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Photon rockets and radiation reactions

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Abstract. We demonstrate how to construct a class of 'photon rockets' from the vacuum future asymptotically simple Robinson-Trautman (RT) space-times. Using Penrose's conformal technique, one can show that in this class of solutions (1) the Bondi-Sachs news function is exactly the same as that in the corresponding vacuum solution, (2) the Bondi-Sachs 4-momentum consists of two terms: one of them is responsible for the generation of gravitational waves while the other gives rise to the null fluid flux, and (3) the Bondi-Sachs mass is manifestly positive and decreases monotonically as gravitational waves and null fluid are emitted.

1 Photon rockets and vacuum RT space-times

Recently there have been interesting discussions in the literature [1, 2, 3, 4] concerning the mechanism that is responsible for the generation of gravitational waves from a class of Robinson-Trautman (RT) pure radiation solutions. These solutions are referred to as 'generalized photon rockets' in [3, 4]. The aims of this paper are (1) to show how to construct a class of RT pure radiation solutions from the vacuum future asymptotically simple RT space-times, and (2) to discuss the physical interpretations of this class of metrics. We select a class of algebraically special pure radiation solutions in the following manner. Assume that the repeated principal null vector ℓ^a of the Weyl conformal tensor coincides with the eigenvector of the energy-momentum tensor of the pure radiation field, which satisfies the weak energy condition [5]. The Bianchi identities then imply that the null vector field ℓ^a is tangent to geodesics but not necessary shearfree. If ℓ^a is further assumed to be shearfree, twistfree and diverging, then these solutions are given by (see also [6]):

$$ds^2 = 2(H du + dr)\, du - \frac{4r^2 e^{2\varpi}\, d\zeta\, d\overline{\zeta}}{(1 + K_0\, \zeta\, \overline{\zeta}\,)^2}, \qquad H = r\, \partial_u \varpi(u, \zeta, \overline{\zeta}\,) + \frac{1}{2} K - \frac{m(u)}{r},$$

$$K = e^{-2\varpi}\,(K_0 - \triangle_0 \varpi\,), \quad \triangle_0 = (1 + K_0\, \zeta\, \overline{\zeta}\,)^2 \partial_\zeta \partial_{\overline{\zeta}}, \quad K_0 = 0, \pm 1\,; \tag{1.1}$$

$$\ell_a = du\,, \quad n_a = H du + dr\,, \quad \Phi_{22} = \frac{\kappa_0 \Phi^2(u, \zeta, \overline{\zeta}\,)}{2r^2}, \quad \Psi_2 = -\frac{m(u)}{r^3},$$

$$\Psi_3 = -\frac{e^{-\varpi}(1 + K_0\, \zeta\, \overline{\zeta}\,) \partial_\zeta K}{2\sqrt{2} r^2}, \quad \Psi_4 = \frac{1}{4r^2}\, \partial_\zeta [\, e^{-2\varpi}\, \partial_\zeta (K + 2r \partial_u \varpi\,)]\,; \tag{1.2}$$

$$e^{-2\varpi} \triangle_0 K - 4 \left\{ e^{-3\varpi}\, \partial_u [\, e^{3\varpi}\, m(u)\,] \right\} = 2\kappa_0 \Phi^2\,. \tag{1.3}$$

Geometrically K represents the Gaussian curvature of a 2-dimensional submanifold of the space-time at a given (u, r). The metric of the submanifold is conformally related to that of a space of constant curvature, whose Laplacian is denoted by $\triangle_0$. The only remaining non-trivial Einstein equation is Equation 1.3, which is called the pure radiation RT equation.

If one eliminates the pure radiation field by setting $\Phi = 0$, redefines the retarded time $\tilde{u} = M_0^{-\frac{1}{3}} \int m(u)^{\frac{1}{3}}\, du$ and rescales the affine parameter $\tilde{r} = (M_0/m(u))^{\frac{1}{3}}\, r$, where M_0 is a constant, then one obtains the RT equation: $e^{-2\varpi} \triangle_0 K - 12 M_0 \partial_u \varpi = 0$, which governs the evolution of the vacuum RT space-times. Theorems that assert the existence, uniqueness and stability of solutions which evolve smoothly to become the Schwarzschild space-time have been firmly established (see [7, 8] and references quoted therein). One can treat the pure radiation RT equation 1.3 as the definition of $\Phi(u, \zeta, \overline{\zeta}\,)$ by prescribing $\varpi(u, \zeta, \overline{\zeta}\,)$ and $m(u)$ with the condition that the left side of the equation must be non-negative. In particular, if ϖ is any solution of the (vacuum) RT equation and $m(u) := M_0 + M(u)$, where $M(u)$ is an arbitrary function of u, then by Equation 1.3 the energy-momentum of the null fluid can be obtained from

$$\Phi(u, \zeta, \overline{\zeta}\,)^2 = -\frac{2}{\kappa_0} \left\{ e^{-3\varpi}\, \partial_u [e^{3\varpi}\, M(u)\,] \right\}. \tag{1.4}$$

Strictly speaking, such $\varpi(u, \zeta, \overline{\zeta}\,)$ and $m(u)$ may not constitute physically acceptable solutions. However given a vacuum future asymptotically simple RT space-time with $S^2 \times \mathcal{R}_+^2$ topology, the global existence and convergence theorem [7, 8] on the RT equation ensures that the conformal factor $e^{-\varpi}$ evolves smoothly to a linear combination of the zeroth and the first order spherical harmonics: $e^{-\varpi}\,|_\infty = v^0 - v^1 \sin\theta\, \cos\phi - v^2 \sin\theta\, \sin\phi - v^3 \cos\phi\,,\ \eta_{ab}\, v^a\, v^b = 1\,,\ v^a \epsilon \mathcal{R}$. Using a similar argument as given in [9], this equilibrium state, which is a representation of the Schwarzschild space-time, can be interpreted as uniform motion of the source with respect to an asymptotic Minkowski coordinate system, η_{ab} (see also [3]). Moreover the constant unit time-like 4-velocity v^a which is tangent to the world-line of the source is the same v^a that occurs in $e^{-\varpi}\,|_\infty$. However when v^a is a function of the retarded time u, the source is interpreted as accelerating. Thus by prescribing such a regular ϖ, the right side of Equation 1.4 is regular and well defined. The weak energy condition becomes

$$\partial_u \ln M(u) \le -3 \max_{S^2} |\partial_u \varpi|\,, \quad M(u) \ge 0\,. \tag{1.5}$$

Since $\partial_u \varpi$ approaches to zero as u goes to infinity, the RT pure radiation solution becomes the Vaidya solution at large u. In general, the null fluid in this class of solutions may contain second and higher order spherical harmonics, but in a certain sense as explained in section 2 below, it does not contribute directly to the emission of gravitational waves. We now state the ansatz as a theorem:

Theorem: Let (i) $\varpi(u,\zeta,\overline{\zeta})$ be a vacuum future asymptotically simple solution of the RT equation ($K_0 = 1, M_0 > 0$) in the sense of [7], (ii) $m(u) = M_0 + M(u)$, where $M(u)$ is a non-negative monotonic decreasing C^1 function satisfying Equation 1.5, and (iii) $\Phi(u,\zeta,\overline{\zeta})^2$ is given by Equation 1.4. Then $\varpi(u,\zeta,\overline{\zeta})$, $m(u)$ and $\Phi(u,\zeta,\overline{\zeta})^2$ determine a future asymptotically simple RT pure radiation space-time which satisfies Equations 1.1, 1.2 and 1.3. Furthermore $\Phi(u,\zeta,\overline{\zeta})^2 \to -\frac{2}{\kappa_0}\partial_u M(u)$ as $u \to \infty$.

2 Future asymptotically simple photon rockets

In this section we use Penrose's conformal technique (see [5]) to show that the RT pure radiation solutions constructed according to the theorem above is future asymptotically simple, and to investigate some special features of the far field region (wave zone) and the near field region (source). We choose the conformal rescaling factor $\Omega = e^{-\varpi}r^{-1} \approx 0$, where '$\approx$' denotes asymptotic equality. The ansatz implies that ϖ is regular everywhere in the unphysical space-time. Hence the future null infinite $\Im^+$ is given by $\Omega = 0$ if it exists. The unphysical metric $\hat{g}_{ab}$ in $\{u, w, \zeta, \overline{\zeta}\}$ coodinates, where $w := e^{-2\varpi}r^{-1} = e^{-\varpi}\Omega \approx 0$ is

$$\begin{aligned} d\hat{s}^2 &= \Omega^2 ds^2 = -2\left[du\,dw + \frac{2d\zeta\,d\overline{\zeta}}{(1+\zeta\overline{\zeta})^2}\right] + 2\left[\hat{L}du - 2w(\partial_\zeta\varpi\,d\zeta + \partial_{\overline{\zeta}}\varpi\,d\overline{\zeta})\right]du\,, \\ 2\hat{L} &= -2w\,\partial_u\varpi + w^2(1-\Delta_0\varpi) - 2w^3e^{4\varpi}(M_0 + M(u)) \approx 0\,. \end{aligned} \tag{2.1}$$

Hence in a neighbourhood of $\Im^+$, $d\hat{s}^2 \approx -2\left[du\,dw + \frac{2d\zeta\,d\overline{\zeta}}{(1+\zeta\overline{\zeta})^2}\right]$. Choose the null tetrad

$$\hat{\ell}_a = du\,,\quad \hat{n}_a = \hat{L}du - dw - 2w(\partial_\zeta\varpi\,d\zeta + \partial_{\overline{\zeta}}\varpi\,d\overline{\zeta}) \approx -dw\,,\quad \hat{m}_a = -\frac{\sqrt{2}\,d\zeta}{(1+\zeta\overline{\zeta})}. \tag{2.2}$$

The null vector field $\hat{\ell}^a$ is geodetic, shearfree, non-diverging, non-twisting and the rays in the direction of $\hat{\ell}^a$ are affinely parametrized; while $\hat{n}^a$ is only so on $\Im^+$ and the associated rays are not affinely parametrized in general. It is straightforward to check that $\hat{N}_a := -\nabla_a\Omega \approx -e^{\varpi}\,dw = e^{\varpi}\,\hat{n}_a \neq 0$. Hence the RT pure radiation space-times constructed by the ansatz, just like their vacuum counterparts, are future asymptotically simple (see the definition in [5]). The Bondi-Sachs News function [5] of these future asymptotically simple RT pure radiation space-times is given by

$$\mathcal{N} \approx \hat{\Phi}_{20} = -\frac{e^{\varpi}}{2}\,\partial_\zeta\left[(1+\zeta\overline{\zeta})^2\,\partial_\zeta(e^{-\varpi})\right]. \tag{2.3}$$

$\mathcal{N}$ gives a measure of the gravitational radiation being transmitted to $\Im^+$. It is exactly the same as that in the corresponding vacuum RT space-time. The total energy-momentum at a given retarded time u on $\Im^+$ is determined by the Bondi-Sachs 4-momentum

$$\mathcal{P}_a = -\frac{1}{4\pi G}\int_{S^2} \Omega^{-1}\,\hat{\Psi}_2\,\xi_a\,\omega_0 = \frac{1}{4\pi G}\int_{S^2} (M_0 + M(u))\,e^{3\varpi}\,\xi_a\,\omega_0\,, \tag{2.4}$$

where $\omega_0 = \frac{2i\,d\zeta\wedge d\bar{\zeta}}{(1+\zeta\bar{\zeta})^2}$ is the surface-area 2-form on S^2. $\xi_a = \left(1, \frac{\zeta+\bar{\zeta}}{1+\zeta\bar{\zeta}}, -\frac{i(\zeta-\bar{\zeta})}{1+\zeta\bar{\zeta}}, \frac{\zeta\bar{\zeta}-1}{1+\zeta\bar{\zeta}}\right) = (1, \sin\theta\,\cos\phi, \sin\theta\,\sin\phi, \cos\phi\,)$ is the generator of translations in an asymptotic Minkowski space-time. $\mathcal{P}_a$ consists of two terms, the first term arising from the corresponding vacuum RT solution is responsible for the generation of gravitational waves, while the other gives rise to the energy-momentum of the null fluid. Using Equations 1.3, 1.4, 2.3 and Stokes' theorem, the derivative of the Bondi-Sachs 4-momentum takes the form

$$\frac{d}{du}\mathcal{P}_a = -\frac{1}{4\pi G}\int_{S^2} (e^{-\varpi}\mathcal{N}\overline{\mathcal{N}} + \frac{\kappa_0}{2}\,e^{3\varpi}\,\Phi^2\,)\,\xi_a\,\omega_0\,. \tag{2.5}$$

The right side of the above equation is composed of two u-dependent 4-vectors:

$$\mathcal{F}_a^{GW} := \frac{1}{4\pi G}\int_{S^2} e^{-\varpi}\mathcal{N}\overline{\mathcal{N}}\,\xi_a\,\omega_0\,, \quad \mathcal{F}_a^{PR} := \frac{1}{4\pi G}\int_{S^2} \frac{\kappa_0}{2}\,e^{3\varpi}\,\Phi^2\,\xi_a\,\omega_0\,.$$

$\mathcal{F}_a^{GW}$ represents the energy-momentum 4-vector of the gravitational waves transmitted to $\Im^+$. The integrand is proportional to the square of the magnitude of the complex Bondi-Sachs news function $\mathcal{N}$, which comes from the corresponding vacuum RT solution. $\mathcal{F}_a^{PR}$ represents the energy-momentum 4-vector of the zero rest-mass pure radiation field transmitted to $\Im^+$, which provides the null fluid flux that contributes to the motion of the 'photon rocket'.

The first (energy) component of $\mathcal{P}_a$ is the Bondi-Sachs mass

$$\mathcal{P}_t := \mathcal{M}_B = \frac{1}{4\pi G}\int_{S^2} (M_0 + M(u))e^{3\varpi}\,\omega_0 > 0\,.$$

Note that the integrand is manifestly non-negative and hence the Bondi-Sachs mass is positive as expected. Taking the first (energy) component of Equation 2.5, it is obvious that the Bondi-Sachs mass $\mathcal{M}_B$ is monotonically decreasing in u; i.e.

$$\frac{d}{du}\mathcal{M}_B := -(\,\mathcal{F}_t^{GW} + \mathcal{F}_t^{PR}\,) = -\frac{1}{4\pi G}\int_{S^2} (e^{-\varpi}\mathcal{N}\overline{\mathcal{N}} + \frac{\kappa_0}{2}\,e^{3\varpi}\,\Phi^2\,)\,\omega_0 \leq 0\,. \tag{2.6}$$

Hence we can interpret the integral $-\mathcal{F}_t^{GW}$ as the energy from radiation reactions of the gravitational waves back on the source; similarly, the integral $-\mathcal{F}_t^{PR}$ can be viewed as the energy from back-reactions on the source due to the null fluid flux. Consequently an inertial observer in an asymptotic Minkowski coordinate system would interpret that radiation reactions of the emitted gravitational waves and back-reactions of the null fluid flux together determine the motion of the 'photon rocket':

$$\frac{d}{du}\mathcal{P}_a := -(\,\mathcal{F}_a^{GW} + \mathcal{F}_a^{PR}\,)\,.$$

Further details and the case where a generalized photon rocket approaches the Kinnersley photon rocket at large retarded time will be discussed elsewhere.

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