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Black Holes and Solitons in String Theory

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Abstract. In this review I discuss various aspects of some of the recently constructed black hole and soliton solutions in string theory. I begin with the axionic instanton and related solutions of bosonic and heterotic string theory. The latter ten-dimensional solutions can be compactified to supersymmetric monopole, string and domain wall solutions which break $1/2$ of the spacetime supersymmetries of $N = 4, D = 4$ heterotic string theory, and which can be generalized to two-parameter charged black hole solutions. The low-energy dynamics of these solutions is also discussed, as well as their connections with strong/weak coupling duality and target space duality in string theory. Finally, new solutions are presented which break $3/4$, $7/8$ and $15/16$ of the spacetime supersymmetries and which also arise in more realistic $N = 1$ and $N = 2$ compactifications.

1 Introduction

In this review I discuss some basic results in the study of classical solitonic and black hole solutions of string theory. One motivation in this endeavour is that the existence of these Plack-scale solitons may shed light on the nature of string theory as a finite theory of quantum gravity. Furthermore, there is the possibility of adapting to string theory nonperturbative methods from the physics of solitons and instantons already employed in field theory. For example, the stringy analogs of Yang-Mills instantons may be used to explore tunneling between string vacua and thus lead to a better understanding of the nature of the vacuum in string theory. Finally, these soliton and black hole solutions point to interesting connections between the various spacetime and worldsheet dualities in string theory.

I begin in section 2 with a description of the axionic instanton solution in the gravitational sector of the string, first discovered in [1,2] and which represents a stringy analog of the 't Hooft ansatz [3–6]. In particular, the generalized curvature of the string solution, with torsion coming from the antisymmetric tensor field strength H_3 (and hence the name "axionic instanton", since in four dimensions H_3 is dual to an axion field) obeys a (anti) self-duality condition identical to that obeyed by the field strength of the Yang-Mills instanton [7,8]. In ten-dimensional heterotic string theory the axionic instanton manifests itself as a $5+1$-dimensional soliton solution, the so-called "fivebrane" [9,10,11] and whose existence was predicted by the string/fivebrane duality conjecture [12,13]. Related solutions with axionic instanton structure are also briefly discussed [14,15].

In section 3 I discuss toroidal compactifications of the axionic instanton/fivebrane to four dimensions and obtain supersymmetric monopole [16], string and domain wall solutions [17] which break half the spacetime supersymmetries of $N=4$, $D=4$ heterotic string theory.

The string soliton solution is singled out in section 4, where it is observed to be the solitonic dual of the fundamental string solution of [18] in four-dimensions. Another way of saying this is that the string/fivebrane duality conjecture manifests itself as (effective) string/string duality in $D=4$ [17]. One attraction of this reduction is that a dual string theory is probably far easier to construct than a fundamental fivebrane theory. More immediately, four-dimensional string/string duality is seen to interchange two other dualities in string theory [19,20,17]: target space duality, already established in various compactifications, and strong/weak coupling duality, shown in the low-energy limit but conjectured to be an exact symmetry of string theory.

The four-dimensional solitons represent extremal limits, saturating a Bogomol'nyi bound [21] between mass and charge, of two-parameter families of black hole solutions [22]. These black hole generalizations, as well as their connections with ten-dimensional and four-dimensional dualities, are discussed in section 5.

In section 6 I study the dynamics of the four-dimensional solitons from two different viewpoints [23]. The first involves computing the Manton metric on moduli space [24], whose geodesics represent the motion of quasi-static solutions in the static solution manifold, and which represent a low-velocity approximation to the actual dynamics of the solitons. The second approach calculates the four-point amplitude for the scattering of winding string states, the nearest approximation in string theory to solitonic string states. Both computations yield trivial scattering to leading order in the velocities (i.e. zero-dynamical force to l eading order) in direct contrast to analogous computations for BPS monopoles [25,21].

In section 7 I present new string solutions corresponding to more intricate toroidal compactifications [26]. Interesting connections are made between the number of preserved supersymmetries and the nature of the target space duality group. The role of the axionic instanton is again seen to be crucial in this respect. Analogous solutions are also seen to arise in more realistic $N = 1$ and $N = 2$ compactifications.

Finally, in section 8 I discuss future directions in this subfield of string theory and suggest some open problems. Earlier reviews which deal more extensively with the conformal field theoretic aspects of soliton and black hole solutions of string theory may be found in [27,28].

2 Axionic Instanton

Consider the four-dimensional Euclidean action

$$S = -\frac{1}{2g^2}\int d^4y \mathrm{tr} F_{mn}F^{mn}, \qquad m,n = 1,2,3,4. \tag{1}$$

For gauge group $SU(2)$, the fields may be written as $A_m = (g/2i)\sigma^a A^a_m$ and $F_{mn} = (g/2i)\sigma^a F^a_{mn}$ (where σ^a, $a = 1,2,3$ are the 2×2 Pauli matrices). The equation of motion derived from this action is solved by the 't Hooft ansatz [3–6]

$$A_{mn} = i\overline{\Sigma}_{mn}\partial_n \ln f, \tag{2}$$

where $\overline{\Sigma}_{mn} = \overline{\eta}^{imn}(\sigma^i/2)$ for $i = 1,2,3$, where

$$\begin{aligned}\overline{\eta}^{imn} = -\overline{\eta}^{inm} &= \epsilon^{imn}, \qquad && m,n = 1,2,3,\\ &= -\delta^{im}, && n = 4\end{aligned} \tag{3}$$

and where $f^{-1}\Box f = 0$. The above solution obeys the self-duality condition

$$F_{mn} = \tilde{F}_{mn} = \frac{1}{2}\epsilon_{mn}{}^{kl}F_{kl}. \tag{4}$$

The ansatz for the anti-self-dual solution $F_{mn} = -\tilde{F}_{mn} = -\frac{1}{2}\epsilon_{mn}{}^{kl}F_{kl}$ is similar, with the δ-term in (3) changing sign. To obtain a multi-instanton solution, one solves for f in the four-dimensional space to obtain

$$f = 1 + \sum_{i=1}^{k}\frac{\rho_i^2}{|\vec{y} - \vec{a}_i|^2}, \tag{5}$$

where ρ_i is the instanton scale size, $\vec{a}_i$ the location in four-space of the ith instanton and

$$k = \frac{1}{16\pi^2}\int_{M^4} trF^2 \tag{6}$$

is the instanton number. Note that this solution has $5k$ parameters, while the most general (anti) self-dual solution has $8k$ parameters, or $8k - 3$ if one excludes the 3 zero modes associated with global $SU(2)$ rotations.

Now consider the bosonic string sigma model action [29]

$$I = \frac{1}{4\pi\alpha'}\int d^2x\sqrt{\gamma}\left(\gamma^{ab}\partial_a X^M\partial_b X^N g_{MN} + i\epsilon^{ab}\partial_a X^M\partial_b X^N B_{MN} + \alpha' R^{(2)}\phi\right), \tag{7}$$

where g_{MN} is the sigma model metric, ϕ is the dilaton and B_{MN} is the antisymmetric tensor, and where γ_{ab} is the worldsheet metric and $R^{(2)}$ the two-dimensional curvature.

A classical solution of bosonic string theory corresponds to Weyl invariance of (7) [30]. It turns out that any dilaton function satisfying $e^{-2\phi}\Box\, e^{2\phi} = 0$ with

$$\begin{aligned} g_{mn} &= e^{2\phi}\delta_{mn} \qquad m,n = 1,2,3,4, \\ g_{\mu\nu} &= \eta_{\mu\nu} \qquad \mu,\nu = 0,5,...,25, \\ H_{mnp} &= \pm 2\epsilon_{mnpk}\partial^k\phi \qquad m,n,p,k = 1,2,3,4, \end{aligned} \tag{8}$$

where $H = dB$, is a tree-level solution of (7). The ansatz (8) in fact possesses a (anti) self-dual structure in the subspace (1234), which can be seen as follows. We define a generalized curvature $\hat{R}^i{}_{jkl}$ in terms of the standard curvature $R^i{}_{jkl}$ and $H_{\mu\alpha\beta}$ [31]:

$$\hat{R}^i{}_{jkl} = R^i{}_{jkl} + \frac{1}{2}\left(\nabla_l H^i{}_{jk} - \nabla_k H^i{}_{jl}\right) + \frac{1}{4}\left(H^m{}_{jk}H^i{}_{lm} - H^m{}_{jl}H^i{}_{km}\right). \tag{9}$$

One can also define $\hat{R}^i{}_{jkl}$ as the Riemann tensor generated by the generalized Christoffel symbols $\hat{\Gamma}^\mu_{\alpha\beta}$ where $\hat{\Gamma}^\mu_{\alpha\beta} = \Gamma^\mu_{\alpha\beta} - (1/2)H^\mu{}_{\alpha\beta}$. Then we can express the generalized curvature in covariant form in terms of the dilaton field as [7]

$$\hat{R}^i{}_{jkl} = \delta_{il}\nabla_k\nabla_j\phi - \delta_{ik}\nabla_l\nabla_j\phi + \delta_{jk}\nabla_l\nabla_i\phi - \delta_{jl}\nabla_k\nabla_i\phi \pm \epsilon_{ijkm}\nabla_l\nabla_m\phi \mp \epsilon_{ijlm}\nabla_k\nabla_m\phi. \tag{10}$$

It easily follows that

$$\hat{R}^i{}_{jkl} = \mp\frac{1}{2}\epsilon_{kl}{}^{mn}\hat{R}^i{}_{jmn}. \tag{11}$$

So the (anti) self-duality appears in the gravitational sector of the string in terms of its generalized curvature thus justifying the name "axionic instanton" for the four-dimensional solution first found in [1,2]. For $e^{2\phi} = e^{2\phi_0}f$, where f is given in (5), we obtain a multi-instanton solution of string theory analogous to the YM instanton.

In the special case of $e^{2\phi} = Q/r^2$, the sigma model decomposes into the product of a one-dimensional CFT and a three-dimensional WZW model with an $SU(2)$ group manifold. This can be seen by setting $u = \ln r$ and rewriting (7) in this case in the form $I = I_1 + I_3$, where

$$I_1 = \frac{1}{4\pi\alpha'}\int d^2x\left(Q(\partial u)^2 + \alpha' R^{(2)}\phi\right) \tag{12}$$

is the action for a Feigin-Fuchs Coulomb gas, which is a one-dimensional CFT with central charge given by $c_1 = 1 + 6\alpha'(\partial\phi)^2$ [32]. The imaginary charge of the Feigin-Fuchs Coulomb gas describes the dilaton background growing linearly in imaginary time. I_3 is the Wess–Zumino–Witten [33] action on an $SU(2)$ group manifold with central charge

$$c_3 = \frac{3k}{k+2} \simeq 3 - \frac{6}{k} + \frac{12}{k^2} + ... \tag{13}$$

where $k = Q/\alpha'$, called the "level" of the WZW model, is an integer. This can be seen from the quantization condition on the Wess-Zumino term [33]

$$\begin{aligned} I_{WZ} &= \frac{i}{4\pi\alpha'} \int_{\partial S_3^{\pm}} d^2x \epsilon^{ab} \partial_a x^m \partial_b x^n B_{mn} \\ &= \frac{i}{12\pi\alpha'} \int_{S_3^{\pm}} d^3x \epsilon^{abc} \partial_a x^m \partial_b x^n \partial_c x^p H_{mnp} \\ &= 2\pi i \left(\frac{Q}{\alpha'}\right). \end{aligned} \tag{14}$$

Thus Q is not arbitrary, but is quantized in units of α'. We use this splitting to obtain exact expressions for the fields by fixing the metric and antisymmetric tensor field in their lowest order form and rescaling the dilaton order by order in α'. The resulting expression for the dilaton is

$$e^{2\phi} = \frac{Q}{r^{\sqrt{\frac{4}{1+\frac{2\alpha'}{Q}}}}}. \tag{15}$$

The above bosonic solution easily generalizes to an analogous solution of heterotic string theory. The bosonic ansatz

$$\begin{aligned} e^{-2\phi} \Box\, e^{2\phi} &= 0, \\ g_{mn} &= e^{2\phi} \delta_{mn} \qquad m, n = 1, 2, 3, 4 \\ g_{\mu\nu} &= \eta_{\mu\nu} \qquad \mu, \nu = 0, 5, 6, 7, 8, 9 \\ H_{mnp} &= \pm 2\epsilon_{mnpk} \partial^k \phi \qquad m, n, p, k = 1, 2, 3, 4 \end{aligned} \tag{16}$$

is a solution of the bosonic sector of the ten-dimensional low-energy heterotic string effective action

$$S_{10} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \left(R + 4(\partial\phi)^2 - \frac{H^2}{12} \right), \tag{17}$$

whose equations of motion are equivalent to Weyl invariance of the sigma-model. Eq.(16) with zero fermi fields, zero gauge field and constant chiral spinor $\epsilon = \epsilon_4 \otimes \eta_6$ in fact preserves half the spacetime supersymmetries stemming from the supersymmetry equations

$$\begin{aligned} \delta\psi_M &= \left(\partial_M + \tfrac{1}{4} \Omega_{MAB} \Gamma^{AB}\right) \epsilon = 0, \\ \delta\lambda &= \left(\Gamma^A \partial_A \phi - \tfrac{1}{12} H_{ABC} \Gamma^{ABC}\right) \epsilon = 0, \\ \delta\chi &= F_{AB} \Gamma^{AB} \epsilon = 0, \end{aligned} \tag{18}$$

where $A, B, C, M = 0, 1, 2, ..., 9$, ψ_M, λ and χ are the gravitino, dilatino and gaugino fields and where $\Omega_M{}^{PQ} = \omega_M{}^{PQ} - 1/2 H_M{}^{PQ}$ is the generalized connection that generates the generalized curvature (9). The (anti) self-duality of the generalized curvature (11) in the (1234) subspace in fact translates into an analogous condition for the generalized connection and is intimately connected to the choice of chirality of ϵ_4 that leads to the preservation of precisely half of the supersymmetry generators. In the above form (16), we recover the tree-level multi-fivebrane solution of [11]. The existence of the fivebrane as a soliton solution of string theory lends support to the string/fivebrane duality conjecture [12,9,10,13], which states that the same physics as superstring theory may be described by a theory of fundamental superfivebranes propagating in ten dimensions. This conjecture is a stringy analog of the Montonen-Olive conjecture [34], which postulates a duality between electrically charged particles and magnetically charged solitons in four-dimensional supersymmetric point field theory.

The simple expedient of equating the gauge connection ${A_M}^{PQ}$ to the generalized connection ${\Omega_M}^{PQ}$ then leads to another solution of (18), which possesses an instanton in both gauge and gravitational sectors. This solution was argued to be an exact solution of heterotic string theory (i.e. a solution to all orders in α') which, in contrast to the purely bosonic solution above, does not require rescaling the dilaton from its tree-level form [14]. In fact, many of the pure gravity sector solutions I will discuss in this paper may be generalized to solutions with nontrivial YM fields and which may be argued to be exact solutions of heterotic string theory by using the above gauge equals generalized connection embedding (first discovered in a somewhat different context in [35]). For the sake of simplicity, however, I will concentrate mainly on the former class of solutions and merely point out where such generalizations may be interesting.

Another related axionic instanton solution of heterotic string theory inspired by conformal field theoretic constructions in [36] is given by the string-like solution [15]

$$\begin{aligned}
e^{-2\phi_1}\Box\, e^{2\phi_1} &= e^{-2\phi_2}\Box\, e^{2\phi_2} = 0, \\
\phi &= \phi_1 + \phi_2, \\
g_{mn} &= e^{2\phi_1}\delta_{mn} \qquad m,n = 2,3,4,5, \\
g_{ij} &= e^{2\phi_2}\delta_{ij} \qquad i,j = 6,7,8,9, \\
g_{\mu\nu} &= \eta_{\mu\nu} \qquad \mu,\nu = 0,1, \\
H_{mnp} &= \pm 2\epsilon_{mnpq}\partial^q\phi \qquad m,n,p,q = 2,3,4,5, \\
H_{ijk} &= \pm 2\epsilon_{ijkl}\partial^k\phi \qquad i,j,k,l = 6,7,8,9,
\end{aligned} \tag{19}$$

which for constant chiral spinors $\epsilon_\pm = \epsilon_2 \otimes \eta_4 \otimes \eta'_4$ solves the supersymmetry equations (18) for zero fermi and gauge fields (or alternatively for $A_M = \Omega_M$). In this case we have two independent axionic instantons, each of which breaks half the spacetime supersymmetries. As a consequence, only 1/4 of the original supersymmetries are preserved. More recently, axionic instanton solutions have been constructed in [37].

3 D=4 Solitons

Let us single out a direction (say x^4) in the transverse four-space (1234) and assume all fields in (16) are independent of this coordinate. Since all fields are already independent of x^5, x^6, x^7, x^8, x^9, we may consistently assume the $x^4, x^5, x^6, x^7, x^8, x^9$ are compactified on a six-dimensional torus, where we shall take the x^4 circle to have circumference $Le^{-\phi_0}$ and the rest to have circumference L, so that $\kappa_4^2 = \kappa_{10}^2 e^{\phi_0}/L^6$. Going back to the 't Hooft ansatz (2), the solution for f satisfying the $f^{-1}\Box f = 0$ has the form

$$f_M = 1 + \sum_{i=1}^{N} \frac{m_i}{|\vec{x} - \vec{a}_i|}, \tag{20}$$

where m_i is proportional to the charge and $\vec{a}_i$ the location in the three-space (123) of the ith instanton string. The solution (16) with $e^{2\phi} = e^{2\phi_0} f_M$ can be reduced to an explicit solution in the four-dimensional space (0123) [17]. The reduction from ten to five dimensions is trivial, as the metric is flat in the subspace (56789). In going from five to four dimensions, one follows the usual Kaluza-Klein procedure [38–40] of replacing g_{44} with a scalar field $e^{-2\sigma}$. The tree-level effective action reduces in four dimensions to

$$I_4 = \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-g} e^{-2\phi-\sigma} \left(R + 4(\partial\phi)^2 + 4\partial\sigma \cdot \partial\phi - e^{2\sigma} \frac{M_{\alpha\beta}M^{\alpha\beta}}{4} \right), \tag{21}$$

where $\alpha, \beta = 0, 1, 2, 3$ and where $M_{\alpha\beta} = H_{\alpha\beta 4} = \partial_\alpha B_{\beta 4} - \partial_\beta B_{\alpha 4}$. The four-dimensional monopole solution for this reduced action is then given by

$$\begin{aligned} e^{2\phi} &= e^{-2\sigma} = e^{2\phi_0} \left(1 + \sum_{i=1}^{N} \frac{m_i}{|\vec{x} - \vec{a}_i|} \right), \\ ds^2 &= -dt^2 + e^{2\phi} \left(dx_1^2 + dx_2^2 + dx_3^2 \right), \\ M_{ij} &= \pm \epsilon_{ijk} \partial_k e^{2\phi}, \qquad i, j, k = 1, 2, 3. \end{aligned} \tag{22}$$

For a single monopole, in particular, we have

$$M_{\theta\phi} = \pm m \sin\theta, \tag{23}$$

which is the magnetic field strength of a Dirac monopole. Note, however, that this monopole arose from the compactified three-form H, and arises in all versions of this solution. In particular, one may obtain a multi-magnetic monopole solution of purely bosonic string theory [7]. A similar reduction of instantons to monopoles was done in [41].

We now modify the solution of the 't Hooft ansatz even further and choose two directions in the four-space (1234) (say x^3 and x^4) and assume all fields are independent of both of these coordinates. We may now consistently assume that $x^3, x^4, x^6, x^7, x^8, x^9$ are compactified on a six-dimensional torus, where we shall take the x^3 and x^4 circles to have circumference $Le^{-\phi_0}$ and the remainder to have circumference L, so that $\kappa_4^2 = \kappa_{10}^2 e^{2\phi_0}/L^6$. Then the solution for f satisfying $f^{-1}\Box f = 0$ has multi-string structure

$$f_S = 1 - \sum_{i=1}^{N} \lambda_i \ln|\vec{x} - \vec{a}_i|, \tag{24}$$

where λ_i is the charge per unit length and $\vec{a}_i$ the location in the two-space (12) of the ith string. By setting $e^{2\phi} = e^{2\phi_0} f_S$, we obtain from (16) a multi-string solution which reduces to a solution in the four-dimensional space (0125). The reduction from ten to six dimensions is trivial, as the metric is flat in the subspace (6789). In going from six to four dimensions, we compactify the x_3 and x_4 directions and again follow the Kaluza-Klein procedure by replacing g_{33} and g_{44} with a scalar field $e^{-2\sigma}$. The tree-level effective action reduces in four dimensions to

$$S_4 = \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-g} e^{-2\phi-2\sigma} \left(R + 4(\partial\phi)^2 + 8\partial\sigma \cdot \partial\phi + 2(\partial\sigma)^2 - e^{4\sigma} \frac{N_\rho N^\rho}{2} \right), \tag{25}$$

where $\rho = 0, 1, 2, 5$, where $N_\rho = H_{\rho 34} = \partial_\rho B$, and where $B = B_{34}$. The four-dimensional string soliton solution for this reduced action is then given by [17]

$$\begin{aligned} e^{2\phi} &= e^{-2\sigma} = e^{2\phi_0} \left(1 - \sum_{i=1}^{N} \lambda_i \ln|\vec{x} - \vec{a}_i| \right), \\ ds^2 &= -dt^2 + dx_5^2 + e^{2\phi} \left(dx_1^2 + dx_2^2 \right), \\ N_i &= \pm \epsilon_{ij} \partial_j e^{2\phi}. \end{aligned} \tag{26}$$

We complete the family of solitons that can be obtained from the solutions of the 't Hooft ansatz by demanding that f depend on only one coordinate, say x^1. We may now consistently assume that $x^2, x^3, x^4, x^7, x^8, x^9$ are compactified on a six-dimensional torus, where we shall take the x^2, x^3 and x^4 circles to have circumference $Le^{-\phi_0}$ and the rest to have circumference L, so that $\kappa_4^2 = \kappa_{10}^2 e^{3\phi_0}/L^6$. Then the solution of $f^{-1}\Box f = 0$ has domain wall structure with the "confining potential"

$$f_D = 1 + \Lambda x_1, \tag{27}$$

where Λ is a constant. By setting $e^{2\phi} = e^{2\phi_0} f_D$ in (16), we obtain a multi-domain wall solution which once more can be explicitly reduced to four dimensions. For the spacetime (0156), the tree-level effective action in $D = 4$ has the form

$$S_4 = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} e^{-2\phi - 3\sigma} \left(R + 4(\partial\phi)^2 + 12\partial\sigma \cdot \partial\phi + 6(\partial\sigma)^2 - e^{6\sigma} \frac{P^2}{2} \right), \tag{28}$$

where P is a cosmological constant. Note that (28) is not obtained by a simple reduction of the ten-dimensional action owing to the nonvanishing of H_{234}. The four-dimensional domain wall solution for this reduced action is then given by [17]

$$\begin{aligned} e^{2\phi} = e^{-2\sigma} &= e^{2\phi_0} \left(1 + \sum_{i=1}^{N} \Lambda_i |x_1 - a_i| \right), \\ ds^2 &= -dt^2 + dx_5^2 + dx_6^2 + e^{2\phi} dx_1^2, \\ P &= \sum_{i=1}^{N} \Lambda_i \left(\Theta(x_1 - a_i) - \Theta(-x_1 + a_i) \right). \end{aligned} \tag{29}$$

A trivial change of coordinates reveals that the spacetime is, in fact, flat. Dilatonic domain walls with a flat spacetime have been discussed in a somewhat different context in [42].

For both strings and domain walls, generalizations to solutions with $A_M = \Omega_M$ are straightforward [43,17]. In all three cases, the multi-soliton solutions break half the spacetime supersymmetries of the $N = 4$ four-dimensional heterotic string theory to which the original $N = 1$ ten-dimensional heterotic string theory is toroidally compactified. The existence of these solutions owes to the saturation of a Bogomol'nyi bound between mass per unit volume and topological charge, and which results in a "zero-force" condition analogous to that found for BPS monopoles [25,21].

4 String/String Duality

Let us focus on the solitonic string configuration (26) in the case of a single source [17]. In terms of the complex field

$$
\begin{aligned}
T &= T_1 + iT_2 \\
&= e^{-2\sigma} - iB_{34} \\
&= \sqrt{\det g^S_{mn}} - iB_{34} \qquad m,n = 3,4,
\end{aligned}
\tag{30}
$$

where g^S_{MN} is the string σ-model metric, the solution takes the form (with $z = x_1 + x_2$)

$$
\begin{aligned}
T &= -\frac{1}{2\pi}\ln\frac{z}{r_0}, \\
ds^2 &= -dt^2 + dx_5^2 - \frac{1}{2\pi}\ln\frac{r}{r_0}dzd\bar{z},
\end{aligned}
\tag{31}
$$

whereas both the four-dimensional (shifted) dilaton $\eta = \phi + \sigma$ and the four-dimensional two-form $B_{\mu\nu}$ are zero. In terms of the canonical metric $g_{\mu\nu}$, T_1 and T_2, the relevant part of the action takes the form

$$
S_4 = \frac{1}{2\kappa_4^2}\int d^4x\sqrt{-g}\left(R - \frac{1}{2T_1^2}g^{\mu\nu}\partial_\mu T\partial_\nu\overline{T}\right) \tag{32}
$$

and is invariant under the $SL(2,R)$ transformation

$$
T \to \frac{aT+b}{cT+d}, \qquad ad - bc = 1. \tag{33}
$$

The discrete subgroup $SL(2,Z)$, for which a, b, c and d are integers, is just a subgroup of the $O(6,22;Z)$ *target space duality*, which can be shown to be an exact symmetry of the compactified string theory at each order of the string loop perturbation expansion.

This $SL(2,Z)$ is to be contrasted with the $SL(2,Z)$ symmetry of the elementary four-dimensional solution of Dabholkar *et al.* [18]. In the latter solution T_1 and T_2 are zero, but η and $B_{\mu\nu}$ are non-zero. The relevant part of the action is

$$
S_4 = \frac{1}{2\kappa_4^2}\int d^4x\sqrt{-g}\left(R - 2g^{\mu\nu}\partial_\mu\eta\partial_\nu\eta - \frac{1}{12}e^{-4\eta}H_{\mu\nu\rho}H^{\mu\nu\rho}\right). \tag{34}
$$

The equations of motion of this theory also display an $SL(2,R)$ symmetry, but this becomes manifest only after dualizing and introducing the axion field a via

$$
\sqrt{-g}g^{\mu\nu}\partial_\nu a = \frac{1}{3!}\epsilon^{\mu\nu\rho\sigma}H_{\nu\rho\sigma}e^{-4\eta}. \tag{35}
$$

Then in terms of the complex field

$$\begin{aligned} S &= S_1 + iS_2 \\ &= e^{-2\eta} + ia \end{aligned} \tag{36}$$

the Dabholkar *et al.* fundamental string solution may be written

$$\begin{aligned} S &= -\frac{1}{2\pi} \ln \frac{z}{r_0}, \\ ds^2 &= -dt^2 + dx_5^2 - \frac{1}{2\pi} \ln \frac{r}{r_0} dz d\bar{z}. \end{aligned} \tag{37}$$

Thus (31) and (37) are the same with the replacement $T \leftrightarrow S$. It has been conjectured that this second $SL(2,Z)$ symmetry may also be a symmetry of string theory [44,45,46,47], but this is far from obvious order by order in the string loop expansion since it involves a strong/weak coupling duality $\eta \rightarrow -\eta$. What interpretation are we to give to these two $SL(2,Z)$ symmetries: one an obvious symmetry of the fundamental string and the other an obscure symmetry of the fundamental string?

Related issues are brought up in the recent interesting papers by Sen [48], Schwarz and Sen [19] and Binétruy [20]. In particular, Sen draws attention to the Dabholkar *et al.* string solution (37) and its associated $SL(2,Z)$ symmetry as supporting evidence in favor of the conjecture that $SL(2,Z)$ invariance may indeed be an exact symmetry of string theory. He also notes that the spectrum of electric and magnetic charges is consistent with the proposed $SL(2,Z)$ symmetry [48].†

All of these observations fall into place if one accepts the proposal of Schwarz and Sen [19]: *under string/fivebrane duality the roles of the target-space duality and the strong/weak coupling duality are interchanged !* This proposal is entirely consistent with an earlier one that under string/fivebrane duality the roles of the σ-model loop expansion and the string loop expansion are interchanged [49]. In this light, the two $SL(2,Z)$ symmetries discussed above are just what one expects. From the string point of view, the T-field $SL(2,Z)$ is an obvious target space symmetry, manifest order by order in string loops whereas the S-field $SL(2,Z)$ is an obscure strong/weak coupling symmetry. From

† Sen also discusses the concept of a "dual string", but for him this is obtained from the fundamental string by an $SL(2,Z)$ transform. For us, a dual string is obtained by the replacement $S \leftrightarrow T$.

the fivebrane point of view, it is the T-field $SL(2,Z)$ which is obscure while the S-field $SL(2,Z)$ is an "obvious" target space symmetry. (This has not yet been proved except at the level of the low-energy field theory, however. It would be interesting to have a proof starting from the worldvolume of the fivebrane.) This interchange in the roles of the S and T field in going from the string to the fivebrane has also been noted by Binétruy [20]. It is made more explicit when S is expressed in terms of the variables appearing naturally in the fivebrane version

$$\begin{aligned} S &= S_1 + iS_2 \\ &= e^{-2\eta} + ia_{034789}, \\ &= \sqrt{\det g^F_{mn}} + ia_{034789}, \qquad m,n = 3,4,6,7,8,9, \end{aligned} \tag{38}$$

where $g^F_{MN} = e^{-\phi/3} g^S_{MN}$ is the fivebrane σ-model metric [13] and a_{MNPQRS} is the 6-form which couples to the 6-dimensional worldvolume of the fivebrane, in complete analogy with (30). For a recent concise description of four-dimensional string/string duality see [50].

It may at first sight seem strange that a string can be dual to another string in $D=4$. After all, the usual formula relating the dimension of an extended object, d, to that of the dual object, $\tilde{d}$, is $\tilde{d} = D - d - 2$. So one might expect string/string duality only in $D=6$ [49]. However, when we compactify n dimensions and allow the dual object to wrap around $m \leq d-1$ of the compactified directions we find $\tilde{d}_{\text{effective}} = \tilde{d} - m = D_{\text{effective}} - d - 2 + (n-m)$, where $D_{\text{effective}} = D - n$. In particular for $D_{\text{effective}} = 4$, $d=2$, $n=6$ and $m=4$, we find $\tilde{d}_{\text{effective}} = 2$.

Thus the whole string/fivebrane duality conjecture is put in a different light when viewed from four dimensions. After all, our understanding of the quantum theory of fivebranes in $D=10$ is rather poor, whereas the quantum theory of strings in $D=4$ is comparatively well-understood (although we still have to worry about the monopoles and domain walls). In particular, the dual string will presumably exhibit the normal kind of mass spectrum with linearly rising Regge trajectories, since the classical ($\hbar$-independent) string expression $\widetilde{T}_6 L^4 \times$ (angular momentum) has dimensions of (mass)^2, whereas the analogous classical expression for an uncompactified fivebrane is $(\widetilde{T}_6)^{1/5} \times$ (angular momentum) which has dimensions (mass)$^{6/5}$ [12]. Indeed, together with the observation that the $SL(2,Z)$ strong/weak coupling duality appears only after compactifying at least 6 dimensions, it is tempting to revive the earlier conjecture [12,51] that the internal consistency of the fivebrane may actually *require* compactification.

In more recent work, Sen [52] argued that the $SL(2,Z)$ S-duality group and the $O(6,22,Z)$ T-duality group may be combined in a larger $O(8,24,Z)$ duality group of three-dimensional heterotic string theory, in which the string-like solitons of four-dimensional heterotic string theory appear as point-like objects. Following the notation of [52], it is straightforward to produce an explicit $O(8,24;Z)$ transformation that takes the S string to the T string. We first find $\mathcal{M}_T$, the 32×32 matrix that corresponds to the T string. It turns out that this is simply what one obtains from $\mathcal{M}_S$, the 32×32 matrix that corresponds to the S string, on exchanging (rows and columns) 1 with 10, 2 with 31, 3 with 8 and 9 with 32, and is thus an explicit $O(8,24;Z)$ transformation (for more details see [26]). An interesting discussion of the connections between duality groups can be found in [53].

5 D=4 Black Holes

We now extend the three solutions of section 3 to two-parameter solutions of the low-energy equations of the four-dimensional heterotic string [22], characterized by a mass per unit p-volume, $\mathcal{M}_{p+1}$, and magnetic charge, g_{p+1}, where $p = 0, 1$ or 2. The solitons discussed in section 3 are recovered in the extremal limit, $\sqrt{2}\kappa\mathcal{M}_{p+1} = g_{p+1}$. The two-parameter solution extending the supersymmetric monopole corresponds to a magnetically charged black hole, while the solution extending the supersymmetric domain wall corresponds to a black membrane. By contrast, the two-parameter string solution does not possess a finite horizon and corresponds to a naked singularity.

All three solutions involve both the dilaton and the modulus fields, and are thus to be contrasted with pure dilaton solutions. In particular, the effective scalar coupling to the Maxwell field, $e^{-\alpha\phi}F_{\mu\nu}F^{\mu\nu}$, gives rise to a new string black hole with $\alpha = \sqrt{3}$, in contrast to the pure dilaton solution of the heterotic string which has $\alpha = 1$ [54–62]. It thus resembles the black hole previously studied in the context of Kaluza-Klein theories [63,64,38–40,65,54,28] which also has $\alpha = \sqrt{3}$, and which reduces to the Pollard-Gross-Perry-Sorkin [38–40] magnetic monopole in the extremal limit. In this connection, we recall [66], according to which $\alpha > 1$ black holes behave like elementary particles!

The fact that the heterotic string admits $\alpha = \sqrt{3}$ black holes also has implications for string/fivebrane duality [12–13]. We shall show that electric/magnetic duality in $D = 4$ may be seen as a consequence of string/fivebrane duality in $D = 10$.

We begin with the two-parameter black hole. The solution of the action (21) is given by

$$\begin{aligned} e^{-2\Phi} &= e^{2\sigma_1} = \left(1 - \frac{r_-}{r}\right), \\ ds^2 &= -\left(1-\frac{r_+}{r}\right)\left(1-\frac{r_-}{r}\right)^{-1} dt^2 + \left(1-\frac{r_+}{r}\right)^{-1} dr^2 + r^2\left(1-\frac{r_-}{r}\right) d\Omega_2^2, \\ F_{\theta\varphi} &= \sqrt{r_+ r_-}\sin\theta \end{aligned} \tag{39}$$

where here, and throughout this section, we set the dilaton vev Φ_0 equal to zero. This represents a magnetically charged black hole with event horizon at $r = r_+$ and inner horizon at $r = r_-$. The magnetic charge and mass of the black hole are given by

$$\begin{aligned} g_1 &= \frac{4\pi}{\sqrt{2}\kappa}(r_+ r_-)^{\frac{1}{2}}, \\ \mathcal{M}_1 &= \frac{2\pi}{\kappa^2}(2r_+ - r_-). \end{aligned} \tag{40}$$

Changing coordinates via $y = r - r_-$ and taking the extremal limit $r_+ = r_-$ yields:

$$\begin{aligned} e^{2\Phi} &= e^{-2\sigma_1} = \left(1 + \frac{r_-}{y}\right), \\ ds^2 &= -dt^2 + e^{2\Phi}\left(dy^2 + y^2 d\Omega_2^2\right), \\ F_{\theta\varphi} &= r_- \sin\theta. \end{aligned} \tag{41}$$

which is just the supersymmetric monopole solution of section 3 which saturates the Bogomol'nyi bound $\sqrt{2}\kappa\mathcal{M}_1 \geq g_1$.

Next we derive a two-parameter string solution which, however, does not possess a finite event horizon and consequently cannot be interpreted as a black string. A two-parameter family of solutions of the action (25) is now given by

$$\begin{aligned} e^{2\Phi} &= e^{-2\sigma_2} = (1 + k/2 - \lambda \ln y), \\ ds^2 &= -(1+k)dt^2 + (1+k)^{-1}(1+k/2-\lambda\ln y)dy^2 + y^2(1+k/2-\lambda\ln y)d\theta^2 + dx_3^2, \\ F_\theta &= \lambda\sqrt{1+k}, \end{aligned} \tag{42}$$

where for $k = 0$ we recover the supersymmetric string soliton solution of section 3 which, as shown in section 4, is dual to the elementary string solution of Dabholkar *et al.*. The solution shown in (42) in fact represents a naked singularity, since the event horizon is pushed out to $r_+ = \infty$, which agrees with the Horowitz-Strominger "no-4D-black-string" theorem [67].

Finally, we consider the two-parameter black membrane solution. The two-parameter black membrane solution of the action (28) is then

$$\begin{aligned} e^{-2\Phi} &= e^{2\sigma_3} = \left(1 - \frac{r}{r_-}\right), \\ ds^2 &= -\left(1 - \frac{r}{r_+}\right)\left(1 - \frac{r}{r_-}\right)^{-1} dt^2 + \left(1 - \frac{r}{r_+}\right)^{-1}\left(1 - \frac{r}{r_-}\right)^{-4} dr^2 + dx_2^2 + dx_3^2, \\ F &= -(r_+ r_-)^{-1/2}. \end{aligned} \tag{43}$$

This solution represents a black membrane with event horizon at $r = r_+$ and inner horizon at $r = r_-$. Changing coordinates via $y^{-1} = r^{-1} - r_-^{-1}$ and taking the extremal limit yields

$$\begin{aligned} e^{2\Phi} &= e^{-2\sigma_3} = \left(1 + \frac{y}{r_-}\right), \\ ds^2 &= -dt^2 + dx_2^2 + dx_3^2 + e^{2\Phi} dy^2, \\ F &= -\frac{1}{r_-}. \end{aligned} \tag{44}$$

which is just the supersymmetric domain wall solution of section 3.

We note that the black hole solution corresponds to a Maxwell-scalar coupling $e^{-\alpha\phi} F_{\mu\nu} F^{\mu\nu}$ with $\alpha = \sqrt{3}$. This is to be contrasted with the pure dilaton black hole solutions of the heterotic string that have attracted much attention recently [54–62] and have $\alpha = 1$ *. The case $\alpha = \sqrt{3}$ also occurs when the Maxwell field and the scalar field ϕ arise from a Kaluza-Klein reduction of pure gravity from $D = 5$ to $D = 4$:

$$\hat{g}_{MN} = e^{\frac{\phi}{\sqrt{3}}} \begin{pmatrix} g_{\mu\nu} + e^{-\sqrt{3}\phi} A_\mu A_\nu & e^{-\sqrt{3}\phi} A_\mu \\ e^{-\sqrt{3}\phi} A_\nu & e^{-\sqrt{3}\phi} \end{pmatrix}, \tag{45}$$

where $\hat{g}_{MN}\,(M, N = 0,1,2,3,4)$ and $g_{\mu\nu}\,(\mu,\nu = 0,1,2,3)$ are the canonical metrics in 5 and 4 dimensions respectively. The resulting action is given by

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left[R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4} e^{-\sqrt{3}\phi} F_{\mu\nu} F^{\mu\nu} \right], \tag{46}$$

* Contrary to some claims in the literature, the pure Reissner-Nordström black hole with $\alpha = 0$ is also a solution of the low energy heterotic string equations. This may be seen by noting that it provides a solution to ($N = 2$, $D = 4$) supergravity which is a consistent truncation of toroidally compactified $N = 1$, $D = 10$ supergravity [68].

and it admits as an "elementary" solution the $\alpha = \sqrt{3}$ black hole metric (39), but with the scalar field

$$e^{-2\phi} = \Delta_-^{\sqrt{3}} \tag{47}$$

and the electric field

$$\frac{1}{\sqrt{2}\kappa} e^{-\sqrt{3}\phi} {}^*F_{\theta\varphi} = \frac{e}{4\pi} \sin\theta \tag{48}$$

corresponding to an electric monopole with Noether charge e. This system also admits the topological magnetic solution with

$$e^{-2\phi} = \Delta_-^{-\sqrt{3}} \tag{49}$$

and the magnetic field

$$\frac{1}{\sqrt{2}\kappa} F_{\theta\varphi} = \frac{g}{4\pi} \sin\theta \tag{50}$$

corresponding to a magnetic monopole with topological magnetic charge g obeying the Dirac quantization rule

$$eg = 2\pi n, \quad n = \text{integer}. \tag{51}$$

In effect, it was for this reason that the $\alpha = \sqrt{3}$ black hole was identified as a solution of the Type II string in [69], the fields A_μ and ϕ being just the abelian gauge field and the dilaton of ($N = 2$, $D = 10$) supergravity which arises from Kaluza-Klein compactification of ($N = 1$, $D = 11$) supergravity.

Some time ago, it was pointed out in [65] that $N = 8$ supergravity, compactified from $D = 5$ to $D = 4$, admits an infinite tower of elementary states with mass m_n and electric charge e_n given by $\sqrt{2}\kappa m_n = e_n$, where e_n are quantized in terms of a fundamental charge e, $e_n = n\, e$, and that these elementary states fall into $N = 8$ supermultiplets. They also pointed out that this theory admits an infinite tower of solitonic states with the masses $\tilde{m}_n$ and magnetic charge g_n given by $\sqrt{2}\kappa\tilde{m}_n = g_n = n\, g$, where e and g obey $eg = 2\pi$, which also fall into the same $N = 8$ supermultiplets. The authors of [65] conjectured, á la Olive-Montonen [34], that there should exist a dual formulation of the theory for which the roles of electric elementary states and magnetic solitonic states are interchanged. It was argued in [69] that this electric/magnetic duality conjecture in $D = 4$ could be reinterpreted as a particle/sixbrane duality conjecture in $D = 10$.

To see this, consider the action dual to S, with $\alpha = -\sqrt{3}$, for which the roles of Maxwell field equations and Bianchi identities are interchanged:

$$\tilde{S} = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}e^{\sqrt{3}\phi}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu}\right], \tag{52}$$

where $\tilde{F}_{\mu\nu} = e^{-\sqrt{3}\phi}\,{}^*F_{\mu\nu}$. This is precisely the action obtained by double dimensional reduction of a dual formulation of $(D = 10,\ N = 2)$ supergravity in which the two-form F_{MN} $(M, N = 0, ..., 9)$ is swapped for an 8-form $\tilde{F}_{M_1..M_8}$, where $\tilde{F}_{\mu\nu} = \tilde{F}_{\mu\nu456789}$. This dual action also admits both electric and magnetic monopole solutions but because the roles of field equations and Bianchi identities are interchanged, so are the roles of electric and magnetic. Since the 1-form and 7-form potentials, which give rise to these 2-form and 8-form field strengths, are those that couple naturally to the worldline of a point particle or the worldvolume of a 6-brane, we see that the Gibbons-Perry $(N = 8,\ D = 4)$ electric/magnetic duality conjecture may be re-expressed as an (Type II, $D = 10$) particle/sixbrane duality conjecture. Indeed, the $D = 10$ black sixbrane of [67] is simply obtained by adding 6 flat dimensions to the $D = 4$, $\alpha = \sqrt{3}$ magnetic black hole.

In general, the $N = 8$ theory will admit black holes with $\alpha = 0, 1$ and $\sqrt{3}$ whose extreme limits preserve $1, 2$ or 4 spacetime supersymmetries, respectively. Defining $\mathcal{M}_1 = M$, $g_1^2 = 4\pi Q^2$ and $\kappa^2 = 8\pi G$, these extreme black holes satisfy the "no-force" condition, i.e. they saturate the Bogomol'nyi bounds

$$G(M^2 + \Sigma^2) = (1 + \alpha^2)GM^2 = N'GM^2 = Q^2 \tag{53}$$

where $\Sigma = \alpha M$ is the scalar charge and N' is the number of unbroken supersymmetries.

The solutions presented in this section now allow us to discuss the $\alpha = \sqrt{3}$ electric/magnetic duality from a totally different perspective from above. For concreteness, let us focus on generic toroidal compactification of the heterotic string. Instead of $N = 8$ supergravity, the four-dimensional theory is now $N = 4$ supergravity coupled to 22 $N = 4$ vector multiplets[†]. The same dual Lagrangians (46) and (52) still emerge but with completely different origins. The Maxwell field $F_{\mu\nu}$ (or $\tilde{F}_{\mu\nu}$) and the scalar field ϕ do not come

[†] Gibbons discusses both the $\alpha = 1$ black hole of pure $N = 4$ supergravity and the $\alpha = \sqrt{3}$ Kaluza-Klein black hole in the same paper [54], as does Horowitz [28]. Moreover, black holes in pure $N = 4$ supergravity are treated by Kallosh *et al.* [58–60]. The reader may therefore wonder why the $\alpha = \sqrt{3}$ $N = 4$ black hole discussed in the present paper was overlooked. The reason is that pure $N = 4$ supergravity does not admit the $\alpha = \sqrt{3}$ solution; it is crucial that we include the $N = 4$ vector multiplets in order to introduce the modulus fields.

from the $D = 10$ 2-form (or 8-form) and dilaton of the Type II particle (or sixbrane), but rather from the $D = 10$ 3-form (or 7-form) and dilaton plus modulus field of the heterotic string (or heterotic fivebrane). Thus, the $D = 4$ electric/magnetic duality can now be re-interpreted as a $D = 10$ string/fivebrane duality!

Because of the non-vanishing modulus field $g_{44} = e^{-2\sigma}$ however, the $D = 10$ black fivebrane solution is not obtained by adding 6 flat dimensions to the $D = 4$ black hole. Rather the two are connected by wrapping the fivebrane around 5 of the 6 extra dimensions [17].

Of course, we have established only that these two-parameter configurations are solutions of the field theory limit of the heterotic string. Although the extreme one-parameter solutions are expected to be exact to all orders in α', the same reasoning does not carry over to the new two-parameter solutions.

It would be also interesting to see whether the generalization of the one-parameter solutions of section 3 to the two-parameter solutions of this section can be carried out when we include the Yang-Mills coupling. This would necessarily involve giving up the self-duality condition on the Yang-Mills field strength, however, since the self-duality condition is tied to the extreme, $\sqrt{2}\kappa\mathcal{M}_{p+1} = g_{p+1}$, supersymmetric solutions.

6 Dynamics of string solitons

All the soliton solutions we have discussed so far have the property, like BPS magnetic monopoles, that they exert zero static force on each other and can be superposed to form multi-soliton solutions with arbitrarily variable collective coordinates. Since these static properties are also possessed by fundamental strings winding around an infinitely large compactified dimension, it was conjectured in [18] that the elementary string is actually the exterior solution for an infinitely long fundamental string. In this section we show that, in contradistinction to the BPS case, the velocity-dependent forces between these string solitons also vanish (i.e. we argue that the scattering is trivial). We also argue that this phenomenon provides further, dynamical evidence for the identification of the elementary string solution with the underlying fundamental string by comparing the scattering of the elementary solutions with expectations from a Veneziano amplitude computation for macroscopic fundamental strings [23].

As shown in [70], the static ansatz leads to a vanishing leading-order velocity-dependent force to for a test string propagating in the background of an elementary string. This result also holds for the other soliton solutions we have discussed in this paper. In particular, test monopoles propagating in the background of a source monopole also do not experience a dynamic force to leading order. As this is a rather surprising result, we would like to compute the metric on moduli space for these solitons. The geodesics of this metric represent the motion of quasi-static solutions in the static solution manifold and in the absence of a full time dependent solution provide a good approximation to the low-energy dynamics of the solitons. In all cases the metric is found to be flat in agreement with the test-soliton approximation, which again implies vanishing dynamical force in the low-velocity limit. Here we summarize the computation for the metric on moduli space for monopoles discussed in [71,43].

Manton's prescription [24] for the study of soliton scattering may be summarized as follows. We first invert the constraint equations of the system. The resultant time dependent field configuration does not in general satisfy the full time dependent field equations, but provides an initial data point for the fields and their time derivatives. Another way of saying this is that the initial motion is tangent to the set of exact static solutions. The kinetic action obtained by replacing the solution to the constraints into the action defines a metric on the parameter space of static solutions. This metric defines geodesic motion on the moduli space [24].

A calculation of the metric on moduli space for the scattering of BPS monopoles and a description of its geodesics was worked out by Atiyah and Hitchin [72]. Several interesting properties of monopole scattering were found, such as the conversion of monopoles into dyons and the right angle scattering of two monopoles on a direct collision course [72,73]. The configuration space is found to be a four-dimensional manifold M_2 with a self-dual Einstein metric.

Here we adapt Manton's prescription to study the dynamics of the heterotic string monopoles discussed in section 3. We follow essentially the same steps that Manton outlined for monopole scattering, but take into account the peculiar nature of the string effective action. Since we work in the low-velocity limit, our kinematic analysis is nonrelativistic.

We first solve the constraint equations for the monopoles. These equations are simply the $(0j)$ components of the tree-level equations of motion

$$\begin{aligned} R_{0j} - \frac{1}{4}H^2_{0j} + 2\nabla_0\nabla_j\phi = 0, \\ -\frac{1}{2}\nabla_k H^k{}_{0j} + H_{0j}{}^k\partial_k\phi = 0 \end{aligned} \qquad (54)$$

which follow from the action (17). We wish to find an $O(\beta)$ solution to the above equations which represents a quasi-static version of the neutral multi-monopole solution (i.e. a multi-monopole solution with time dependent $\vec{a}_i$). Here we use the uncompactified solution (16) with $e^{2\phi} = e^{2\phi_0} f_M$, with f_M given in (20), as opposed to the explicitly compactified version (22) (in particular, we do not make the replacement $g_{44} = e^{-2\sigma}$) although the results in both cases are identical, in order to more easily keep track of the terms in the former case. We give each monopole an arbitrary transverse velocity $\vec{\beta}_n$ in the (123) subspace of the four-dimensional transverse space and see what corrections to the fields are required by the constraints. The vector $\vec{a}_n$ representing the position of the nth monopole in the three-space (123) is given by

$$\vec{a}_n(t) = \vec{A}_n + \vec{\beta}_n t, \qquad (55)$$

where $\vec{A}_n$ is the initial position of the nth monopole. Note that at $t = 0$ we recover the exact static multi-monopole solution. Our solution to the constraints will adjust our quasi-static approximation so that the initial motion in the parameter space is tangent to the initial exact solution at $t = 0$. The $O(\beta)$ solution to the constraints is given by [71]

$$\begin{aligned} e^{2\phi(\vec{x},t)} &= 1 + \sum_{n=1}^{N} \frac{m_n}{|\vec{x} - \vec{a}_n(t)|}, \\ g_{00} &= -1, \qquad g^{00} = -1, \qquad g_{ij} = e^{2\phi}\delta_{ij}, \qquad g^{ij} = e^{-2\phi}\delta_{ij}, \\ g_{0i} &= -\sum_{n=1}^{N} \frac{m_n\vec{\beta}_n \cdot \hat{x}_i}{|\vec{x} - \vec{a}_n(t)|}, \qquad g^{0i} = e^{-2\phi} g_{0i}, \\ H_{ijk} &= \epsilon_{ijkm}\partial_m e^{2\phi}, \\ H_{0ij} &= \epsilon_{ijkm}\partial_m g_{0k} = \epsilon_{ijkm}\partial_k \sum_{n=1}^{N} \frac{m_n\vec{\beta}_n \cdot \hat{x}_m}{|\vec{x} - \vec{a}_n(t)|}, \end{aligned} \qquad (56)$$

where $i, j, k, m = 1, 2, 3, 4$, all other metric components are flat, all other components of H vanish, the $\vec{a}_n(t)$ are given by (55) and we use a flat space ϵ-tensor. Note that g_{00}, g_{ij} and

H_{ijk} are unaffected to order β. Also note that we can interpret the monopoles as either strings in the space (01234) or point objects in the three-dimensional subspace (0123).

The kinetic Lagrangian is obtained by replacing the expressions for the fields in (56) into the string σ-model action (17) *. Since (56) is a solution to order β, the leading order terms in the action (after the quasi-static part) are of order β^2. The $O(\beta)$ terms in the solution give $O(\beta^2)$ terms when replaced in the kinetic action. Collecting all $O(\beta^2)$ terms in (17) we get the following kinetic Lagrangian density for the volume term:

$$\begin{aligned}\mathcal{L}_{kin} = -\frac{1}{2\kappa^2}\Bigg(& 4\dot{\phi}\vec{M}\cdot\vec{\nabla}\phi - e^{-2\phi}\partial_i M_j \partial_i M_j - e^{-2\phi}M_k\partial_j\phi\left(\partial_j M_k - \partial_k M_j\right) \\ & + 4M^2 e^{-2\phi}(\vec{\nabla}\phi)^2 + 2\partial_t^2 e^{2\phi} - 4\partial_t(\vec{M}\cdot\vec{\nabla}\phi) - 4\vec{\nabla}\cdot(\dot{\phi}\vec{M})\Bigg),\end{aligned} \tag{57}$$

where $\vec{M} \equiv -\sum_{n=1}^{N} \frac{m_n\vec{\beta}_n}{|\vec{x}-\vec{a}_n(t)|}$. Henceforth let $\vec{X}_n \equiv \vec{x} - \vec{a}_n(t)$. The last three terms in (57) are time-surface or space-surface terms which vanish when integrated over the uncompactified four-space (0123). The kinetic Lagrangian $L_{kin} = \int d^3x\mathcal{L}_{kin}$ for monopole scattering converges everywhere. This can be seen simply by studying the limiting behaviour of L_{kin} near each monopole. For a single monopole at $r = 0$ with magnetic charge m and velocity β, we collect the logarithmically divergent pieces and find that they cancel:

$$\frac{m\beta^2}{2}\int r^2 dr d\theta \sin\theta d\phi \left(-\frac{1}{r^3} + \frac{3\cos^2\theta}{r^3}\right) = 0. \tag{58}$$

We now specialize to the case of two identical monopoles of magnetic charge $m_1 = m_2 = m$ and velocities $\vec{\beta}_1$ and $\vec{\beta}_2$. Let the monopoles be located at $\vec{a}_1$ and $\vec{a}_2$. Our moduli space consists of the configuration space of the relative separation vector $\vec{a} \equiv \vec{a}_2 - \vec{a}_1$. The most general kinetic Lagrangian can be written as

$$\begin{aligned}L_{kin} =& h(a)(\vec{\beta}_1\cdot\vec{\beta}_1 + \vec{\beta}_2\cdot\vec{\beta}_2) + p(a)\left((\vec{\beta}_1\cdot\hat{a})^2 + (\vec{\beta}_2\cdot\hat{a})^2\right) \\ & + 2f(a)\vec{\beta}_1\cdot\vec{\beta}_2 + 2g(a)(\vec{\beta}_1\cdot\hat{a})(\vec{\beta}_2\cdot\hat{a}).\end{aligned} \tag{59}$$

Now suppose $\vec{\beta}_1 = \vec{\beta}_2 = \vec{\beta}$, so that (59) reduces to

$$L_{kin} = (2h + 2f)\beta^2 + (2p + 2g)(\vec{\beta}\cdot\hat{a})^2. \tag{60}$$

* Strictly speaking one must add to (17) a surface term to cancel the double derivative terms in the action [74–76,77,71] however the addition of this term introduces only flat kinetic terms and thus presents no nontrivial contribution to the metric on moduli space.

This configuration, however, represents the boosted solution of the two-monopole static solution. The kinetic energy should therefore be simply

$$L_{kin} = \frac{M_T}{2}\beta^2, \tag{61}$$

where $M_T = M_1 + M_2 = 2M$ is the total mass of the two-monopole solution. It then follows that the anisotropic part of (60) vanishes and we have

$$\begin{aligned} g + p &= 0, \\ 2(h+f) &= \frac{M_T}{2}. \end{aligned} \tag{62}$$

It is therefore sufficient to compute h and p. This can be done by setting $\vec{\beta}_1 = \vec{\beta}$ and $\vec{\beta}_2 = 0$. The kinetic Lagrangian then reduces to

$$L_{kin} = h(a)\beta^2 + p(a)(\vec{\beta}\cdot\hat{a})^2. \tag{63}$$

Suppose for simplicity also that $\vec{a}_1 = 0$ and $\vec{a}_2 = \vec{a}$ at $t = 0$. The Lagrangian density of the volume term in this case is given by

$$\begin{aligned} \mathcal{L}_{kin} = \frac{-1}{2\kappa^2}\Bigg(&\frac{3m^3e^{-4\phi}}{2r^4}(\vec{\beta}\cdot\vec{x})\left[\frac{\vec{\beta}\cdot\vec{x}}{r^3} + \frac{\vec{\beta}\cdot(\vec{x}-\vec{a})}{|\vec{x}-\vec{a}|^3}\right] - \frac{e^{-2\phi}m^2\beta^2}{r^4} \\ &- \frac{e^{-4\phi}m^3\beta^2}{2r^4}\left(\frac{1}{r} + \frac{\vec{x}\cdot(\vec{x}-\vec{a})}{|\vec{x}-\vec{a}|^3}\right) + \frac{e^{-6\phi}m^4\beta^2}{r^2}\left(\frac{1}{r^4} + \frac{1}{|\vec{x}-\vec{a}|^4} + \frac{2\vec{x}\cdot(\vec{x}-\vec{a})}{r^3|\vec{x}-\vec{a}|^3}\right)\Bigg). \end{aligned} \tag{64}$$

The integration of the kinetic Lagrangian density in (64) over three-space yields the kinetic Lagrangian from which the metric on moduli space can be read off. For large a, the nontrivial leading order behaviour of the components of the metric, and hence for the functions $h(a)$ and $p(a)$, is generically of order $1/a$. In fact, for Manton scattering of BPS monopoles, the leading order scattering angle is $2/b$ [24], where b is the impact parameter. Here we restrict our computation to the leading order metric in moduli space. A tedious but straightforward collection of $1/a$ terms in the Lagrangian yields

$$\frac{-1}{2\kappa^2}\frac{1}{a}\int d^3x\left[-\frac{3m^4e^{-6\phi_1}}{r^7}(\vec{\beta}\cdot\vec{x})^2 + \frac{m^3e^{-4\phi_1}}{r^4}\beta^2 + \frac{m^4e^{-6\phi_1}}{r^5}\beta^2 - \frac{3m^5e^{-8\phi_1}}{r^6}\beta^2\right], \tag{65}$$

where $e^{2\phi_1} \equiv 1 + m/r$. The first and third terms clearly cancel after integration over three-space. The second and fourth terms are spherically symmetric. A simple integration yields

$$\int_0^\infty r^2dr\left(\frac{e^{-4\phi_1}}{r^4} - \frac{3m^2e^{-8\phi_1}}{r^6}\right) = \int_0^\infty \frac{dr}{(r+m)^2} - 3m^2\int_0^\infty \frac{dr}{(r+m)^4} = 0. \tag{66}$$

The $1/a$ terms therefore cancel, and the leading order metric on moduli space is flat. This implies that to leading order the dynamical force is zero and the scattering is trivial, in agreement with the test-soliton result. In other words, there is no deviation from the initial trajectories to leading order in the impact parameter. Analogous computations for elementary strings in $D = 4$ [78] and fivebranes in $D = 10$ [77,79] lead to the same result of a flat metric. From $S \leftrightarrow T$ duality (see section 4) it follows that the metric on moduli space for solitonic strings in $D = 4$ is also flat.

We now address the scattering problem from the string theoretic point of view. In particular, we calculate the string four-point amplitude for the scattering of macroscopic winding state strings in the infinite winding radius limit. In this scenario, we can best approximate the soliton scattering problem considered above but in the case of elementary strings in $D = 4$. We find that the Veneziano amplitude obtained also indicates trivial scattering in the large winding radius limit, thus providing evidence for the identification of the elementary strings with infinitely long macroscopic fundamental strings. The fivebrane analog of this computation awaits the construction of a fundamental fivebrane theory. However, a vertex operator representation of fivebrane solitons (and also of string monopoles) should in principle be possible. The computation of the fivebrane Veneziano amplitude would then represent a dynamical test for string/fivebrane duality.

The scattering problem is set up in four dimensions, as the kinematics correspond essentially to a four dimensional scattering problem, and strings in higher dimensions generically miss each other anyway [80]. The precise compactification scheme is irrelevant to our purposes.

The winding state strings then reside in four spacetime dimensions (0123), with one of the dimensions, say x_3, taken to be periodic with period R, called the winding radius. The winding number n describes the number of times the string wraps around the winding dimension:

$$x_3 \equiv x_3 + 2\pi R n, \tag{67}$$

and the length of the string is given by $L = nR$. The integer m, called the momentum number of the winding configuration, labels the allowed momentum eigenvalues. The momentum in the winding direction is thus given by

$$p^3 = \frac{m}{R}. \tag{68}$$

The number m is restricted to be an integer so that the quantum wave function $e^{ip\cdot x}$ is single valued. The total momentum of each string can be written as the sum of a right momentum and a left momentum

$$p^\mu = p^\mu_R + p^\mu_L, \tag{69}$$

where $p^\mu_{R,L} = (E, E\vec{v}, \frac{m}{2R} \pm nR)$, $\vec{v}$ is the transverse velocity and R is the winding radius. The mode expansion of the general configuration $X(\sigma, \tau)$ in the winding direction satisfying the two-dimensional wave equation and the closed string boundary conditions can be written as the sum of right moving pieces and left moving pieces, each with the mode expansion of an open string [81]

$$\begin{aligned} X(\sigma,\tau) &= X_R(\tau-\sigma) + X_L(\tau+\sigma) \\ X_R(\tau-\sigma) &= x_R + p_R(\tau-\sigma) + \frac{i}{2}\sum_{n=0}\frac{1}{n}\alpha_n e^{-2in(\tau-\sigma)} \\ X_L(\tau+\sigma) &= x_L + p_L(\tau+\sigma) + \frac{i}{2}\sum_{n=0}\frac{1}{n}\tilde{\alpha}_n e^{-2in(\tau+\sigma)}. \end{aligned} \tag{70}$$

The right moving and left moving components are then essentially independent parts with corresponding vertex operators, number operators and Virasoro conditions.

The winding configuration described by $X(\sigma, \tau)$ describes a soliton string state. It is therefore a natural choice for us to compare the dynamics of these states with the soliton-like solutions of the previous sections (including the elementary solutions) in order to determine whether we can identify the elementary string solutions of the supergravity field equations with infinitely long fundamental strings. Accordingly, we study the scattering of the winding states in the limit of large winding radius.

Our kinematic setup is as follows. We consider the scattering of two straight macroscopic strings in the CM frame with winding number n and momentum number $\pm m$ [81,80]. The incoming momenta in the CM frame are given by

$$\begin{aligned} p^\mu_{1R,L} &= (E, E\vec{v}, \frac{m}{2R} \pm nR) \\ p^\mu_{2R,L} &= (E, -E\vec{v}, -\frac{m}{2R} \pm nR). \end{aligned} \tag{71}$$

Let $\pm m'$ be the outgoing momentum number. For the case of $m = m'$, the outgoing momenta are given by

$$\begin{aligned} -p^\mu_{3R,L} &= (E, E\vec{w}, \frac{m}{2R} \pm nR) \\ -p^\mu_{4R,L} &= (E, -E\vec{w}, -\frac{m}{2R} \pm nR), \end{aligned} \tag{72}$$

where conservation of momentum and winding number have been used and where $\pm\vec{v}$ and $\pm\vec{w}$ are the incoming and outgoing velocities of the strings in the transverse $x-y$ plane. The outgoing momenta winding numbers are not *a priori* equal to the initial winding numbers, but must add up to $2n$. Conservation of energy for sufficiently large R then results in the above answer. This is also in keeping with the soliton scattering nature of the problem (i.e. the solitons do not change "shape" during a collision).

For now we have assumed no longitudinal excitation ($m = m'$). We will later relax this condition to allow for such excitation, but show that our answer for the scattering is unaffected by this possibility. It follows from this condition that $v^2 = w^2$. For simplicity we take $\vec{v} = v\hat{x}$ and $\vec{w} = v(\cos\theta\hat{x} + \sin\theta\hat{y})$, and thus reduce the problem to a two-dimensional scattering problem.

As usual, the Virasoro conditions $L_0 = \widetilde{L}_0 = 1$ must hold, where

$$\begin{aligned} L_0 &= N + \tfrac{1}{2}(p_R^\mu)^2 \\ \widetilde{L}_0 &= \widetilde{N} + \tfrac{1}{2}(p_L^\mu)^2 \end{aligned} \qquad (73)$$

are the Virasoro operators [81] and where N and $\widetilde{N}$ are the number operators for the right- and left-moving modes respectively:

$$\begin{aligned} N &= \sum \alpha_{-n}^{\mu}\alpha_{n\mu} \\ \widetilde{N} &= \sum \tilde{\alpha}_{-n}^{\mu}\tilde{\alpha}_{n\mu}, \end{aligned} \qquad (74)$$

where we sum over all dimensions, including the compactified ones. It follows from the Virasoro conditions that

$$\begin{aligned} \widetilde{N} - N &= mn \\ E^2(1-v^2) &= 2N - 2 + \left(\frac{m}{2R} + nR\right)^2. \end{aligned} \qquad (75)$$

In the following we set $n = 1$ and consider for simplicity the scattering of tachyonic winding states. For our purposes, the nature of the string winding states considered is irrelevant. A similar calculation for massless bosonic strings or heterotic strings, for example, will be slightly more complicated, but will nevertheless exhibit the same essential behaviour. For tachyonic winding states we have $N = \widetilde{N} = m = 0$. Equation (75) reduces to

$$E^2(1-v^2) = R^2 - 2. \qquad (76)$$

The Mandelstam variables (s,t,u) are identical for right and left movers and are given by

$$\begin{aligned} s &= 4\left[\frac{(R^2-2)v^2}{1-v^2} - 2\right] \\ t &= -2\left[\frac{(R^2-2)v^2}{1-v^2}\right](1+\cos\theta) \\ u &= -2\left[\frac{(R^2-2)v^2}{1-v^2}\right](1-\cos\theta). \end{aligned} \tag{77}$$

It is easy to see that $p_{iR}\cdot p_{jR} = p_{iL}\cdot p_{jL}$ holds for this configuration so that the tree level 4-point function reduces to the usual Veneziano amplitude for closed tachyonic strings [80]

$$\begin{aligned} A_4 &= \frac{\kappa^2}{4}B(-1-s/2,-1-t/2,-1-u/2) \\ &= (\frac{\kappa^2}{4})\frac{\Gamma(-1-s/2)\Gamma(-1-t/2)\Gamma(-1-u/2)}{\Gamma(2+s/2)\Gamma(2+t/2)\Gamma(2+u/2)}. \end{aligned} \tag{78}$$

This can be seen as follows. In the standard computation of the four point function for closed string tachyons, we rely on the independence of the right and left moving open strings. For the tachyonic winding state, we also separate the right and left movers with vertex operators given by $V_R = e^{ip_R\cdot x_R}$ and $V_L = e^{ip_L\cdot x_L}$ respectively to arrive at the following expression for the amplitude

$$A_4 = \frac{\kappa^2}{4}\int d\mu_4(z)\prod_{i<j}|z_i - z_j|^{p_{iR}\cdot p_{jR}}|z_i - z_j|^{p_{iL}\cdot p_{jL}}. \tag{79}$$

From $p_{iR}\cdot p_{jR} = p_{iL}\cdot p_{jL}$, (79) reduces to the expression for the four-point amplitude of a nonwinding closed tachyonic string, from which the standard Veneziano amplitude in (78) results.

To compare the implications of A_4 with the results of the Manton calculation, we take $R\to\infty$. It is convenient to define $x \equiv \frac{(R^2-2)v^2}{1-v^2} = s/4+2$, since the Mandelstam variables can be expressed solely in terms of x and θ. We now have $A_4 = A_4(x,\theta)$, which can be explicitly written as

$$A_4 = (\frac{\kappa^2}{4})\frac{\Gamma(3-2x)\Gamma(-1+x(1+\cos\theta))\Gamma(-1+x(1-\cos\theta))}{\Gamma(-2+2x)\Gamma(2-x(1+\cos\theta))\Gamma(2-x(1-\cos\theta))}. \tag{80}$$

The problem reduces to studying A_4 in the limit $x\to\infty$. We now use the identity $\Gamma(1-a)\Gamma(a)\sin\pi a = \pi$ to rewrite A_4 as

$$\begin{aligned} A_4 = (\frac{\kappa^2}{4\pi})&\left[\frac{\Gamma(-1+x(1+\cos\theta))\Gamma(-1+x(1-\cos\theta))}{\Gamma(-2+2x)}\right]^2 \\ &\times\left(\frac{\sin(\pi x(1+\cos\theta))\sin(\pi x(1-\cos\theta))}{\sin 2\pi x}\right). \end{aligned} \tag{81}$$

From the Stirling approximation $\Gamma(u) \sim \sqrt{2\pi} u^{u-1/2} e^{-u}$ for large u, we obtain in the limit $x \to \infty$

$$A_4 \sim \left[\frac{(x(1+\cos\theta))^{x(1+\cos\theta)} (x(1-\cos\theta))^{x(1-\cos\theta)}}{(2x)^{2x}} \right]^2 \times \left(\frac{\sin(\pi x(1+\cos\theta)) \sin(\pi x(1-\cos\theta))}{\sin 2\pi x} \right). \tag{82}$$

Note that the exponential terms cancel automatically. From (82) we notice that the powers of x in the first factor also cancel. A_4 then reduces in the limit $x \to \infty$ to

$$A_4 \sim \left(\frac{1+\cos\theta}{2} \right)^{2x(1+\cos\theta)} \left(\frac{1-\cos\theta}{2} \right)^{2x(1-\cos\theta)} \times \left(\frac{\sin(\pi x(1+\cos\theta)) \sin(\pi x(1-\cos\theta))}{\sin 2\pi x} \right). \tag{83}$$

The poles in the third factor in (83) are just the usual s-channel poles. It follows from (83) that for $\theta \neq 0, \pi$ $A_4 \to e^{-f(\theta)x}$ as $x \to \infty$, where f is some positive definite function of θ. Hence the 4-point function vanishes exponentially with the winding radius away from the poles.

In general, for finite R and fixed v the strings may scatter into longitudinally excited final states, *i.e.* states not satisfying the above assumption that $m' = m$. The 4-point amplitude for each transition still vanishes exponentially with R. A simple counting argument shows that the total number of possible final states for a given R is bounded by a polynomial function of R. This counting argument proceeds as follows.

Without loss of generality, we may assume that our incoming states have $N = \widetilde{N} = m = 0$ with fixed R and v. We relax the assumption of no logitudinal excitation to obtain outgoing states with nonzero m. We still consider $n = 1$ winding states for simplicity. Our scattering configuration can now be described by the incoming momenta

$$\begin{aligned} p^{\mu}_{1R,L} &= (E, E\vec{v}, \pm R) \\ p^{\mu}_{2R,L} &= (E, -E\vec{v}, \pm R). \end{aligned} \tag{84}$$

and the outgoing momenta

$$\begin{aligned} -p^{\mu}_{3R,L} &= (E_1, E_1\vec{w}_1, \frac{m}{2R} \pm R) \\ -p^{\mu}_{4R,L} &= (E_2, -E_2\vec{w}_2, -\frac{m}{2R} \pm R). \end{aligned} \tag{85}$$

Note that in general E_1 and E_2 are not equal to E. Without loss of generality, we take m to be positive. From conservation of momentum, however, we have

$$\begin{aligned} E_1 + E_2 &= 2E \\ E_1 \vec{w}_1 &= E_2 \vec{w}_2 . \end{aligned} \tag{86}$$

It follows from the energy momentum relations for the ingoing and outgoing momenta that

$$\begin{aligned} E^2(1-v^2) &= R^2 - 2 \\ E_1^2(1-w_1^2) &= 2N_1 - 2 + \left(\frac{m}{2R} + R\right)^2 \\ E_2^2(1-w_2^2) &= 2N_2 - 2 + \left(-\frac{m}{2R} + R\right)^2 , \end{aligned} \tag{87}$$

where N_1 and N_2 are the number operators for the the right movers of the outgoing states.

Subtracting the third equation in (87) from the second equation and using (86) we obtain the relation

$$N_1 - N_2 + m = (E_1 - E_2)E. \tag{88}$$

From the first equation in (87) it follows that E is bounded by some multiple of R for fixed v. It then follows from the first equation in (86) that both E_1 and E_2 are bounded by a multiple of R. So from (88) we see that $N_1 - N_2 + m$ is bounded by some quadratic polynomial in R. We now add the last two equations in (87) to obtain

$$E_1^2(1-w_1^2) + E_2^2(1-w_2^2) = 2N_1 + 2N_2 + 2R^2 + \frac{m^2}{2R^2} - 4. \tag{89}$$

The left hand side of (89) is clearly bounded by a quadratic polynomial in R. It follows that $N_1 + N_2$ is also bounded by a quadratic polynomial, and that so are N_1 and N_2 and also, then, $N_1 - N_2$. From the boundedness of $N_1 - N_2 + m$ it therefore follows that m is bounded by a polynomial in R. Therefore the total number of possible distinct excited states (numbered by m) is bounded by a polynomial in R. The above argument also goes through for the case of a nonzero initial momentum number. For each transition, however, one can show that the Veneziano amplitude is dominated by an exponentially vanishing function of R, from a calculation entirely analogous to the zero-longitudinal excitation case worked out above. Hence the total square amplitude of the scattering (obtained by summing the square amplitudes of all possible transitions) is still dominated by a factor which vanishes exponentially with the radius, except at the poles at $\theta = 0, \pi$ corresponding to forward and backward scattering, which are physically equivalent for identical bosonic strings. This is in agreement with the trivial scattering found above and provides further evidence for the identification of the elementary string with the fundamental string.

The above argument can be repeated for any other type of string, including the heterotic string [82]. The kinematics differ slightly from the tachyonic case but the 4-point function is still dominated by an exponentially vanishing factor in the large radius limit. Hence the scattering is trivial, again in agreement with the result found above.

The Veneziano amplitude result in fact holds for arbitrary incoming winding states. A considerably more tedious calculation for the general case shows that in the large winding radius limit the outgoing strings always scatter trivially and with no change in their individual winding numbers [83]. In this limit, then, these states scatter as true solitons. It would be interesting to see if this result holds for the full quantum string loop expansion.

7 String Solitons and Supersymmetry

In this section I summarize some recent results found in [26]. Let (0123) be the four-dimensional spacetime, $z = x_2 + ix_3 = re^{i\theta}$, (456789) the compactified directions, $S = e^{-2\Phi} + ia = S_1 + iS_2$, where Φ and a are the four-dimensional dilaton and axion and

$$\begin{aligned} T^{(1)} &= T_1^{(1)} + iT_2^{(1)} = e^{-2\sigma_1} - iB_{45} = \sqrt{\det g_{mn}} - iB_{45}, \qquad m,n = 4,5, \\ T^{(2)} &= T_1^{(2)} + iT_2^{(2)} = e^{-2\sigma_2} - iB_{67} = \sqrt{\det g_{pq}} - iB_{67}, \qquad p,q = 6,7, \\ T^{(3)} &= T_1^{(3)} + iT_2^{(3)} = e^{-2\sigma_3} - iB_{89} = \sqrt{\det g_{rs}} - iB_{89}, \qquad r,s = 8,9. \end{aligned} \tag{90}$$

Throughout this section we assume dependence only on the coordinates x_2 and x_3. For an $N = 4$, $D = 4$ toroidal compactification of the ten-dimensional heterotic string with three independent moduli, the four-dimensional action for the massless gravitational fields (i.e. without YM terms) can be written in terms of $g_{\mu\nu}$ $(\mu, \nu = 0, 1, 2, 3)$, S and $T^{(a)}, a = 1, 2, 3$ as

$$\begin{aligned} S_4 = \int d^4x\sqrt{-g}\Big(R &- \frac{g^{\mu\nu}}{2S_1^2}\partial_\mu S \partial_\nu \bar{S} \\ &- \frac{g^{\mu\nu}}{2T_1^{(1)^2}}\partial_\mu T^{(1)}\partial_\nu \bar{T}^{(1)} - \frac{g^{\mu\nu}}{2T_1^{(2)^2}}\partial_\mu T^{(2)}\partial_\nu \bar{T}^{(2)} - \frac{g^{\mu\nu}}{2T_1^{(3)^2}}\partial_\mu T^{(3)}\partial_\nu \bar{T}^{(3)}\Big). \end{aligned} \tag{91}$$

Then a solution for this action for $S = 1$ ($\Phi = a = 0$) is given by the metric

$$ds^2 = -dt^2 + dx_1^2 + T_1^{(1)} T_1^{(2)} T_1^{(3)} (dx_2^2 + dx_3^2), \tag{92}$$

where three cases with different $T-$duality arise depending on the number n of nontrivial moduli:

$$\begin{aligned} n = 2: \qquad & T^{(1)} = -\frac{1}{2\pi} \ln \frac{z}{r_0}, \qquad T^{(2)} = T^{(3)} = 1, \\ n = 4: \qquad & T^{(1)} = T^{(2)} = -\frac{1}{2\pi} \ln \frac{z}{r_0}, \qquad T^{(3)} = 1, \\ n = 6: \qquad & T^{(1)} = T^{(2)} = T^{(3)} = -\frac{1}{2\pi} \ln \frac{z}{r_0}. \end{aligned} \tag{93}$$

In each of the expressions for $T^{(a)}$ z may be replaced by $\bar{z}$ independently (i.e. the axionic instanton in each $T^{(a)}$ may be either self-dual or anti-self-dual). Note that the $n = 2$ case is the string solution of section 3. Since S_4 has manifest $SL(2, R)$ duality in each of the moduli, we can generate from the $n = 4$ case an $SL(2, R)^2$ family of solutions and from the $n = 6$ case an $SL(2, R)^3$ family of solutions.

From the ten-dimensional viewpoint, the $n = 6$ solution, for example, can be rewritten as

$$\begin{aligned} e^{2\phi} &= (-\frac{1}{2\pi} \ln \frac{r}{r_0})^3, \\ ds^2 &= -dt^2 + dx_1^2 + e^{2\phi}(dx_2^2 + dx_3^2) + e^{2\phi/3}(dx_4^2 + ... + dx_9^2), \\ B_{45} &= \pm B_{67} = \pm B_{89} = \pm \frac{\theta}{2\pi}, \end{aligned} \tag{94}$$

where $\phi = -\sigma_1 - \sigma_2 - \sigma_3$ is the ten-dimensional dilaton.

The above solutions for the massless fields in the gravitational sector solve the tree-level supersymmetry equations (18) of the heterotic string for zero fermi and Yang-Mills fields. It further follows that the $n = 2, 4$ and 6 solutions break $1/2, 3/4$ and $7/8$ of the spacetime supersymmetries respectively. I will show this to be true for the most difficult case of $n = 6$.

$\delta\lambda = 0$ follows from scaling, since the dilaton can be written as the sum of three parts (the moduli) each of which produces a contribution which cancels against the contribution of the H term coming from the appropriate four-dimensional subspace. In other words, each of the subspaces (2345), (2367) and (2389) effectively contains a four-dimensional axionic instanton with the appropriate (anti) self-duality in the generalized connection. Another way of saying this is that there are three independent parts, each of which vanishes as in the simple $n = 2$ case. $\delta\psi_M = 0$ is a little more subtle. For the $n = 2$ case, the generalized connection is an instanton, and for constant chiral spinor ϵ with chirality in the four-space of the instanton opposite to that of the instanton (e.g. negative for instanton and positive for anti-instanton), it is easy to show that $\Omega_M{}^{AB}\Gamma_{AB}\epsilon = 0$. In the more general case of $n = 6$ we proceed as follows. It is sufficient to show that $\delta\psi_M = 0$ for, say, $M = 2$ and $M = 4$ (i.e. for a spacetime and for a compactified index, the argument for the other indices being identical to one or the other). For $M = 2$ this can be written out explicitly as

$$\begin{aligned}4\delta\psi_2 = &\left(\frac{1}{3}\omega_2^{23}\Gamma_{23} + \omega_2^{24}\Gamma_{24} + \omega_2^{25}\Gamma_{25} - \frac{1}{2}H_2{}^{45}\Gamma_{45}\right)\epsilon \\ &+ \left(\frac{1}{3}\omega_2^{23}\Gamma_{23} + \omega_2^{26}\Gamma_{26} + \omega_2^{27}\Gamma_{27} - \frac{1}{2}H_2{}^{67}\Gamma_{67}\right)\epsilon \\ &+ \left(\frac{1}{3}\omega_2^{23}\Gamma_{23} + \omega_2^{28}\Gamma_{28} + \omega_2^{29}\Gamma_{29} - \frac{1}{2}H_2{}^{89}\Gamma_{89}\right)\epsilon.\end{aligned} \tag{95}$$

Each line in (95) acts on only a four-dimensional component of ϵ and can be shown (this is the crucial point) to exactly correspond to the contribution of the supersymmetry equation of a single $n = 2$ axionic instanton. So in effect, the configuration carries three such instantons in the generalized curvature in the spaces (2345), (2367) and (2389). So for the appropriate chirality of the four-dimensional components of ϵ (depending on the choices of the signs of $T_2^{(a)}$), $\delta\psi_2 = 0$. Since we are making three such choices, only 1/8 of the spacetime supersymmetries are preserved. Another perhaps simpler way to see this is to write $\epsilon = \epsilon_{(01)} \oplus \epsilon_{(23)} \oplus \epsilon_{(45)} \oplus \epsilon_{(67)} \oplus \epsilon_{(89)}$. Then the chiralities of $\epsilon_{(45)}, \epsilon_{(67)}$ and $\epsilon_{(89)}$ are all correlated with that of $\epsilon_{(23)}$, so it again follows that only 1/8 of the supersymmetries are broken, and 7/8 are broken.

We still need to check $\delta\psi_4 = 0$. In this case, it is easy to show that the whole term reduces exactly to the contribution of a single $n = 2$ axionic instanton:

$$4\delta\psi_4 = \left(\omega_4^{42}\Gamma_{42} + \omega_4^{43}\Gamma_{43} - \frac{1}{2}H_4{}^{25}\Gamma_{25} - \frac{1}{2}H_4{}^{35}\Gamma_{35}\right)\epsilon = 0, \tag{96}$$

as in this case there is only the contribution of the instanton in the (2345) subspace.

When the above solution is combined with a nonzero Yang-Mills field given by $A_M{}^{PQ} = \Omega_M{}^{PQ}$ (the usual gauge equals generalized connection embedding), we still need to show that $\delta\chi = 0$. This can be easily seen by noting that as in the $\delta\psi_M$ case, the term $F_{23}\Gamma^{23}$ splits into three equal pieces, each of which combines with the rest of a $D = 4$

instanton (since the Yang-Mills connection is equated to the generalized connection and is also effectively an instanton in each of the three four-dimensional subspaces) to give a zero contribution.

For the $n = 4$ case, it is even easier to show that 1/4 of the supersymmetries are preserved, and thus that 3/4 are broken.

It turns out that these solutions generalize even further to solutions including a non-trivial S field. The net result of adding a nontrivial S (with $SL(2, Z)$ symmetry) is to break half of whatever remaining spacetime supersymmetries preserved by the corresponding T configuration with trivial S. In particular, a solution of the action (91) with one nontrivial S and three nontrivial T's has the form

$$\begin{aligned} ds^2 &= -dt^2 + dx_1^2 + S_1 T_1^{(1)} T_1^{(2)} T_1^{(3)} (dx_2^2 + dx_3^2), \\ S &= T^{(1)} = T^{(2)} = T^{(3)} = -\frac{1}{2\pi} \ln \frac{z}{r_0}. \end{aligned} \tag{97}$$

This solution preserves only 1/16 of the spacetime supersymmetries (i.e. half of what the three T solution did) since the nontrivial S field breaks half of the remaining supersymmetries by imposing a chirality choice on the (01) subspace of the ten-dimensional space. Note that the real parts of the S and T fields can be arbitrary as long as they satisfy the box equation (e.g. $S^{-1}\Box\, S = 0$) in the two-dimensional subspace (23). In particular, all of the solutions discussed in this section can be generalized to multi-string configurations independently in each of the moduli, with arbitrary number of strings each with arbitrary winding number.

A special case of the generalized S and T solution is the one with only one S and one T. This is in fact a "dyonic" solution. An interesting feature of this special case is that in going to higher dimensions, one still has a solution even if the box equation covers the whole transverse four-space (2345) (the remaining four directions are flat even in $D = 10$, as we have only one T in this case). The $D = 10$ form in fact reduces to a $D = 6$ dyonic solution ($i = 2, 3, 4, 5$)

$$\begin{aligned} \phi &= \Phi_E + \Phi_M, \\ ds^2 &= e^{2\Phi_E}(-dt^2 + dx_1^2) + e^{2\Phi_M} dx_i dx^i, \\ e^{-2\Phi_E} &= 1 + \frac{Q_E}{y^2}, \qquad e^{2\Phi_M} = 1 + \frac{Q_M}{y^2}, \\ H_3 &= 2Q_M \epsilon_3, \qquad e^{-2\phi *} H_3 = 2Q_E \epsilon_3, \end{aligned} \tag{98}$$

for the special case of a single electric and single magnetic charge at $y = 0$ (again this generalizes to arbitrary–but quantized– charges at arbitrary locations). For $Q_M = 0$ this is just the Dabholkar et al. string. For $Q_E = 0$ this is just the $D = 6$ dual string (which can be obtained from the fivebrane by compactifying four flat directions). This solution breaks 3/4 of the spacetime supersymmetries. The self-dual limit of $Q_E = Q_M$ was already found [69] in a different context.

It turns out that the above solutions that break more than 1/2 of the spacetime supersymmetries in $N = 4$ break only 1/2 the spacetime supersymmetries when their analogs arise in $N = 1$ or $N = 2$ compactifications to $D = 4$. In fact, analogs of all of these solutions arise in $N = 1$ compactifications, and all but the 3 T solutions arise in $N = 2$.

For the $N = 1$ compactification, (the $N = 2$ case is similar), it turns out that the number of T and S fields does not affect the number of supersymmetries broken, as in the supersymmetry equations the contribution of each field is independent. In particular, the presence of an additional field produces no new condition on the chiralities, so that the number of supersymmetries broken is the same for any number of fields. So there remains to check that for, say, one T field, 1/2 of the supersymmetries are broken. Details may be found in [26]. The relationship between duality and supersymmetry is also discussed in [84,85].

8 Future Directions

As mentioned in the introduction, an important possible application of the instanton solutions in string theory (in particular the axionic instanton discussed in this paper) is the exploration of vacuum tunnelling in string theory. The true stringy analogs of instanton computations in field theory probably arise within the context of string field theory [86,87]. However, it is possible to perform simpler computations in the low-energy supergravity limit of string theory [88]. Another possibility is to try to obtain vertex-operator representations of states corresponding to string instanton solutions and compute transition amplitudes between vacuum states in the low-energy limit. Comparisons with analogous results in point field theory may then be quite illuminating.

The construction of vertex operators may also be useful as a dynamical test of the various dualities discussed in this paper. For example, a vertex operator representation of fivebranes would allow us to repeat the Veneziano amplitude calculation of section 6 for fivebranes and again compare with expectations from the Manton metric on moduli space. As in the string case, one would still expect a vanishing leading order dynamical force in the limit of infinitely long fivebranes. A similar result should also hold for the monopoles considered in this paper.

The fact that these extremal $\alpha = \sqrt{3}$ black holes/monopoles scatter trivially to leading order in the impact parameter is in direct contrast to not only field-theoretic analogs, such as the scattering of BPS monopoles (as pointed out in section 6), but also to analogous computations in general relativity (see, e.g., [89], where the metric on moduli space is computed for extreme Reisnner-Nordstrom black holes and is found to be non-flat). Whether the flatness of the metric is an intrinsically stringy property is far from clear, but it would worthwhile to do such a calculation for $\alpha = 1$ extreme black holes. What is especially interesting in this latter case is that this class of solutions, described by exact WZW coset conformal field theories in the throat limit [62], are suggested to be possible stable remnants at the endpoint of Hawking radiation [90,91] in the context of a string theory of quantum gravity. In this regard, a vanishing dynamical force between two $\alpha = 1$ extreme black holes to leading order in the impact parameter would suggest that such states would interact very weakly at any scale larger than the Planck length and would therefore most likely be stable against coalescence. A flat metric result in this case may therefore support the plausibility of the stable remnant approach to the resolution of the Hawking radiation information paradox.

Finally, given the encouraging success of the application of stringy techniques in statistical physics and QCD, the question arises as to whether stringy methods may be useful in relativity. An open problem in this regard is the representation of the more astrophysically realistic Schwarzschild black hole as a conformal field theory. Such a representation would certainly lead to important insights into the nature of string theory as a theory of quantum gravity, but may in addition lead to a greater understanding of the nature of singularities in relativity. A (slightly) less ambitious project is to try to adapt stringy scattering techniques, in conjunction with the Manton approach and the various dualities [28,92,93], to analyze the low-velocity interactions of Schwarzschild black holes.

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