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Incompressible Quantum Liquid versus Two–Dimensional Electron Solid

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Abstract. A simple but rigorous approach of sum–rule techniques being applied to the density and displacement fluctuations of the centers of cyclotron orbits of Coulomb–interacting electrons provides a study of the nature of two respective condensed phases: an incompressible quantum liquid of Bose condensed charge–vortex composites and a two–dimensional quantum solid with a lattice–periodic structure due to broken magnetic translational invariance.

Introduction

Interacting electrons of charge $-e$ being confined to a plane perpendicular to a magnetic field are considered to compete between two condensed phases: an incompressible quantum liquid [1] and a two–dimensional (2D) quantum solid, respectively, if the magnitude B of the field is sufficiently large that the mean number ν^{-1} of magnetic flux quanta ch/e per electron exceeds one. For a given area A of that plane this then corresponds to a fractional filling factor of the lowest Landau level $\nu = 2\pi r_L^2 n_e < 1$, i.e., a fractional ratio of the mean areal density of electrons, $N_e/A = n_e$, and of flux quanta, $N_s/A = (2\pi r_L^2)^{-1}$, respectively, if $r_L = (c\hbar/eB)^{1/2}$ denotes the Larmor radius. For certain rational values of $\nu = p/q < 1$, with p and q denoting mutual primes and q usually being odd, the incompressible quantum liquid (IQL) phase of Bose condensed charge–vortex composites (p charges and q flux quanta) may be stabilized because of a novel broken symmetry [2]. This IQL exhibits a quantized Hall conductance $\sigma_{12} = \nu\, e^2/h$, being accompanied by minima in the longitudinal conductance σ_{11} [3]. Below a critical filling factor ν_c, the expectation values $R_j \equiv \langle X_j \rangle$ of the centers of the cyclotron orbits (guiding centers) with non–commuting Cartesian components are assumed to possibly form a lattice–periodic structure with a non–vanishing shear modulus $\mu > 0$ due to broken magnetic translational invariance. Particularly, in the dilute electron ($\nu \to 0$) or high–magnetic–field ($r_L \to 0$) limit, the triangular lattice of the classical 2D Wigner crystal [4] may be identified with the ground–state of this sort of a 2D quantum solid, to be called quantum Hall crystal (QHC) [5]. Reentrance phenomena of the correlated electron system may finally allow the IQL–phase to exist even at certain rational values $p/q < \nu_c$.

Outline of Approach and Summary of Results

Differences and similarities in the IQL–phase and the QHC–phase can now be associated quite generally with differences in their respective symmetries and various results are obtained by the use of sum–rule techniques without employing approximate wave functions as having been discussed in detail previously [5]. First, the finite–gap versus gapless behavior of collective modes $\omega(k)$ was shown to exist rigorously within the lowest Landau level in the $k \to 0$ limit of these two phases [6]. An important role in obtaining these results is played by the respective restoring forces of the non–commuting density fluctuations $\Delta(k,t) = \Sigma_j \exp(-ik \cdot X_j(t))$ of the guiding centers $X_j(t)$ and the fluctuations $u(k,t) = N_e^{-1/2}\, \Sigma_j\, [X_j(t) - R_j] \exp(ik \cdot R_j)$ of their displacements $X_j(t) - R_j$. The mentioned restoring forces are related to frequency moments which determine the non–dispersive parts in the spectral representations of the inverse of the response functions χ^{-1}. By the use of sum rules, these collisionless terms of χ^{-1} can therfore be calculated via those frequency moments of the spectral functions $\chi''(k,\omega)$ without making reference to the nature of the single–particle excitations.

In the IQL–phase of the uniform system of interacting electrons that restoring force at k=0 induces a k–independent ratio of the third and first frequency moment of the spectral function of guiding center density fluctuations $\chi''_{\Delta\Delta}$, giving rise to the gap at k=0 in the excitation spectrum $\omega_L(k=0)\neq 0$, derived from Re $\chi^{-1}_{\Delta\Delta}(k,\omega_L(k))\equiv 0$. Combined with a sum rule, the k^4–behavior actually obtained for both frequency moments in the $k\to 0$ limit provides a rigorous proof that also the static susceptibility $\chi_{\Delta\Delta}(k,0)\sim k^4$, i.e., the longwave length density fluctuations are strongly suppressed in that IQL–phase. In single–mode approximation [7], this then implies moreover the magneto–roton minimum to exist at some finite k of ω_L, whose softening may induce the transition to the QHC–phase.

In the QHC–phase, combinations of the first and zeroth frequency moment of the spectral function of guiding center displacements play a similar role for the quasi–transverse restoring force being related to the dynamical matrix of conventional lattice dynamics. For the long–wavelength limit of the gapless Goldstone mode, being isotropic for a triangular lattice, the exact expression $\omega_S^2(k)=(2\pi e^2/m^{*2}\omega_C^2\epsilon)\ \mu\ k^3$ is then obtained from Re $\chi^{-1}_{uu}(k,\omega_S(k))\equiv 0$ (m^*: effective mass, ϵ: background dielectric constant, $\omega_C = \hbar/m^*\ r_L^2$: cyclotron frequency). The isothermal shear modulus μ is rigorously related to the second derivative of the free energy with respect to displacement deformations according to the generalized elastic sum rule [8]. A non–vanishing shear modulus $\mu > 0$, therefore, reveals the crystalline–like nature of that phase rather generally.

A minimum, found in the ν–dependence of μ in an approach allowing for quantum statistical features and anharmonicities to one–loop order [6],[9], is possibly indicating a competition between the QHC–phase and the IQL–phase in the vicinity of fractional filling factors exhibiting quantized Hall conductance as revealed by experiment [10]. This, finally, would also be consistent with a diverging longitudinal resistivity, having been observed experimentally in a narrow region around $\nu=1/5$ [11], as well as with our recent estimate for the critical filling factor ν_c slightly higher than 1/5, obtained from comparing total energy calculations in both phases, i.e., $E_L(\nu_c)=E_S(\nu_c)$ [12].

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