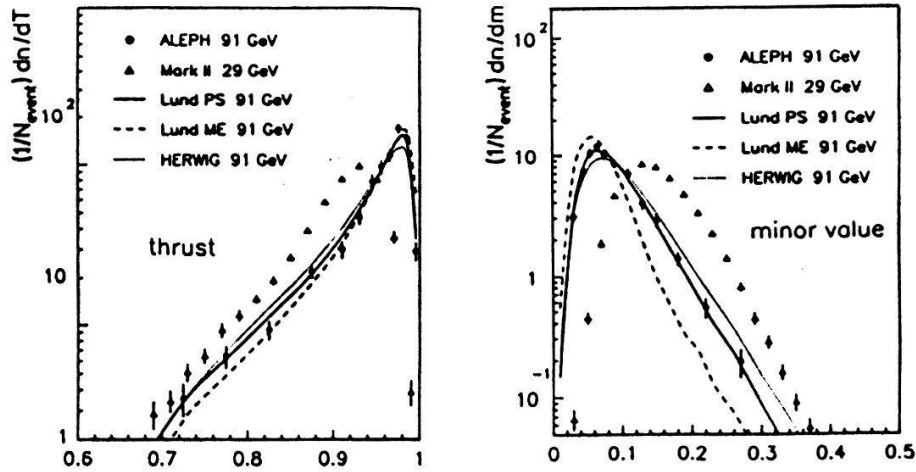
Figure 5: Thrust and Minor distributions at LEP (91 GeV) and at PEP (29 GeV). Note the success of cascade models and the shortcoming of a matrix element approach.
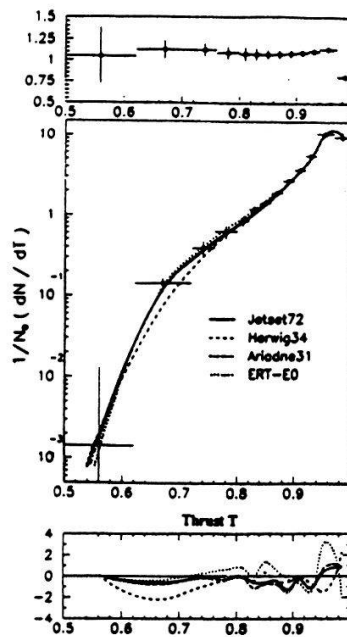

Figure 6: The convergence and success of model calculation. (Thrust distributions at the Z mass).
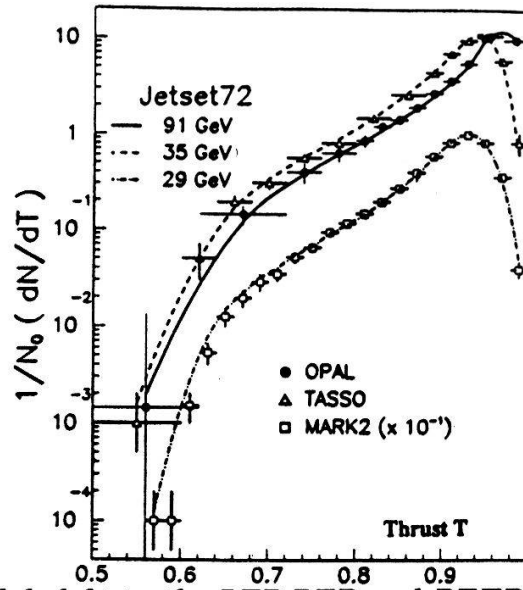
Figure 7: A global fit to the LEP,PEP and PETRA results.

Fig. 7 shows the thrust distribution fine-fitted at LEP and then predicted at PEP and PETRA energy, not changing the value of any adjustable parameter. The achievements of present models are already impressive. They will further improve in complexity and precision when high statistics data are available on baryon pair production and on heavy flavours production.

Having a good prediction of the event structure, one can turn to a determination of $\alpha_s(M_Z)$. For the distribution of some of the shape parameters, which start as α, namely $1 - T$, O, C...., one can write a generic perturbation expansion [8]:

$$\frac{1}{\sigma}U\frac{d\sigma}{dU} = \frac{\alpha_s(\mu)}{2\pi}U_0 + \left[\frac{\alpha_s(\mu)}{2\pi}\right]^2 \left[\frac{U_0}{2}b_f \ln\left(\frac{\mu^2}{s}\right) + U_1\right] + \cdots . \tag{8}$$

The lowest order term is scale dependent. So is the second one but the coefficient now brings a correction according to the scale chosen. All coefficients are known. The full calculation, to all orders, should not depend upon the scale μ but, at second order, we have to aknowledge a possible dependence. Fig. 8 then shows the values thus obtained for the mean values of $1 - T$, O and C as functions of the annihilation energy, E, to leading order (LO), and to next to leading order (NL) [9]. In each of the cases, the width translates the variations associated with a change in scale μ, varying it between $E/4$ and E. One may be tempted to conclude that some parameters are safer than others which could then be disregarded. The correct attitude is however to take them all and to collect confidence from the fact that they all give converging values for $\alpha_s(M_Z)$. This is shown in Fig. 9 which puts together the values found at leading order and at next to leading order [9]. In the latter case one gets a conforting convergence. Also shown is the relative rate for 3-jets (R_3), to which we shall later turn. Most of the quoted errors corresponds to the scale ambiguity.

At present there still seems to be a marked preference for AEEC. Using an asymmetry is of course better since some errors cancel out. It also appears that in that case there is but a rather weak scale dependence. DELPHI and OPAL used it to reach rather precise values
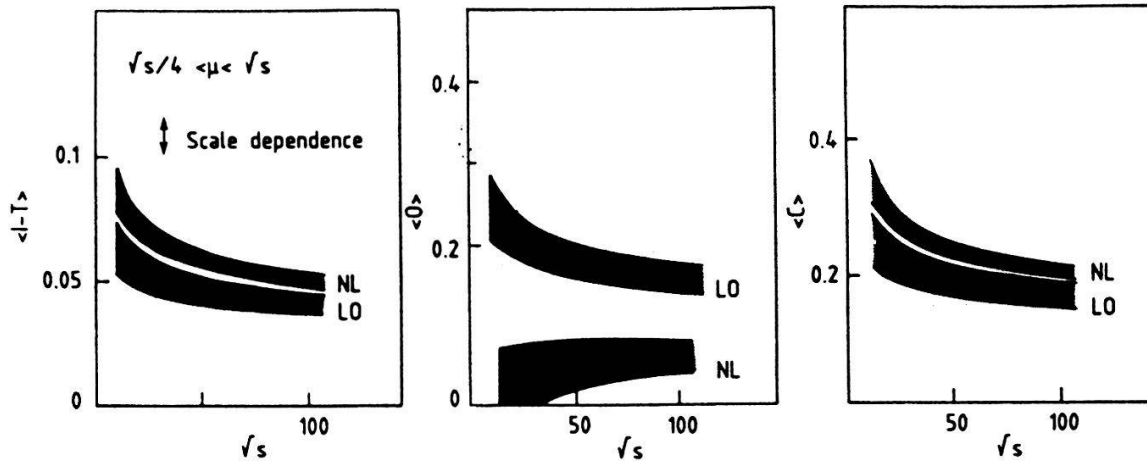
Figure 8: The determination of the mean value of some shape parameters in perturbation theory (Ref [9]): as a function of annihilation energy, varying the scale (leading and next to leading orders).
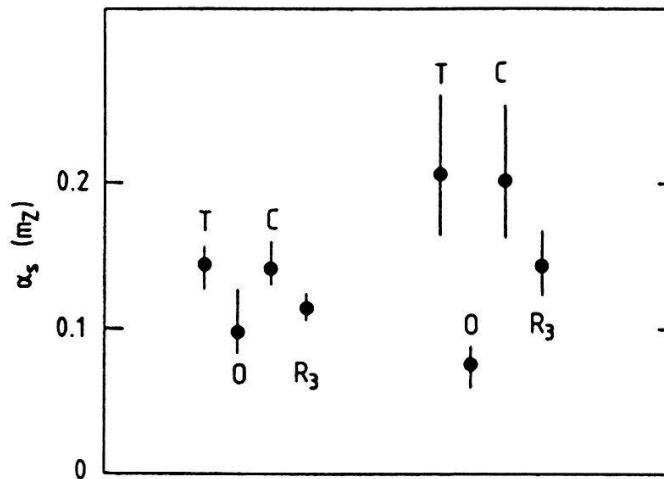

Figure 9: The determination of the mean value of some shape parameters in perturbation theory (Ref [9]): the values at LEP energy, leading order (right) and next to leading order (left), together with the three-jet relative rate R_3.
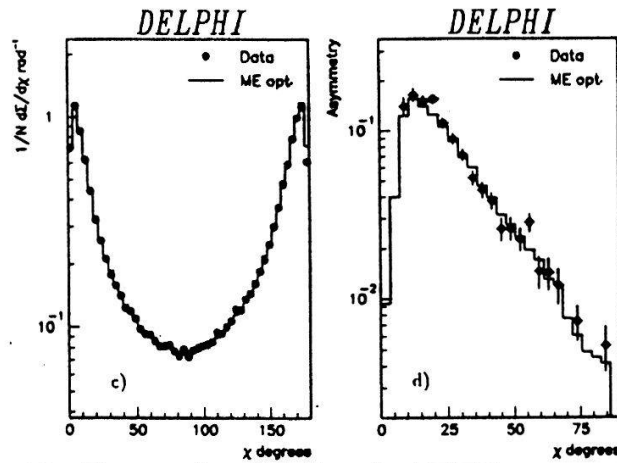
Figure 10: Fits to the *EEC* and *AEEC* parameters (DELPHI).

of $\alpha_s(M_Z)$, namely [10]:

$$\text{DELPHI} \qquad \alpha_s(M_Z) = 0.106 \pm 0.003 \pm 0.006 \pm 0.003$$

$$\text{OPAL} \qquad \alpha_s(M_Z) = 0.116 \pm 0.014 \left\{ \begin{array}{c} +0.011 \\ -0.019 \end{array} \right. \tag{9}$$

Such values correspond actually to a determination of Λ, with

$$\text{DELPHI} \qquad \Lambda = 104 \pm 25 \left\{ \begin{array}{c} +45 \\ -10 \end{array} \right. \pm 30 \text{ MeV} . \tag{10}$$

These values are both in agreement with the expected one [4],[5]. Fig. 10 shows the impressive fit obtained by DELPHI. Yet, at present, there may be some missplaced optimism in the estimation of the "theoretical" error. The fit actually combines a matrix element approach with a partonic shower and,whilst there is a stability with respect to the scale μ, the best fit appears at the limit of this stability range. I therefore prefer to stress again, at this stage, the compatibility between values extracted from different parameters calculated in the matrix element approach. The present safest determination of $\alpha_s(M_Z)$ has then still to be found elsewhere. This is with jet counting.

3.3   Jet counting

With shape parameters we could avoid a precise definition of a jet. With jet counting we cannot. We therefore face new ambiguities. Yet this is at present the safest way to determine $\alpha_s(M_Z)$ most precisely and all four LEP experiment have converging results on that [4]. Jet counting has to rely on a particular algorithm. We have to define how we group particles in an observed event into different jets and also how we group partons, issued from a calculation, into jets, before hadronization, or after it. The most convenient cluster algorithm is the one developed at PETRA [8][11]. For each event, one calculates the centre of mass energy square of any pair of two particles i, j, scaled to the full energy square. These quantities are called y_{ij}. If the lowest y_{ij} is smaller than a certain value, y_{cut}, one replaces the two corresponding particles i and j by a pseudo particle with momentum $p_i + p_j$. One then repeats the process
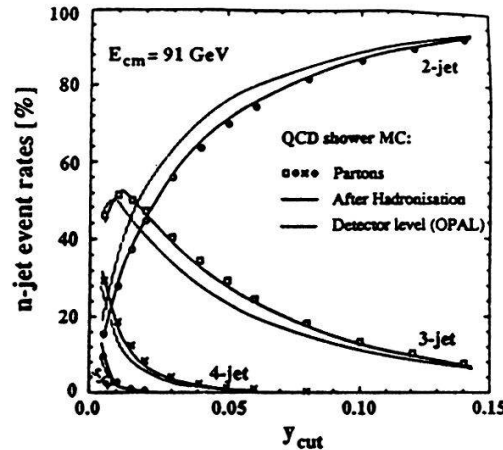
Figure 11: Jet counting as a function of the invariant mass cut. Jet simulation (OPAL). Parton level (dots), hadron level (solid curves) and detector hits (dashed curves).

with particles and pseudo particles until one exceeds y_{cut}. The corresponding pseudo particle is then called a jet. After all initial particles have been dealt with, one counts the jets. The procedure can be equaly well applied to particles and to partons. In both cases there is however an ambiguity since we begin with particles (partons) of practically zero mass but start taking them on the same footing as pseudo particles with a mass. This of course reflects the basic problem that the fragmentation of a massless parton into a jet (Fig. 4 -a) does not fully conserve momentum and energy. It conserves either of them, or both, only in some approximate way. One can then follow different recombination schemes, forcing or not the reconstructed pseudo particles to a massless kinematics. In each case one may end with a different number of jets when all particles have been included in different clusters with masses limited by y_{cut}. The morale is to follow different equaly approximate schemes and assess the error from the dispersion obtained. There are four popular ones [1],[12]. In the E_0 one, for instance, the energy of the pseudoparticle is the sum of the two energies but the momentum is then rescaled according to zero mass kinematics.

Fig. 11 shows the number of jets obtained as a function of y_{cut} in a Monte Carlo programme (OPAL). The dots correspond to a clustering of partons. The solid curves correspond to the same process after hadronization and the dashed curves to the results after the response of the detector has been included. There is here little difference between the first and second stages but this is scheme dependent. The E_0 scheme is used in that case. One sees that, as expected, the number of jets depends upon y_{cut}. As y_{cut} decreases, some events first labelled as two-jet become three-jet events. Some three-jet events eventually feed the four-jet sample and so on and so forth.. One could be inclined to take a rather small value of y_{cut} in order to better match the full complexity of the event. However, if one takes too small a value, one gets mired into collinear problems. A compromise has to be found. Fig. 12 -a shows the OPAL data (dots). The distribution looks very similar to that of Fig. 11. This is conforting. The respective rate for 2,3 and 4 jets can be calculated in perturbation theory [13]

$$
\begin{aligned}
R_2(y) &= 1 + C_1^{(2)}(y)\alpha_s(\mu) + C_2^{(2)}(y,\mu)\alpha_s^2(\mu) + \cdots, \\
R_3(y) &= \quad\quad C_1^{(3)}(y)\alpha_s(\mu) + C_2^{(3)}(y,\mu)\alpha_s^2(\mu) + \cdots, \\
R_4(y) &= \quad\quad\quad\quad\quad\quad C_2^{(4)}(y,\mu)\alpha_s^2(\mu) + \cdots.
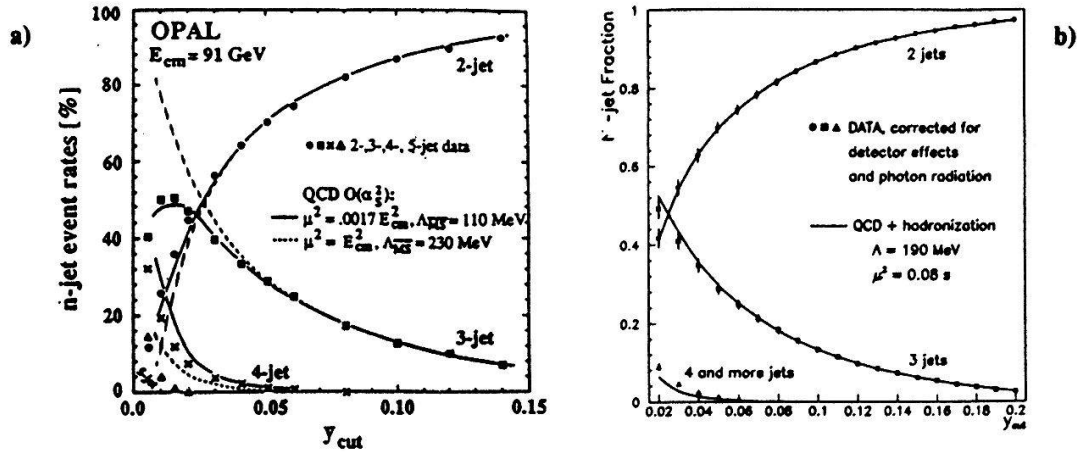\end{aligned}
\tag{11}
$$

Figure 12: Jet counting in "real life". (a) OPAL results with fits with $f = 1$ (dashed curves) and using f as a parameter (solid curves). (b) L3 fit with $f = 0.08$.

Here we see again that, whilst the full result should be independent of the scale μ, at which one defines α_s, the second order result depends on it. The next to leading order coefficients indeed depend on μ through the quantity $f = s/\mu^2$, in an attempt to compensate for the freedom of scale. It is most natural to take $\mu = M_Z$. The calculation then gives the dashed curves in Fig. 12. The data are well reproduced as long as y_{cut} is not too small ($y_{cut} > 0.04$). For smaller values, one gets too few 4-jet events and, accordingly too many three- jet ones. One may be tempted to remedy that playing on the scale, in the hope that this can mimick, to some extent, the contribution of higher order terms. Varying f down from 1, one can indeed obtain a much better fit. This correspond to the solid curve in Fig. 12 -a. The problem is that this is here achieved with a rather low value of f, $f = 0.0017$! The corresponding value of the scale is still reasonable to justify a perturbation approach but it is ambarassly small since we could expect f to remain a priori close to 1. Yet the fits obtained are indeed impressive. Fig. 12 -b gives the L3 result with now $f = 0.08$. This is still small! One should however not overplay these beautiful fits. It is clear that lowering f, or decreasing μ, one increases the effective value of α_s. This increases the probability to get four jets, at the expense then to that to get three. This is indeed what is needed when comparing the dashed curve of Fig. 12 -a to the data (dots). It may however be missplaced optimism to reproduce the 4-jet rate starting from a perturbation calculation which gives it to the tree approximation only. It may also be too bold to reproduce the 3-jet rate down to small values of y_{cut}, when the definition of a jet becomes then difficult because of collinear problems. A full cascade calculation would be necessary but a perturbation calculation in term of α_s only would then be lost. It therefore seems more modest but more reasonable to limit oneself to a moderate value of y_{cut}, $y_{cut} = 0.08$ say, and not to try to match the data with more accuracy when comparing them to a perturbative calculation to second order. In that case a value of f of order one is, as expected, satisfactory. At the same time however, one has to limit the analysis to the two and three jet events, focussing on R_3, the probability to have three jets.

The different probabilities R_2, R_3, R_4, $\cdots$ have to sum up to one and are therefore not independent. Each event enter also several times in Fig. 12 as y_{cut} cut is varied. Rather that measuring R_2, R_3, $\cdots$, one therefore prefers to measure independent quantities such
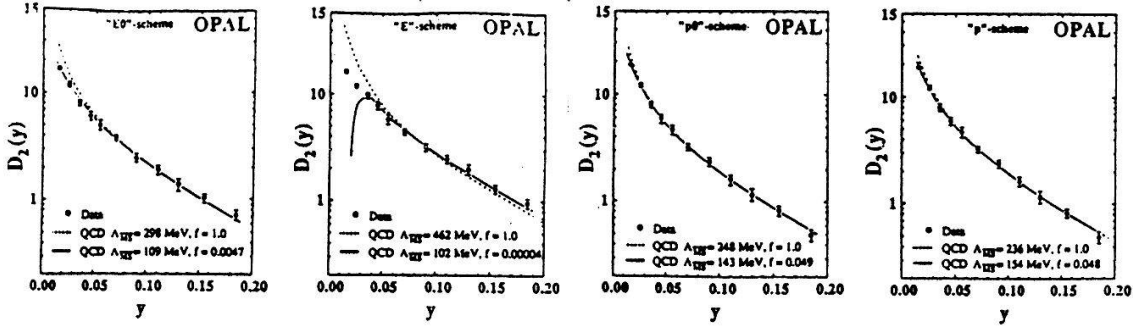
Figure 13: The parameter D_2 fitted from the jet reconstruction obtained according to four different recombination schemes. In each case f is either kept at 1 or fitted to reproduce better the behaviour at low y. The acknowledged error on the scale parameter Λ comes mainly from the open choice of recombination scheme.

as $D_2(y)$

$$D_2(y) = \frac{R_2(y) - R_2(y - \Delta y)}{\Delta y} \quad \text{at} \quad y = y_{\text{cut}} \qquad (12)$$

This is the rate at which 2-jet configurations turn into 3-jet configurations as a function of y (short for y_{cut}). The data (OPAL) are compared to calculation (9) in Fig. 13. In each case, corresponding to a particular recombination scheme, a fit is obtained with $f = 1$ and also opimized according to f, with, as already mentionned, a rather low value of f imposed by the fit at low y. In each case one determines the corresponding value of Λ or of $\alpha_s(M_Z)$. The error on Λ deduced from each particular fit is relatively low. Yet the values obtained differ far more than allowed within such error bars according to the scheme which is followed. This is this larger dispersion which gives the present error to be conceded on the determination of Λ and therefore $\alpha_s(M_Z)$. The 10% error which we now have on α results indeed mainly from the ambiguity which we have when defining a jet according to a particular algorithm. Whilst we have used OPAL results to illustrate the procedure (Fig.s 12 -a and 13), all four LEP experiments have converging results. Quoting for all $\alpha_s(M_Z)$, one has [4],[19]:

$$\text{OPAL}: \qquad \alpha_s(M_Z) = 0.118 \pm 0.003 \pm 0.004 \pm 0.007$$

$$\text{DELPHI}: \qquad \alpha_s(M_Z) = 0.114 \pm 0.005 \pm 0.003 \pm 0.011$$

$$\text{L3}: \qquad \alpha_s(M_Z) = 0.115 \pm 0.005 \pm 0.003 \pm 0.010$$

$$\text{ALEPH}: \qquad \alpha_s(M_Z) = 0.121 \pm 0.004 \pm 0.007 \pm 0.008$$

$$\text{with an average of}: \qquad \alpha_s(M_Z) = 0.119 \pm 0.008 \qquad (13)$$

The quoted errors are experimental, due to hadronization, due to scale.

All these results agree among themselves. They also agree with the expected value (5). The coupling constants behaves as predicted. Whilst SLC does not have the statistics of LEP, the direct comparison of the MARK2 data at PEP and SLC is highly worthwhile. There is good agreement with the more precise value found at LEP [1].

Since R_3 (at $y = 0.08$) is to a good approximation proportional to α_s, one may use it to illustrate its running, combining PEP,PETRA,TRISTAN and LEP(SLC) results. This
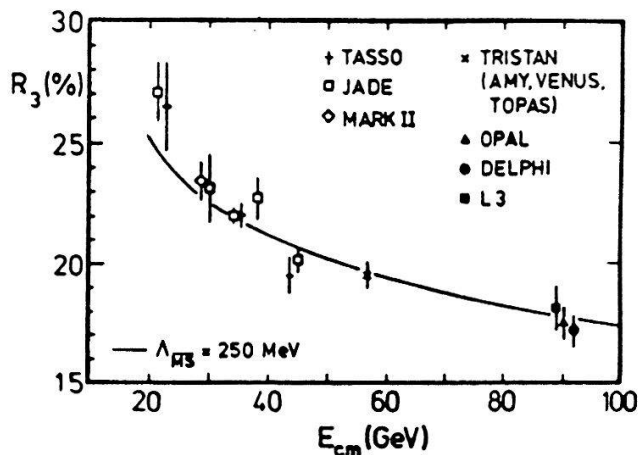
Figure 14: The three-jet relative rate as a function of annihilation energy. As a function of E.

is shown in Fig. 14. The solid curve corresponds here to a running according to a Λ value of 250 MeV. One may illustrate the asymptotic freedom which this implies displaying the results according to $1/\ln(E)$. In this case aymptopia is at the origin. The data more than suggest asymptotic freedom and the new LEP data are instrumental at that. This is shown in Fig. 15. This concludes what can be done now with jet counting.

3.4   The three-gluon coupling

One contribution relevant to the four-jet configuration involves the three-gluon coupling. It is shown in Fig. 16 -a. Its relative importance can be amplified through some specific choice of variables. This is in particular the case for the Nachtmann-Reiter angle, as defined in Fig. 16 -b. Fig. 16 -c gives the observed and predicted double distribution for the two angles defined by Fig. 16 -b. The measurements are those of DELPHI. From the overall agreement found, one can conclude that the famous QCD ratio of 9/4, which determines the predicted strength of the three-gluon coupling is compatible with the experimental value $2.05 \pm 0.4 + 0.7(-0.1) \pm 0.4$. OPAL and L3 have similar results [14]. This is a further test of QCD, not yet very precise but which will be refined more with increased statistics. One can but test that experimental distributions are in agreement with QCD. There are no alternative theory.

3.5   Particle spectra

The calculation of particle spectra involves hadronization models but, at LEP energy, the key ingredient turns out to be the partonic cascade (Fig. 4 -a), which can be calculated in the modified leading log approximation, summing double and single leading log contributions. An extensive amount of theoretical work has been done along that line [15]. Coherence effects imposed by QCD lead to a remarkable suppression of the production of soft partons. A soft parton may indeed originates coherently from different colour sources. One can efficiently implement the effects of destructive interferences imposing angular ordering, successsive partons being produced with lower and lower angles down the fragmentation chain of a primordial parton. At the same time however, parton production time, which has to remain
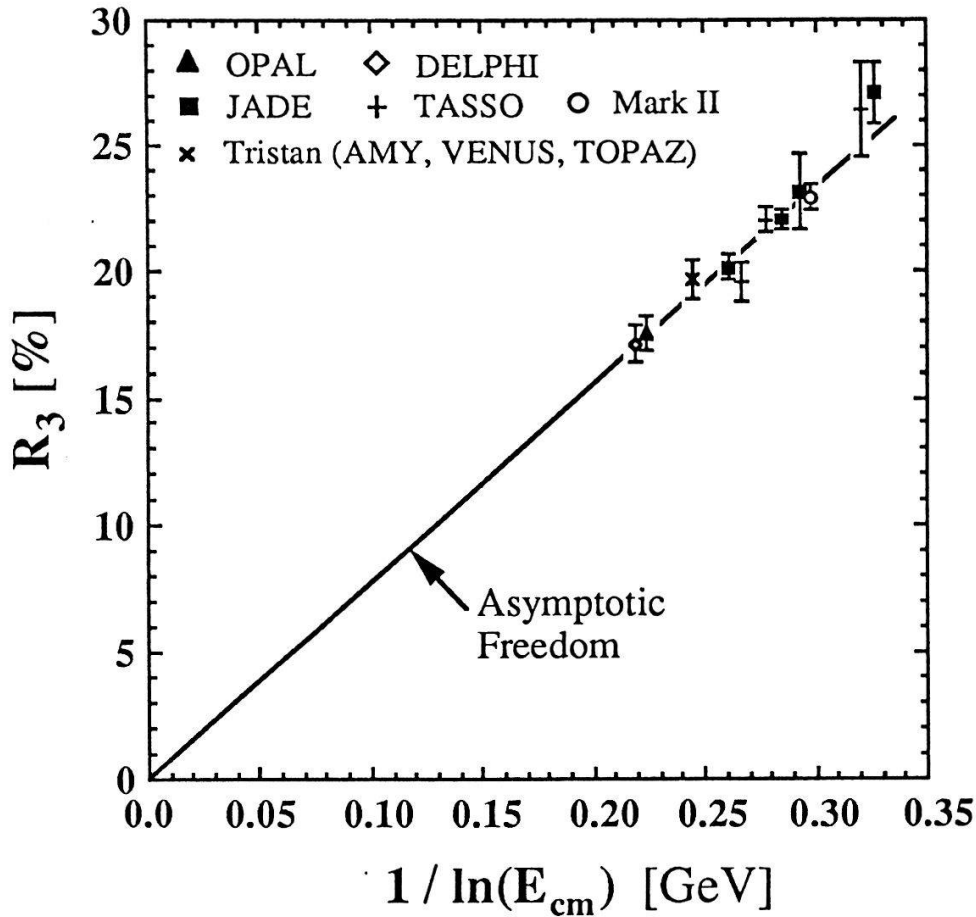
Figure 15: The three-jet relative rate as a function of annihilation energy. As a function of $(\ln E)^{-1}$. This more than suggests asymptotic freedom!
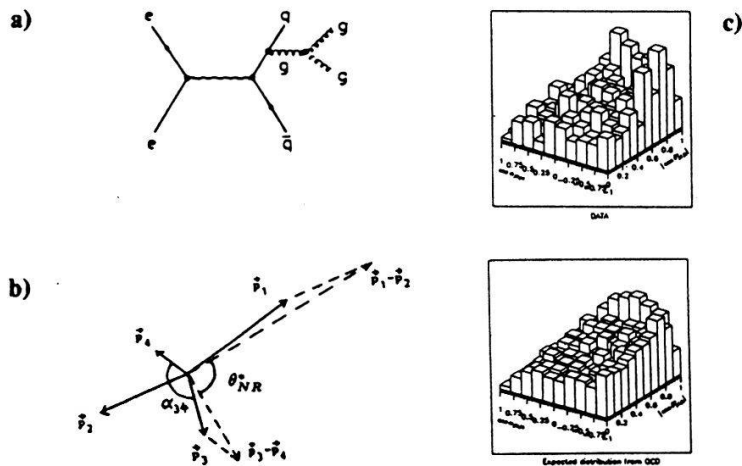

Figure 16: The three gluon coupling determination of DELPHI (Ref [14]) (a) The specific contribution to the amplitude. (b) The definition of specific angles. (c) The predicted and measured double distributions.
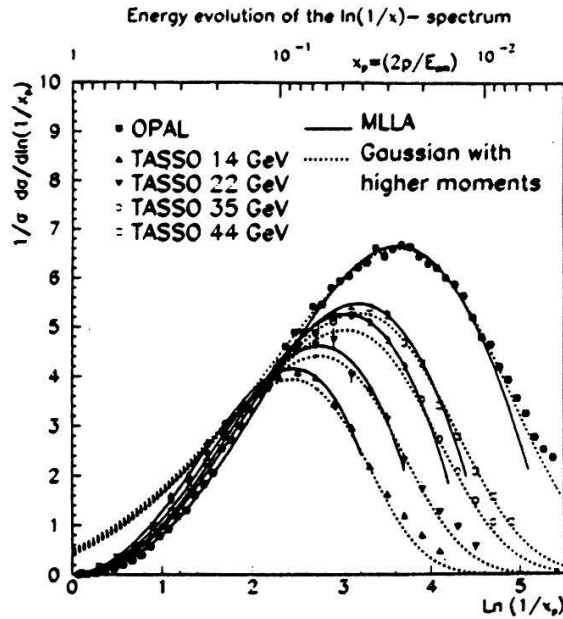
Figure 17: The inclusive $\ln(1/x_p)$ spectrum at different energies, showing the rise with energy and the depletion of the soft parton yield.

smaller than some typical hadronization time, is the longer the smaller the production angle is. Soft partons are therefore supressed. This is in very nice agreement with the OPAL data of Fig. 17. The quantity x_p is the parton momentum devided by $E/2$. The $\ln(1/x_p)$ spectrum shows an expected depletion at small values but also a clear depletion at large values, which remains with increasing energy. It is not a mere kinematical effect! In between, the spectrum show an almost Gaussian shape, with a maximum moving in location as $\ln E$ whilst it increases as a result of the development of the parton shower. The result of the leading log calculation at different energies (PETRA and LEP) are in good agreement with observation. The energy dependence of the maximum of the Gaussian peak follows the linear expectation for the logarithm (Fig. 18). There is but a small difference when considering either the parton distribution or the hadron distribution obtained using specific models. This reflects the local character (in term of rapidity) of the parton-hadron relationship. Reproducing well the particle spectra, one should reproduce their integral, namely the multiplicity. This is shown in Fig. 19. The rapidity distribution shows a rising "plateau" well understood in terms of an increasing parton branching with increasing energy. Multiplicity distributions are also well reproduced by model calculations. The complexity of the QCD cascade (Fig. 4 -a) is such that the distributions even include intermittency.

Interference effects among relatively soft partons also affect those partons for which jet assignement is not clear. This is the case for soft particles produced in the regions of phase space where jets overlap. Fig. 20 shows the angular energy flow observed at LEP for three-jet events (DELPHI). There is the famous dip in between the two leading jets which are associated with quark jets, while the third, softer one, is the natural candidate for a gluon jet. This is called the "string" effect since this was an early prediction of the Lund string model. Its association with destructive interferences from the quark and antiquark sources is however more closely associated with the present more detailed QCD approach [16]. Calculations (solid curve) match the data very well. This agreement, which strongly
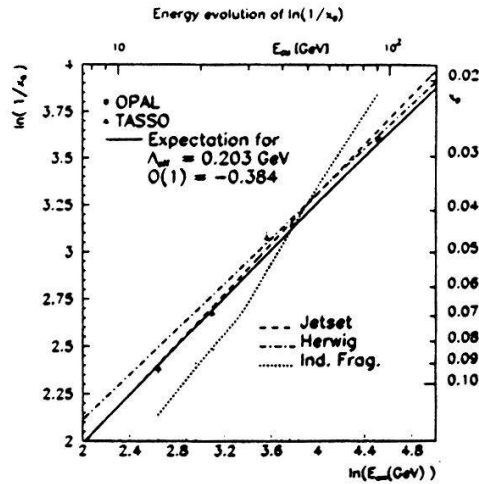
Figure 18: The variation of the location of the "Gaussian" maximum in Fig. 17 as a function of energy. Hadronization models do not improve much over the parton distribution. Most of the effect is with the parton shower of Fig. 4 -a. However, the whole pattern has to be considered. An independent jet model fails.
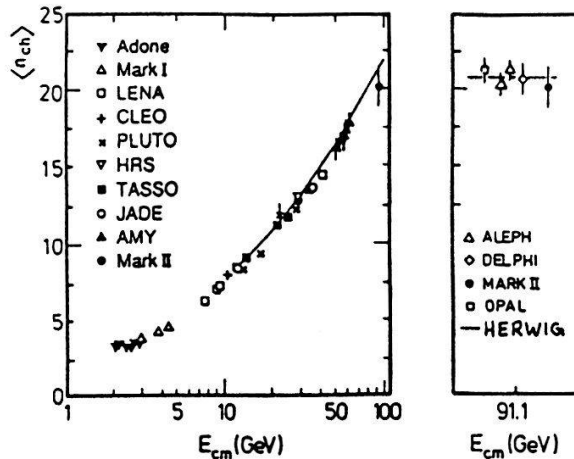

Figure 19: The mean charged multiplicity as a function of annihilation energy. Predicted and observed values at LEP.
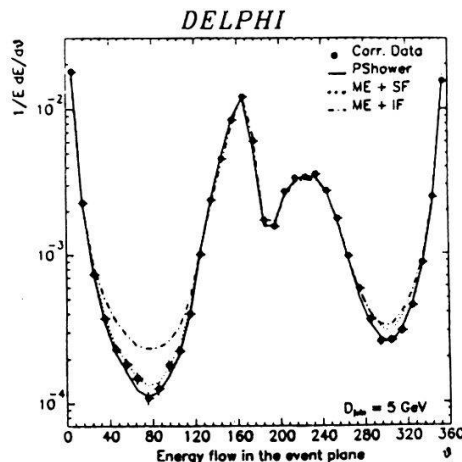

Figure 20: The energy flow in three-jet events (DELPHI). Note the "string effect" between the two leading (quark) jets.
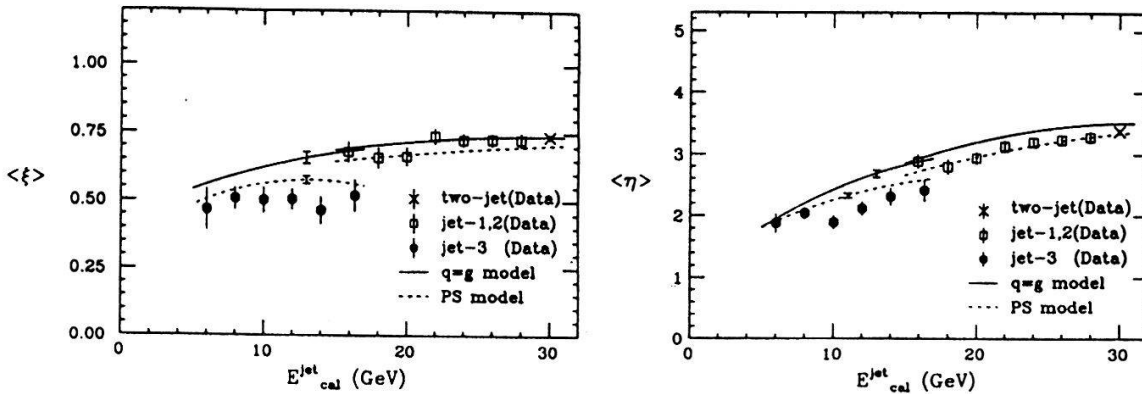
Figure 21: Distinguishing between quark and gluon jets using sensitive variables (AMY-TRISTAN).
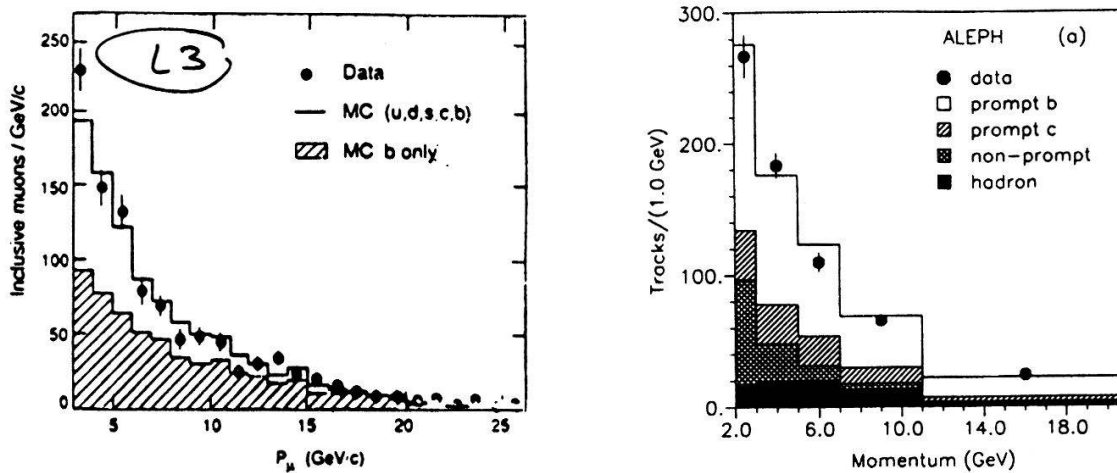

Figure 22: The lepton yields and its heavy flavour origin. L3 (muons) and ALEPH (electrons) results.

relies upon interference effects, underlines the fact that it is the whole energy flow pattern which is successfully calculated and not those associated separately with quark and gluon jets. It is of course tempting to try to distinguish the "fatter" and "softer" distribution associated with a gluon jet from that typical of a quark jet at the same energy. However, as expected in QCD, a simple superposition of jets does not work (dashed curve in Fig. 20. The complexity of the parton shower is also such that part of the first expected differences are to some extent washed away. It is still possible to distinguish quark from gluon jets using specific variables particularily sensitive to the expected differences such as the mean core energy and the mean rapidity of the leading particle. Fig. 21 shows the values measured by AMY at TRISTAN [17], comparing gluon and quark enrich samples obtained from 3-jet events to a pure quark sample (2-jet events) for the same jet energy. The differences are not dramatic but they are sizeable and in agreement with expectation.

4   Conclusion

LEP has already provided us with a beautiful set of jet data. The power of the detectors together with the important preparatory work, both theoretical and experimental (PEP,

PETRA) have jointly allowed for quick and detailed results which altogether provide a beautiful series of successful tests of QCD. For the most direct one, namely the measure of α_s on the Z, one can already achieve a 10% accuracy which presently confronts us with the basic ambiguities associated with the definition of a jet. Whilst progress on some questions such as the three-gluon coupling will follow with increasing statistics, the major newest development should be with heavy flavour production. Fig. 22, which shows the muon and electron distributions already obtained by L3 and ALEPH respectively, stands as a preview of far more extensive results to come out soon [18]. This will allow one to implement a full new set of tests for QCD.

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