

**Zeitschrift:** Helvetica Physica Acta

**Band:** 65 (1992)

**Heft:** 2-3

**Artikel:** Magnetic properties of the Hubbard model with infinite interaction

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**DOI:** <https://doi.org/10.5169/seals-116480>

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## MAGNETIC PROPERTIES OF THE HUBBARD MODEL

## WITH INFINITE INTERACTION

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Abstract. The problem of the ground state multiplicity of the Hubbard model with the infinite interaction is considered for the systems of segments of length  $n$ .

Introduction.

The simplest model of strong correlated systems is the Hubbard model with the infinite repulsion described by the Hamiltonian

$$H = \sum t_{ij} (c_{i\sigma}^+ c_{j\sigma} + c_{j\sigma}^+ c_{i\sigma}) (1 - c_{i,-\sigma}^+ c_{i,-\sigma}) (1 - c_{j,-\sigma}^+ c_{j,-\sigma}) \quad (1)$$

One of the most important problems is to find the dependence of the ground state multiplicity on the electron density  $\rho$ . (Nagaoka problem [1]). We consider this problem for special kind of systems, consisting of ladders of  $n$ -sites segments with different intra- and intersegment hopping integrals,  $t$  and  $t_{\perp}$ .

Spin Hamiltonian (SH).

If  $\alpha = t_{\perp}/t \ll 1$  one can make use of the perturbation theory in  $\alpha$ . For  $t_{\perp} = 0$  energetic levels are spin degenerated. The resolve of degeneracy (up to  $\alpha^2$ ) leads to Hamiltonian acting upon the spin variables of the neighbouring segments [2]. It turns out that the energy of two neighbouring segments with equal (inequal) non-zero numbers of electrons is minimal at total minimal (maximal) spin. Thus, SH describes the competing interactions of ferro (F)- and antiferromagnetic (AF) types. For  $\rho < 1/n$  there cannot be more than one electron at each segment and the ground state is singlet. For  $\rho > 1/n$  the competition interactions leads to a separation of phases with F and AF ordering. For example, for the system with  $n=2$  (ladders of dimers) and for  $\rho > 1/2$  F-polaron is created. Its size is proportional to  $\alpha^{-1/3}$ , and at  $\rho_c = (2\pi)^{-1} (3\pi\alpha \ln 2)^{1/3}$  is equal to the ladder length. Thus, the system is saturated ferromagnet. For the case with  $n > 2$  polarons may be both F- and AF-type leading to the "cascade" of transitions with the alteration of total spin. Equally to the ladders a more complicated system may be considered. It

consists of dimers packed up a 2d lattice. For  $\rho \leq 1/2$  SH is equivalent to Hamiltonian of the 2d t-J model which magnetic properties have been discussed in [3]. As in the case of the ladder model for  $\rho > 1/2$  the F polaron appears as well as the critical density  $\rho_c$  above which the system becomes a saturated ferromagnet. The variational approach.

For ladders with arbitrary  $\alpha$  we seek the wave function  $\Psi$  in the following form (for simplicity we consider the model with  $n=2$ ):

$$\Psi = \prod_i N_{i\alpha} N_{i\beta} (x_i + y_{i\alpha} \psi_{\alpha\alpha}(i) + y_{i\beta} \psi_{\beta\beta}(i) + \psi_{\alpha\beta}(i) b_{i\alpha}^+ b_{i\beta}^+ + b_{i\alpha}^+ \psi_{\beta}(i) + b_{i\beta}^+ \psi_{\alpha}(i)),$$

where

$$\psi_{\sigma\sigma}(i) = c_{1\sigma}^+ c_{2\sigma}^+; \quad \psi_{\alpha\beta}(i) = \lambda_s \varphi_s(i) + \lambda_t \varphi_t(i); \quad \varphi_{s,t} = (2)^{-1/2} (c_{1\alpha}^+ c_{2\beta}^+ \mp c_{1\beta}^+ c_{2\alpha}^+); \quad \psi_{\sigma}(i) = (2)^{-1/2} (c_{1\sigma}^+ + c_{2\sigma}^+).$$

1,2 are the numbers of sites of a  $i$ -th segment;  $\lambda_s, \lambda_t$  are the variational parameters at  $\lambda_s^2 + \lambda_t^2 = 1$ ;  $x_i^2 + y_{i\alpha}^2 + y_{i\beta}^2 = 1$  ( $y_{i\sigma} = 0$  or  $1$ ).  $N_{i\sigma} = b_{i\sigma}^+ b_{i\sigma}$ ;  $b_{i\sigma}^+$  are the operators of pseudo-fermi-particles.

Effective Hamiltonian for pseudoparticles is  $H_{\text{eff}} = \langle \Psi | H | \Psi \rangle$  where  $\langle \dots \rangle$  means an average on  $c$ -operators. Analysing  $H_{\text{eff}}$  we have reproduced the results obtained by SH method for  $\alpha \ll 1$ . In isotropic case,  $\alpha=1$ , and for  $\rho < 1/2$  a ground state is singlet. At  $\rho \rightarrow 1$  a ground state spin is maximal. At  $\rho=0.67$  this state becomes unstable with respect to overturning single up-spin electron. However, it turns out that at  $\rho=0.73$  F state is unstable against multimagnon excitations. Note that in Gutzwiller approximation F state in this model is stable at all values of  $\rho$ .

We also studied the dependence  $S(\rho)$  using the exact diagonalization of small ladders of dimers (up to 16 sites). The agreement between analytical and numerical results is fairly good.

References.

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