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# Inflation with a Complex Scalar Field

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*Abstract.* We discuss the coupled Einstein-Klein-Gordon equations for a complex scalar field with and without a quartic self-interaction in a zero curvature Friedman-Lemaître Universe. The complex scalar field, as well as the metric, is decomposed in a homogeneous, isotropic part (the background) and in first order gauge invariant scalar perturbation terms. The background equations can be written as a set of four coupled first order non-linear differential equations. These equations are analyzed using modern theory of dynamical system. It is shown that, in all singular points where inflation occurs, the phase of the complex scalar field is asymptotically constant. The analysis of the first order equations is done for the inflationary phase. For the short wavelength regime the first order perturbation term of the complex scalar field is smeared out and the Bardeen potential oscillates around a nearly constant mean value. Whereas for the long wavelength regime the first order perturbed quantities increase.

## 1 Basic equations

We will consider the linear scalar mode perturbations since they are the only ones which contribute to the energy density fluctuations [1]. We will, as usual, expand the scalar perturbations in terms of a complete set of harmonic functions $Y_k$, which are the eigenfunctions with eigenvalue $-k^2$ of the Laplace-Beltrami operator $\Delta$ defined on constant time slices $\Sigma_\eta$. In the following, we omit for simplicity to write the subscript $k$ on $Y$.

Taking the zero curvature Friedmann-Lemaître metric—including the first order scalar mode perturbations—given in Ref. [1] and using the following action

$$S = \int \sqrt{-g} d^4x \left[ -\frac{R}{16\pi G} + \frac{1}{2} g^{\mu\nu} \left( e_\mu \Phi e_\nu \Phi^* + e_\nu \Phi e_\mu \Phi^* \right) + m^2 \Phi \Phi^* + \lambda \left( \Phi \Phi^* \right)^2 \right], \quad (1.1)$$

the equations of motion of the background as well as the first order perturbations equations can be derived. To obtain the latter, the scalar field has to be expanded in a background part, $\phi$ and in a first order term, $\delta\phi$. We get for the background

$$H^2 = \frac{8\pi G}{3}(\dot{\phi}\dot{\phi}^* + m^2\phi\phi^* + \lambda(\phi\phi^*)^2)\ , \tag{1.2}$$

$$-2\dot{H} - 3H^2 = 8\pi G(\dot{\phi}\dot{\phi}^* - m^2\phi\phi^* - \lambda(\phi\phi^*)^2)\ . \tag{1.3}$$

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi + 2\lambda(\phi\phi^*)\phi = 0\ , \tag{1.4}$$

where $H = \dot{a}/a$ and dot means derivative with respect to the real time, $t$.

Expressing the first order perturbation equations directly with respect to gauge invariant quantities [2] we obtain

$$6h\psi' + \left(2k^2 + 6h^2\right)\psi = 8\pi G\left(\phi^{*'}\delta\varphi' + \phi'\delta\varphi^{*'} + 2\phi'\phi^{*'}\psi + a^2\frac{\partial V}{\partial\phi}\delta\varphi + a^2\frac{\partial V}{\partial\phi^*}\delta\varphi^*\right), \tag{1.5}$$

$$\psi' + h\psi = -4\pi G\left(\phi^{*'}\delta\varphi + \phi'\delta\varphi^*\right), \tag{1.6}$$

$$\psi'' + 6h\psi' + \left(k^2 + 16\pi G a^2 V\right)\psi = 8\pi G a^2\left(\frac{\partial V}{\partial\phi}\delta\varphi + \frac{\partial V}{\partial\phi^*}\delta\varphi^*\right), \tag{1.7}$$

$$\delta\varphi'' + 2h\delta\varphi' + k^2\delta\varphi + 4\phi'\psi' - 2a^2\frac{\partial V}{\partial\phi^*}\psi + a^2\frac{\partial^2 V}{\partial\phi^{*2}}\delta\varphi^* + a^2\frac{\partial^2 V}{\partial\phi^*\partial\phi}\delta\varphi = 0, \tag{1.8}$$

where $\psi$ is the Bardeen potential, $h = a'/a$, $\delta\varphi$ is the gauge invariant quantity corresponding to $\delta\phi$ and prime means derivation with respect to the conformal time, $\eta$.

Since the matter action is U(1)-globally invariant we get the conservation with respect to the conformal time of the bosonic charge

$$\Xi = \frac{i\,a^2}{2}\left(\phi^{*'}\phi - \phi'\phi^*\right)\ . \tag{1.9}$$

It should be noticed that the first order terms are entirely determined by eqs.(1.5)-(1.6), eq.(1.8) and their complex conjugate. The background solution is established by eq.(1.2), eq.(1.4) and their complex conjugate.

# 2 The background

We will only consider here the case where $m \neq 0$. For a complete discussion of the background see Ref.[3]. The only singular point not lying at infinity of the phase space is the coordinate origin. This singular point is an asymptotically stable winding point. It correspond to the oscillatory phase of the complex scalar field.

In order to find the singular points lying at infinity we have to perform a transformation which maps them on the boundary of a unit three-sphere. We extend the phase space to the boundary to analyze their behavior. It turns out that, whatever the value of $\lambda$ is, there is a line of singular points lying at infinity which meet the criteria of inflation. For $\lambda = 0$, the asymtotical behavior of the scalar field and the Hubble parameter near line of singular points is given by

$$\varphi = \frac{-M_p m t}{\sqrt{3}}\, e^{i\vartheta_{30}}\ , \qquad H = \frac{-m^2 t}{3}\ , \tag{2.1}$$

for $t \to -\infty$. Setting $\vartheta_{30} = 0$, we recover the result found in Ref.[4]. Similarly, for the case $\lambda \neq 0$, we have

$$\varphi = \varphi_0 e^{i\vartheta_{30}} \exp\left(-2M_p\sqrt{\frac{\lambda}{3}}\, t\right) , \tag{2.2}$$

where $t \to -\infty$ and $\varphi_0$ is a negative integration constant. Using eq.(1.2), one gets the asymptotic behavior of the Hubble parameter. We see that the above given asymptotic solutions correspond to outgoing separatrices in phase space. The fact that along these separatrices the phase of $\varphi$ remains constant is important and shows that inflation is essentially driven by one component of the field. Notice that this conclusion is also valid for the massless case.

# 3 Perturbation during inflation

We will consider the long wavelength and the short wavelength limits separately. To solve the Einstein equations, we first define the complex valued function $U(\eta)$ as the solution of the following system of differential equations

$$U' + hU = -4\pi G\phi^{*'}\delta\varphi, \tag{3.1}$$

$$U'' + 2\left(h - \frac{\phi^{*''}}{\phi^{*'}}\right)U' + \left(k^2 + 2h' - 2h\frac{\phi^{*''}}{\phi^{*'}}\right)U = 0. \tag{3.2}$$

To simplify the notation we omit to write the explicit k dependence on $U$. As long as $|\phi^{*'}| \gtrsim |\delta\varphi|$, one easily sees that the sum $(U + U^*)$ fulfills the two Einstein eqs.(1.5)-(1.6) and thus can be identified with $\psi$. Setting $U = \frac{\phi^{*'}u}{a}$, $g = \frac{h}{a\phi^{*'}}$ and using the background equations, eq.(3.2) can be rewritten as

$$u'' + k^2u + \left[\frac{1}{h}\left(\frac{\phi^{*''}}{\phi^{*'}} - \frac{\phi''}{\phi'}\right)\left(h^2 - h'\right) - \frac{g''}{g}\right]u = 0. \tag{3.3}$$

By solving this last equation we obtain $\psi$ and $\delta\varphi$. In order to solve some problems of the standard cosmological model (e.g. the flatness problem ...) a sufficiently long inflationary stage is needed. Hence, inflation starts with $\vartheta$ close to a constant. Writing $\phi = |\phi|\, e^{i\vartheta}$, we get from eq.(1.9)

$$a^2|\phi|^2\vartheta' = \Xi, \tag{3.4}$$

where $\Xi$ is the constant bosonic charge. On the separatrix, where inflation occurs, we have, whatever the values of $m$ and $\lambda$ are, that $a^2|\phi|^2$ is growing exponentially. From eq.(3.4) we see that, immediately after the beginning of inflation, $\vartheta'$ can be taken to be zero within first order approximation. Thus, $\vartheta$ will be constant as long as inflation lasts. As a consequence eq.(3.3), reduces to

$$u'' + k^2u - \frac{g''}{g}u = 0. \tag{3.5}$$

Since during inflation, $a^2 H \gg g''/g$, we get immediately for wavelength perturbations smaller than the Hubble radius that the dominant terms of $\psi$ and $\delta\varphi$ are given by [5, 6]

$$\psi = 2Re\left[\dot{\phi}\left(\gamma_1 \cos\left(k\int\frac{dt}{a}\right) + \gamma_2 \sin\left(k\int\frac{dt}{a}\right)\right)\right], \tag{3.6}$$

$$\delta\varphi \simeq -\frac{k}{4\pi G\, a}\left[\gamma_2^* \cos(k\eta) - \gamma_1^* \sin(k\eta)\right], \tag{3.7}$$

where $Re$ denotes the real part and $\gamma_1, \gamma_2$ are complex integration constants. The slow-rolling approximation, required for having a sufficiently long inflationary stage, leads to a slowly variation of the mean value of $\psi$. The behavior of $\delta\varphi$ is governed by the $1/a$ factor, which decreases rapidly. It follows, as expected, that the short wavelength fluctuations of the scalar field are smeared out.

For $k \ll g''/g$—the long wavelength perturbations— eq.(3.5) can also be solved. Hence, we obtain for the dominant terms

$$\psi = 2Re\left(\tilde{k}\right)\left[1 - \frac{H}{a}\int^t a\, dt\right] \simeq -2Re\left(\tilde{k}\right)\frac{\dot{H}}{H^2}, \quad \delta\varphi \simeq -\frac{k_2\dot{\phi}^*}{4\pi G\, H}. \tag{3.8}$$

with $k_2$ and $\tilde{k} = k_2 e^{-2i\vartheta}/8\pi G$ being complex integration constants. We consider an initial perturbation with wavelength inside the Hubble radius, which will be outside it at the end of inflation. There are wavelengths that fulfill these conditions. At the end of inflation, the evolution of the gauge invariant metric potential is given by eq.(3.8). Later, when the universe is dominated by relativistic particles, the scale factor scales as $a \propto t^\mu$. Hence, the Hubble radius increases more rapidly than the fixed comoving wavelength. The metric perturbation can re-enter inside the Hubble radius and induce fluctuations on the ordinary matter. At Hubble radius crossing, using eq.(3.8), the metric perturbation is given by

$$\psi = 2Re\left(\tilde{k}\right)\frac{1}{\mu + 1}. \tag{3.9}$$

During the inflationary phase the behavior of the perturbations is similar to the one of the real scalar field. This is not surprising, since along the separatrices the phase of the complex scalar field remains constant and thus inflation is essentially driven by one component of the field.

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