

**Zeitschrift:** Helvetica Physica Acta  
**Band:** 65 (1992)  
**Heft:** 2-3

**Artikel:** Order and fluctuations in coupled XY planes  
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**DOI:** <https://doi.org/10.5169/seals-116468>

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## Order and Fluctuations in Coupled XY Planes

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**Abstract.** A three-dimensional array of classical XY spins with couplings  $J_{\parallel}$  in the planes and  $J_{\perp}$  between planes is studied both analytically and numerically. For an infinite number of planes, the magnetization is shown to be extremely sensitive to the interplane coupling. For a finite number  $N$  of planes, the magnetization vanishes, but there is a (Kosterlitz-Thouless) transition. The critical temperature increases as a function of  $N$  and rapidly approaches the bulk value.

### Introduction

It is well established that the two-dimensional XY model undergoes a phase transition of a particular type, as described first by Berezinskii and by Kosterlitz and Thouless [1]. According to Mermin-Wagner theorem, below the critical temperature the order parameter remains zero, while the susceptibility is infinite. Here we consider a collection of coupled XY planes defined by the Hamiltonian

$$H = -J_{\parallel} \sum_{\ell} \sum_{\langle ij \rangle} \vec{s}_{i\ell} \cdot \vec{s}_{j\ell} - J_{\perp} \sum_{\ell i} \vec{s}_{i\ell} \cdot \vec{s}_{i, \ell+1} \quad (1)$$

where  $\vec{s}_{i\ell}$  is a unit vector on site  $i$  in the  $\ell$ 'th plane. We find that any small interplane coupling,  $J_{\perp} \ll J_{\parallel}$ , changes the nature of the transition [2].

### Order Parameter for an Infinite Number of Planes

At low temperatures we can use the spin wave approximation for the magnetization, giving

$$m = \exp \left( -\frac{k_B T}{2J_{\parallel}} \alpha \right), \quad (2)$$

where

$$\alpha = \frac{1}{(2\pi)^3} \int_{BZ} d^3 q \frac{J_{\parallel}}{\omega(\vec{q})} \quad (3)$$

and the spin wave spectrum is

$$\omega(\vec{q}) = 4J_{\parallel} \left[ \sin^2 \frac{q_x}{2} + \sin^2 \frac{q_y}{2} \right] + 4J_{\perp} \sin^2 \frac{q_z}{2}. \quad (4)$$

For  $J_{\perp} \rightarrow 0$ , the parameter  $\alpha$  diverges logarithmically and the magnetization tends to zero. This is in agreement with the upper bound

$$m \leq \left( \frac{J_{\parallel}}{k_B T \alpha} \right)^{1/2}, \quad (5)$$

derived as a straightforward generalization of the Mermin - Wagner theorem [3]. Fig. 1 shows that an extremely small  $J_{\perp}$  is sufficient to efficiently suppress the fluctuations, thus stabilizing the long-range order.

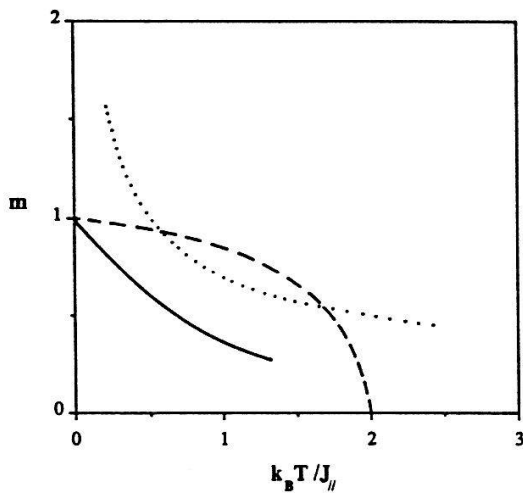


Figure 1 : Order parameter for an infinite number of planes and a ratio  $J_{\perp}/J_{\parallel} = 10^{-10}$ . The dashed line represents the mean-field approximation, the solid line Eq. (2) and the dotted line the r.h.s. of Eq. (5).

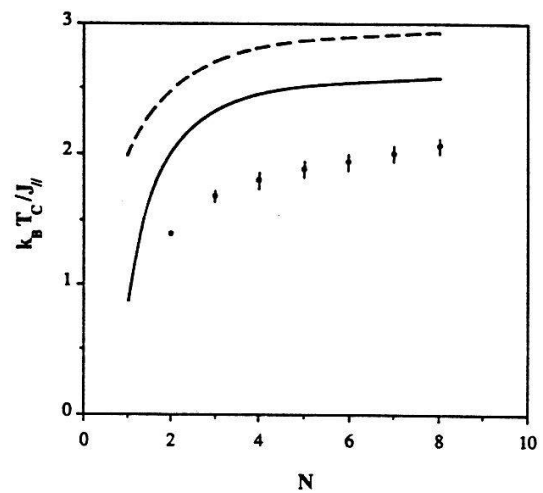


Figure 2 : Critical temperature for a finite number of planes in the isotropic case,  $J_{\perp} = J_{\parallel}$ . The dashed-line represents the mean field result, the full line Eq. (6) and the dots the Monte Carlo simulations.

### Critical Temperature for a Finite Number of Planes

It is easy to show that for any finite number  $N$  of planes the magnetization vanishes for  $T > 0$ . In order to estimate the critical temperature  $T_c$  where the susceptibility diverges we treat the coupling between planes in a mean-field approximation and find the implicit relation

$$1 = 2J_{\perp} \cos\left(\frac{\pi}{N+1}\right) \chi_{2D}(T_c). \quad (6)$$

Here  $\chi_{2D}(T)$  is the susceptibility of the two-dimensional XY model for which we use the high-temperature expansion of Butera et al. [4]. We have also performed Monte Carlo calculations, using a cluster algorithm and a reweighting procedure. The critical temperature was determined on the basis of the fourth cumulant of the magnetization. The results are shown in Fig. 2 for the isotropic case,  $J_{\perp} = J_{\parallel}$ . We see that  $T_c$  first increases rapidly as a function of  $N$  and then approaches the limiting value for the 3D XY model. For smaller values of  $J_{\perp}$  the overall increase of  $T_c$  is smaller. The comparison between the results based on Eq. (6) and the Monte Carlo data indicates that inter-plane fluctuations considerably reduce the critical temperature.

### References

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