Zeitschrift: Helvetica Physica Acta

Band: 69 (1996)

Heft: 3

Artikel: On wormholes in low energy string theory

Autor: Lavrelashvili, George

DOI: https://doi.org/10.5169/seals-116930

Helv Phys Acta
Vol. 69 (1996)

0018-0238/96/030245-04$1.50+0.20/0

On wormholes in low energy string theory

By George Lavrelashvili

Institute for Theoretical Physics, University of Zurich, Winterthurerstrasse 190,
CH-8057 Zurich, Switzerland [1]

Abstract. In the present contribution we briefly discuss some facts about euclidean wormholes, present new results and indicate some unsolved problems.

1. Wormholes – solutions to the euclidean Einstein equations with non-trivial topology – may cause observable effects in the macroscopic low energy world [1, 2, 3, 4]. Some time ago it was advocated that they can lead to interesting phenomena such as loss of coherence, non-conservation of global quantum numbers, etc. [1, 2]. It was suggested later that instead of a loss of coherence wormholes might lead to "dynamical" coupling constants [4]. These arguments were disputed in a recent study of a string theory as a model for parent and baby universes [5]. In the present short contribution we do not discuss physical consequences of wormhole physics, rather we will concentrate on wormhole solutions itself.

2. An example of a wormhole solution was found by Giddings and Strominger in the gravity-axion system [3]

$$S = \int d^4x\sqrt{g}(-R + H_{\mu\nu\rho}H^{\mu\nu\rho}), \tag{1}$$

where $H_{\mu\nu\rho}$ is the field strength of an antisymmetric tensor $B_{\mu\nu}$, $H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}$. In what follows we will use units for which $M_{Pl}^2/16\pi = 1$. We are interested in spherically symmetric solutions, so the metric is parameterized as follows

$$ds^2 = d\tau^2 + a^2(\tau)d\Omega_3^2, \tag{2}$$

where $d\Omega_3^2 = \eta_{ij}dx^i dx^j$ is the line element on the three sphere. The ansatz

$$H_{0ij} = 0, H_{ijk} = q\sqrt{\eta}\epsilon_{ijk} \tag{3}$$

[1]On leave of absence from Tbilisi Mathematical Institute, 380093 Tbilisi, Georgia

solves the equation of motion for the axion field. The constant q determines the axionic charge flowing trough the wormhole. In terms of a new variable x, defined by the relation $dx = d\tau/a^3$ one obtains the following simple equation for a

$$\frac{\dot{a}^2}{a^2} = a^4 - a_0^4, \tag{4}$$

where $\dot{} \equiv \frac{d}{dx}$. Eq.(4) can be integrated with the result

$$a^2(x) = \frac{a_0^2}{cos(2a_0^2 x)}. \tag{5}$$

The corresponding metric (2) is asymptotically flat since $a^2(\tau) = \tau^2$ for $\tau \to \pm\infty$ ($x \to \frac{\pi}{2}/2a_0^2$) and has a funnel shape with a throat in the region $\tau \to 0$ ($x \to 0$). The integration constant $a_0 = a(\tau)|_{\tau=0}$ fixes the size of the wormhole throat.

Such a wormhole solution is interpreted as a kind of gravitational instanton leading to a (small) baby universe splitting off from a (big) parent universe [1, 2, 3, 4].

In the same paper [3] Giddings and Strominger considered a string-inspired theory consisting of axion, dilaton and gravity fields:

$$S = \int d^4x\sqrt{g}(-R + 2(\nabla\Phi)^2 + e^{-4\gamma\Phi}H^2). \tag{6}$$

The value $\gamma = 1$ of the dilaton coupling constant is predicted by string theory. The corresponding equations of motion can still be integrated. The solution for the scale factor remains the same, whereas dilaton field is given by

$$e^{2\gamma\Phi} = \frac{q}{a_0^2}cos(\sqrt{3}\,\gamma\; arccos(\frac{a_0^2}{a^2})). \tag{7}$$

This expression shows that the regular wormhole solution exists only for certain values of the dilatonic coupling constant γ, $0 \leq \gamma < \gamma_{critical}$. If $\gamma \geq \frac{1}{\sqrt{3}}$ the dilaton field is diverging, $\Phi \to -\infty$, when $a \to \pm\infty$. Since the value $\gamma_{critical} = \frac{1}{\sqrt{3}}$ is less than the string value $\gamma_{string} = 1$ the conclusion is that there are no regular stringy wormholes in this simple model.

It was shown [6] that if one takes a model with more then one dilaton field the condition to find a regular wormhole becomes even worth, namely

$$(\Sigma\gamma_i^2)^{\frac{1}{2}} < \gamma_{critical}, \tag{8}$$

where the γ_i is a i^{th} dilatonic coupling constant.

We are developing [7] an approach based on tools in the theory of dynamical systems (similar to [8]). Our aim is to understand the appearance of a critical value of γ in a general context. In this way we hope to find a modified model in which regular wormholes exist up to the string value of the dilatonic coupling constant γ.

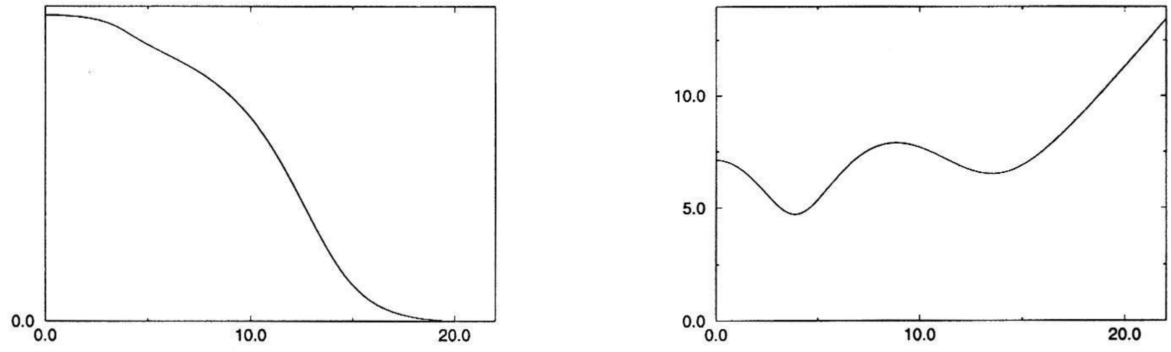

Figure 1: Scalar field $\varphi(\tau)$ and scale factor $a(\tau)$ for a wormhole with "double neck".

3. The GS wormhole describes nucleation of a baby universe at its maximal radius [3, 9]. The analytic continuation of the GS wormhole to the Minkowski time shows [9] that it leads to the creation of a **contracting** baby universe.

The wormholes leading to **expanding** baby universes were found [9, 10] in a theory consisting of an axion, a dilaton and a self-interacting scalar field which is minimaly coupled to gravity:

$$S = \int d^4x\sqrt{g}(-R + H^2 + \frac{1}{2}(\nabla\varphi)^2 + V(\varphi)). \tag{9}$$

Recent careful analysis of the model (9) showed that in addition to the wormholes found already in 1988 in [9] there are wormholes with "double necks" (see Figure 1).

We investigated numerically a more general (stringy) version of (9), defined by the action

$$S = \int d^4x\sqrt{g}(-R + 2(\nabla\Phi)^2 + e^{-4\gamma\Phi}H^2 + \frac{1}{2}(\nabla\varphi)^2 + e^{-2\gamma\Phi}V(\varphi)), \tag{10}$$

in a wide range of parameters and found that the generic solutions in this theory suffers from a singularity at some value of the dilatonic coupling constant $\gamma = \gamma_{critical} < 1$.

4. There are two different types of solutions associated tunneling processes: instanton and bounce. An instanton (at least in quantum mechanics and field theory) describes tunneling between degenerate vacua, whereas the bounce describes decay processes. It was widely believed that the euclidean wormholes are (gravitational) analogues of the instantons.

Recently a new and important property, namely a negative mode about the GS wormhole was found by Rubakov and Shvedov [11]. These authors claim that wormholes describe decay process and are similar to the bounces rather than instantons.

It is an interesting question whether the existence of a (single!) negative mode about a wormhole is generic. To answer this question (at least partly) one can try to analyze the spectrum of small perturbations in more complicated theory, *e.g.* to investigate the negative modes about wormholes leading to expanding baby universes [9]. Unfortunately, application of the same prescription for a conformal factor rotation, as was used by Rubakov and Shvedov [11], leads to a **complex** quadratic action for the model (9). At this stage it is not quite clear how one can overcome this problem.

5. New kinds of wormholes (with double necks) were found. It was demonstrated (numerically) that in the theory defined by the action (10), similarly to the GS case, there is a critical value of the dilatonic coupling constant above which no regular wormhole solutions exist. An interesting open problem is the question about the number of negative modes of different wormholes. We plan to discuss the above problems in more detail in further publications.

The author is thankful to M.Heusler, D.Scialom and N.Straumann for helpful discussions and to the Swiss National Foundation for the financial support.

References

[1] S.W. Hawking, *Phys. Lett*, **195B** (87) 337;
and in: Quantum Gravity. Proc. 4th Int. Seminar, Moscow, 1987, eds. M.A. Markov, V.A. Berezin and V.P. Frolov (*World Scientific, Singapore*, 1988).

[2] G.V. Lavrelashvili, V.A. Rubakov and P.G. Tinyakov, *JETP Lett.*, **46** (1987) 167;
and in: Quantum Gravity. Proc. 4th Int. Seminar, Moscow, 1987, eds. M.A. Markov, V.A. Berezin and V.P. Frolov (*World Scientific, Singapore*, 1988).

[3] S.B.Giddings and A. Strominger, *Nucl. Phys.*, **B306** (1988) 890.

[4] S. Coleman, *Nucl. Phys.*, **B307** (1988) 867.

[5] V.A.Rubakov, *Nucl. Phys.*, **B453** (1995) 395;
Kh.S. Nirov, V.A.Rubakov, *Mod. Phys. Lett.*, **A10** (1995) 3059, hep-th/9510107.

[6] D.H. Coule and Kei-ichi Maeda, *Class. Quantum Grav.*, **7** (1990) 955.

[7] G. Lavrelashvili and D. Scialom, work in progress.

[8] R.Eggenschwiler, D.Scialom and N.Straumann, *Nucl. Phys.*, **B** (1996) to appear.

[9] G.V. Lavrelashvili, V.A. Rubakov and P.G. Tinyakov, *Mod. Phys. Lett.*, **A3** (1988) 1231.

[10] V.A. Rubakov and P.G. Tinyakov, *Phys. Lett.*, **B214** (1988) 334.

[11] V.A.Rubakov, O.Yu. Shvedov, Preprint gr-qc/960438.