Helv Phys Acta
Vol. 69 (1996)

0018-0238/96/030293-04$1.50+0.20/0

FRW model with vector fields in N=1 supergravity

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Abstract. A FRW model obtained from N=1 supergravity with supermatter is analysed in this paper. The matter content is restricted to a vector supermultiplet. The Lorentz and supersymmetry constraints are derived. Non-trivial solutions (no-boundary and wormhole states) are then found.

1 Introduction

Research in supersymmetric quantum cosmology using canonical methods started about 10 years or so [1]. A review on quantum N=1,2 supergravity is found in ref. [2].

Finding and identifying physical states in minisuperspaces obtained from N=1 supergravity with supermatter constitutes an important assignement. A FRW model in the presence of a scalar supermultiplet (constituted by complex scalar fields, $\phi, \bar{\phi}$ and their spin$-\frac{1}{2}$ partners, $\chi_A, \bar{\chi}_{A'}$) and a vector supermultiplet (formed by a gauge vector field $A_\mu^{(a)}$ and its supersymmetric partner) was analysed in ref. [3]. However, the results found there were disapointing: the only allowed physical state was $\Psi = 0$.

The main purpose of this paper and ref. [4] is to initiate a discussion on the parodoxical situation found in ref. [3]. We will study a FRW model where supermatter is restricted to a vector supermultiplet. In section 2 we will address the ansätze for the field variables employed in ref. [3]. In section 3 we derive the quantum constraints. In contrast with ref. [3], *non-trivial* solutions are obtained. We identify a *component* of the Hartle-Hawking (no-boundary) solution [8]. These results support our approach. Finally, our discussions and conclusions close this paper in section 4.

2 Ansätze for the field variables

The action for our model is obtained from the *more general* theory of N=1 supergravity with gauged supermatter [6]. We put all scalar fields and corresponding supersymmetric partners equal to zero. Our field variables will be the tetrad, $e_{\mu}^{AA'}$, the gravitino fields, $\psi_{\mu}^{A}, \bar{\psi}_{\nu}^{A'}$, a gauge spin−1 field, $A_{\mu}^{(a)}$, ((a) is a gauge group index) and the spin-$\frac{1}{2}$ partners, $\lambda_{A}^{(a)}, \bar{\lambda}_{A'}^{(a)}$. The restriction to a closed FRW model requires specific ansätze for these fields.

We choose the geometry to be that of a $k = +1$ Friedmann model with S^3 spatial sections. The ansatz for the tetrad can then be written as

$$e_{a\mu} = \mathrm{diag}\,(N(\tau), a(\tau)) \ , \tag{2.1}$$

where $\hat{a}$ and i run from 1 to 3. $E_{\hat{a}i}$ is a basis of left-invariant 1-forms on the unit S^3 with volume $\sigma^2 = 2\pi^2$. The Lagrange multipliers ${\psi^A}_0$ and ${\bar{\psi}^{A'}}_0$ are taken to be functions of time only. The ansatz for the gravitino field further includes

$${\psi^A}_i = {e^{AA'}}_i \bar{\psi}_{A'} \ , \ {\bar{\psi}^{A'}}_i = {e^{AA'}}_i \psi_A \ , \tag{2.2}$$

where we introduce the new spinors ψ_A and $\bar{\psi}_{A'}$ which are functions of time only.

In the case of pure N=1 supergravity, ansätze (2.1), (2.2) are preserved by a combination of local coordinate, Lorentz and supersymmetry transformations. This holds provided that the generators of Lorentz, coordinate and supersymmetry transformations satisfy specific conditions. The Lorentz constraint $J_{AB} == 0$ also has to be imposed.

Let us then consider the case of a gauge group $\hat{G} = SU(2)$.

The simplest choice would be to take $A_{\mu}^{(a)}, \lambda_{A}^{(a)}, \bar{\lambda}_{A'}^{(a)}$ as time-dependent only. However, this is not sufficient in ordinary quantum cosmology with Yang-Mills fields. Special ansätze are required for $A_{\mu}^{(a)}$ [8, 5]. The ansatz described in ref. [8, 5] for $A_{\mu}^{(a)}$ (and also employed in [3]) is the simplest one that allows vector fields to be present in a FRW geometry. The spin-1 field is taken to be

$$\mathbf{A}_{\mu}(t)\ \omega^{\mu} = \left(\frac{f(t)}{4} \varepsilon_{(a)i(b)} \mathcal{T}^{(a)(b)} \right) \omega^i \ . \tag{2.3}$$

Here $\{\omega^{\mu}\} = \{dt, \omega^i\}$, $\omega^i = {\hat{E}^i}_{\hat{c}} dx^{\hat{c}}$ $(i, \hat{c} = 1, 2, 3)$ and $\mathcal{T}_{(a)(b)}$ are the generators of the $SU(2)$ gauge group. We will use the more general choice for the fermionic partner of $A_{\mu}^{(a)}$ as $\lambda_{A}^{(a)} = \lambda_{A}^{(a)}(t)$. Is then possible to see that these choice of field configurations are invariant for a specific and rather restrictive combination of Lorentz, gauge, supersymmetry and local coordinate transformations (see ref. [4] for more details.

3 Quantum constraints and solutions

We chose $\left(\bar{\lambda}^{(a)}_{A'}, \psi_A, a, f\right)$ to be the coordinates of the configuration space and $\left(\lambda^{(a)}_{A}, \bar{\psi}_A, \pi_a, \pi_f\right)$ to be the momentum operators in this representation. Hence $\lambda^a{}_A \to -\frac{\partial}{\partial \bar{\lambda}^{(a)A}}$, $\bar{\psi}_A \to \frac{\partial}{\partial \psi^A}$, $\pi_a \to \frac{\partial}{\partial a}$, $\pi_f \to -i\frac{\partial}{\partial f}$.

The supersymmetry constraints for our FRW model have the differential operator form

$$\begin{aligned}
S_A \;=\;& -\frac{1}{2\sqrt{6}}a\psi_A\frac{\partial}{\partial a} - \sqrt{\frac{3}{2}}\sigma^2 a^2\psi_A - \frac{1}{8\sqrt{6}}\psi_B\psi^B\frac{\partial}{\partial\psi^A} \\
& - \frac{1}{4\sqrt{6}}\psi^C\bar{\lambda}^{(a)}_C\frac{\partial}{\partial\bar{\lambda}^{(a)A}} + \frac{1}{3\sqrt{6}}\sigma^a{}_{AB'}\sigma^{bCC'}n_D^{\ B'}n^B{}_{C'}\bar{\lambda}^{(a)D}\psi_C\frac{\partial}{\partial\bar{\lambda}^{(b)B}} \\
& + \frac{1}{6\sqrt{6}}\sigma^a{}_{AB'}\sigma^{bBA'}n_D^{\ B'}n^E{}_{A'}\bar{\lambda}^{(a)D}\bar{\lambda}^{(b)}_B\frac{\partial}{\partial\psi^E} - \frac{1}{2\sqrt{6}}\psi_A\bar{\lambda}^{(a)C}\frac{\partial}{\bar{\lambda}^{(a)C}} \\
& + \sigma^a{}_{AA'}n^{BA'}\bar{\lambda}^{(a)}_B\left(-\frac{\sqrt{2}}{3}\frac{\partial}{\partial f} + \frac{1}{8\sqrt{2}}(1-(f-1)^2)\sigma^2\right) + \frac{3}{8\sqrt{6}}\bar{\lambda}^a{}_A\lambda^{(a)C}\frac{\partial}{\partial\psi^C}
\end{aligned} \tag{3.1}$$

and Hermitian conjugate.

When matter fields are taken into account we have $J_{AB} = \psi_{(A}\bar{\psi}^{B'}n_{B)B'} - \lambda^{(a)}_{(A}\bar{\lambda}^{(a)B'}n_{B)B'} = 0$. The Lorentz constraint J_{AB} implies that a physical wave function should be a Lorentz scalar:

$$\begin{aligned}
\Psi = A \;&+\; B\psi^C\psi_C + d_a\lambda^{(a)C}\psi_C + c_{ab}\bar{\lambda}^{(a)C}\bar{\lambda}^{(b)}_C + e_{ab}\bar{\lambda}^{(a)C}\bar{\lambda}^{(b)}_C\psi^D\psi_D \\
&+\; c_{abc}\bar{\lambda}^{(a)C}\bar{\lambda}^{(b)}_C\bar{\lambda}^{(c)D}\psi_D + c_{abcd}\bar{\lambda}^{(a)C}\bar{\lambda}^{(b)}_C\bar{\lambda}^{(c)D}\bar{\lambda}^{(d)}_D + d_{abcd}\bar{\lambda}^{(a)C}\bar{\lambda}^{(b)}_C\bar{\lambda}^{(c)D}\bar{\lambda}^{(d)}_D\psi^E\psi_E \\
&+\; \mu_1\bar{\lambda}^{(2)C}\bar{\lambda}^{(2)}_C\bar{\lambda}^{(3)D}\bar{\lambda}^{(3)}_D\bar{\lambda}^{(1)E}\psi_E \\
&+\; \mu_2\bar{\lambda}^{(1)C}\bar{\lambda}^{(1)}_C\bar{\lambda}^{(3)D}\bar{\lambda}^{(3)}_D\bar{\lambda}^{(2)E}\psi_E + \mu_3\bar{\lambda}^{(1)C}\bar{\lambda}^{(1)}_C\bar{\lambda}^{(2)D}\bar{\lambda}^{(2)}_D\bar{\lambda}^{(3)E}\psi_E \\
&+\; F\bar{\lambda}^{(1)C}\bar{\lambda}^{(1)}_C\bar{\lambda}^{(2)D}\bar{\lambda}^{(2)}_D\bar{\lambda}^{(3)E}\bar{\lambda}^{(3)}_E + G\bar{\lambda}^{(1)C}\bar{\lambda}^{(1)}_C\bar{\lambda}^{(2)D}\bar{\lambda}^{(2)}_D\bar{\lambda}^{(3)E}\bar{\lambda}^{(3)}_E\psi^F\psi_F \,.
\end{aligned} \tag{3.2}$$

where A, B,...,G are functions of a, f only.

From $S_A\Psi = 0$, $\bar{S}_A\Psi = 0$ we obtain

$$A = e^{-3\sigma^2a^2}e^{\frac{3}{16}\sigma^2\left(-\frac{f^3}{3}+f^2\right)}, \quad G = e^{3\sigma^2a^2}e^{\frac{3}{16}\sigma^2\left(\frac{f^3}{3}-f^2\right)}. \tag{3.3}$$

The last solution in (3.3) is present in the Hartle-Hawking (no-boundary) solution of ref. [8]. However, the wave function (3.3) represents only one of the components of the wave function in ref. [8]. The first solution in (3.3) has wormhole features. The Dirac bracket of the supersymmetry constraints induces an expression whose bosonic sector corresponds to the (*decoupled*) gravitational and vector field components of the Hamiltonian constraint in ref. [8].

4 Discussions and Conclusions

Summarizing our work, we considered the canonical formulation of the more general theory of $N = 1$ supergravity with supermatter [6] subject to a $k = +1$ FRW geometry. Ansätze for the the gravitational and gravitino fields, the gauge vector field A^a_μ and fermionic partners were introduced. The scalar fields and their partners were set equal to zero.

Concerning the ansätze employed here (and also in ref. [3]), the form of the tetrad and gravitinos were consistent with the FRW geometry. Supersymmetry invariance was achieved for $A^{(a)}_\mu$ and $\lambda^{(a)}_A$ if further conditions were imposed.

Interesting physical features were derived in secction 3. After a dimensional reduction, we obtained the supersymmetric constraints. We found a non-trivial solution that can be interpreted as a (Hartle-Hawking) no-boundary solution. This result were quite supportive. Namely, the Hartle-Hawking solution found here corresponded to a component of a solution found from a Wheeler-DeWitt equation in ordinary quantum cosmology [8] .

ACKNOWLEDGEMENTS

The author is grateful to O. Bertolami, A.D.Y. Cheng, R. Graham, S.W. Hawking and O. Obregon for useful comments and suggestions. This work was supported by a JNICT/PRAXIS–XXI Fellowship BPD/6095/95. The author would also like to express his gratitude to the organizers of *Jounées Relativistes 96.* On the one hand, for providing financial support and on the other hand by creating a marvellous atmosphere in a splendorous environment.

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