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Constant Cutoff Approach to the Eta Photoproduction in the Skyrme Model

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Abstract. We suggest a quantum stabilization method for the SU(2) σ-model, based on the constant-cutoff limit of the cutoff quantization method developed by Balakrishna et al., which avoids the difficulties with the usual soliton boundary conditions pointed out by Iwasaki and Ohyama. We investigate the baryon number B=1 sector of the model and show that after the collective coordinate quantization it admits a stable soliton solution which depends on a single dimensional arbitrary constant. We then study Eta photoproduction from the nucleon in the constant-cutoff approach to Skyrme model at treshold regions, where N(1535) Born terms are separated from the total amplitude by identifying the resonance with an η bound state to the soliton. Thereby we find that the results obtained using the constant-cutoff approach are of the same order of magnitude as those obtained using the complete Skyrme model and are somewhat closer to the results estimated in a phenomenologocal model.

1 Introduction

It was shown by Skyrme [1] that baryons can be treated as solitons of a nonlinear chiral theory. The original Lagrangian of the chiral SU(2) σ-model is given by

$$L = \frac{F_\pi^2}{16} \; Tr \; \partial_\mu U \partial^\mu U^+ \tag{1.1}$$

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where

$$U = \frac{2}{F_\pi}(\sigma + i\vec{\tau}\cdot\vec{\pi}) \tag{1.2}$$

is a unitary operator ($UU^+ = 1$) and F_π is the pion-decay constant. In (1.2) $\sigma = \sigma(\vec{r})$ is a scalar meson field and $\vec{\pi} = \vec{\pi}(\vec{r})$ is the pion-isotriplet.

The classical stability of the soliton solution to the chiral σ-model lagrangian requires the additional ad-hoc term, proposed by Skyrme [1], to be added to (1.1)

$$L_{sk} = \frac{1}{32e^2} Tr \; [\; U^+\partial_\mu U \;,\; U^+\partial_\nu U \;]^2 \tag{1.3}$$

with a dimensionless parameter e and where [A,B] = AB - BA. It was shown by several authors [2] that, after the collective coordinate quantization using the spherically symmetric ansatz

$$U_o = exp[i\vec{\tau}\cdot\vec{r}_o F(r)] \quad , \quad \vec{r}_o = \vec{r}/r \quad , \tag{1.4}$$

the chiral model, with both (1.1) and (1.3) included, gives a good agreement with the experiment for several important physical quantities. However, the introduction of the Skyrme stabilizing term makes the analytical structure of the results complicated and in many cases difficult to handle.

Mignaco and Wulck (MW) [3] indicated therefore a possibility to build a stable single baryon (n = 1) quantum state in the simple chiral theory, with Skyrme stabilizing term (1.3), omitted. MW have shown that the chiral angle F(r) is indeed a function of a dimensionless variable $s = \frac{1}{2}\chi''(0)r$, where $\chi''(0)$ is an arbitrary dimensional parameter intimately connected to the usual stability argument against the soliton solution for the non- linear σ-model lagrangian.

Using the adiabatically rotated ansatz $U(\vec{r},t) = A(t)U_o(\vec{r})A^+(t)$, where $U_o(\vec{r})$ is given by (1.4), MW obtained the total energy of the non-linear σ-model soliton in the form

$$E = \frac{\pi}{4}F_\pi^2\frac{1}{\chi''(0)}a + \frac{1}{2}\frac{[\chi''(0)]^3}{\frac{\pi}{4}F_\pi^2 b}J(J+1) \tag{1.5}$$

where

$$a = \int_0^\infty [\; \frac{1}{4}s^2(\frac{d\mathcal{F}}{\mathrm{ds}})^2 + 8\sin^2(\frac{1}{4}\mathcal{F}) \;]\; \mathrm{ds} \tag{1.6}$$

$$b = \int_0^\infty ds \;\; \frac{64}{3}s^2\sin^2(\frac{1}{4}\mathcal{F})] \tag{1.7}$$

and $\mathcal{F}$(s) is defined by

$$F(r) = F(s) = -n\pi + \frac{1}{4}\;\mathcal{F}(\mathrm{s}) \tag{1.8}$$

The stable minimum of the function (1.5), with respect to the arbitrary dimensional scale parameter $\chi''(0)$, is

$$E = \frac{4}{3}\;F_\pi\;[\;\frac{3}{2}\;(\frac{\pi}{4})^2\;\frac{a^3}{b}\;J(J+1)\;]^{\frac{1}{4}} \tag{1.9}$$

Despite the non-existence of the stable classical soliton solution to the non-linear σ-model, it is possible, after the collective coordinate quantization, to build a stable chiral soliton at the quantum level, provided that there is a solution F=F(r) which satisfies the soliton boundary conditions, i.e. $F(0) = -n\pi$, $F(infty) = 0$, such that the integrals (1.6) and (1.7) exist.

However, as pointed out by Iwasaki and Ohyama [4], the quantum stabilization method in the form proposed by MW [3] is not correct since in the simple σ-model the conditions $F(0) = -n\pi$ and $F(\infty) = 0$ cannot be satisfied simultaneously. In other words, if the condition $F(0) = -\pi$ is satisfied, Iwasaki and Ohyama obtained numerically $F(\infty) = -\pi/2$, and the chiral phase F=F(r) with correct boundary conditions does not exist.

Iwasaki and Ohyama also proved analytically that both boundary conditions $F(0) = -n\pi$ and $F(\infty) = 0$ can not be satisfied simultaneously. Introducing a new variable y = 1/r into the differential equation for the chiral angle F=F(r) we obtain

$$\frac{d^2F}{dy^2} = \frac{1}{y^2} \sin\ 2F \tag{1.10}$$

There are two kinds of asymptotic solutions to the equation (1.10) around the point $y = 0$, which is called a regular singular point if $\sin 2F \approx 2F$. These solutions are

$$F(y) = \frac{m\pi}{2} + c\,y^2 \quad , \quad m = even\ integer \tag{1.11}$$

$$F(y) = \frac{m\pi}{2} + \sqrt{cy}\cos\left[\frac{\sqrt{7}}{2}\ln{(cy)} + \alpha\right] \quad , \quad m = odd\ integer \tag{1.12}$$

where c is an arbitrary constant and α is a constant to be chosen adequately. When $F(0) = -n\pi$ then we want to know which of these two solutions is approached by $F(y)$ when $y \to 0$ $(r \to \infty)$? In order to answer to that question we multiply (1.10) by $y^2F'(y)$, integrate with respect to y from y to ∞ and use $F(0) = -n\pi$. Thus we get

$$y^2F'(y) + \int_y^\infty dy\,2y[F'(y)]^2 = 1 - \cos[2F(y)]\ . \tag{1.13}$$

Since the left-hand side of (1.13) is always positive, the value of $F(y)$ is always limited to the interval $n\pi - \pi < F(y) < n\pi + \pi$. Taking the limit $y \to 0$, (1.13) is reduced to

$$\int_0^\infty dy\,2y[F'(y)]^2 = 1 - (-1)^m\ , \tag{1.14}$$

where we used (1.11-1.12). Since the left-hand side of (1.14) is strictly positive, we must choose an odd integer m. Thus the solution satisfying $F(0) = -n\pi$ approaches (1.12) and we have $F(\infty) \neq 0$. The behaviour of the solution (1.11) in the asymptotic region $y \to \infty$ $(r \to 0)$ is investigated by multiplying (1.10) by $F'(y)$, integrating from 0 to y and using (1.11). The result is

$$[F'(y)]^2 = \frac{2\sin^2 F(y)}{y^2} + \int_0^y dy\,\frac{2\sin^2 F(y)}{y^3}\ . \tag{1.15}$$

From (1.15) we see that $F'(y) \to$ constant as $y \to \infty$, which means that $F(r) \simeq 1/r$ for $r \to 0$. This solution has a singularity at the origin and can not satisfy the usual boundary condition $F(0) - -n\pi$.

In [5] the present author suggested a method to resolve this difficulty by introducing a radial modification phase $\varphi = \varphi(r)$ in the Ansatz (1.4), as follows

$$U(\vec{r}) = \exp[i\vec{\tau} \cdot \vec{r_0} F(r) + i\varphi(r)]. \qquad (1.16)$$

Such a method provides a stable chiral quantum soliton but the resulting model is an entirely non-covariant chiral model, different from the original chiral σ-model.

In the present paper we use the constant-cutoff limit of the cutoff quantization method developed by Balakrishna, Sanyuk, Schechter and Subbaraman [6] to construct a stable chiral quantum soliton within the original chiral σ-model. We then study Eta photoproduction from the nucleon in the constant-cutoff approach to Skyrme model at treshold regions, where N(1535) Born terms are separated from the total amplitude by identifying the resonance with an η bound state to the soliton. Thereby we find that the results obtained using the constant-cutoff approach are of the same order of magnitude as those obtained using the complete Skyrme model and are somewhat closer to the results estimated in a phenomenologocal model.

The reason why the cutoff-approach to the problem of chiral quantum soliton works is connected to the fact that the solution $F = F(r)$ which satisfies the boundary condition $F(\infty) = 0$ is singular at $r = 0$. From the physical point of view the chiral quantum model is not applicable to the region about the origin, since in that region there is a quark-dominated bag of the soliton.

However, as argued in [6], when a cutoff ϵ is introduced then the boundary conditions $F(\epsilon) = -n\pi$ and $F(\infty) = 0$, can be satisfied. In [6] an interesting analogy with the damped pendulum has been discussed, showing clearly that as long as $\epsilon > 0$, there is a chiral phase $F = F(r)$ satisfying the above boundary conditions. The asymptotic forms of such a solution are given by Eq. (2.2) in [6]. From these asymptotic solutions we immediately see that for $\epsilon \to 0$ the chiral phase diverges at the lower limit.

Different applications of the constant-cutoff approach have been discussed in [7].

2 Constant-Cutoff Stabilization

The chiral soliton with baryon number $n = 1$ is given by (1.4), where $F = F(r)$ is the radial chiral phase function satisfying the boundary conditions $F(0) = -\pi$ and $F(\infty) = 0$.

Substituting (1.4) into (1.1) we obtain the static energy of the chiral baryon

$$M = \frac{\pi}{2} F_\pi^2 \int_{\epsilon(t)}^{\infty} dr [r^2 (\frac{dF}{dr})^2 + 2\sin^2 F]. \qquad (2.1)$$

In (2.1) we avoid the singularity of the profile function $F = F(r)$ at the origin by introducing the cutoff $\epsilon(t)$ at the lower boundary of the space interval $r \in [0, \infty]$, i.e. by working with the interval $r \in [\epsilon, \infty]$. The cutoff itself is introduced following [6] as a dynamic time-dependent variable.

From (2.1) we obtain the following differential equation for the profile function $F = F(r)$

$$\frac{d}{dr}(r^2 \frac{dF}{dr}) = \sin(2F), \tag{2.2}$$

with the boundary conditions $F(\epsilon) = -\pi$ and $F(\infty) = 0$, such that the correct soliton number is obtained. The profile function $F = F[r; \epsilon(t)]$ now depends implicitly on time t through $\epsilon(t)$. Thus in the nonlinear σ-model Lagrangian

$$L = \frac{F_\pi^2}{16} \int d^3x \, Tr(\partial_\mu U \partial^\mu U^+), \tag{2.3}$$

we use the Ansätze

$$U(\vec{r}, t) = A(t) U_0(\vec{r}, t) A^+(t) \ , \quad U^+(\vec{r}, t) = A(t) U_0^+(\vec{r}, t) A^+(t) \ , \tag{2.4}$$

where

$$U_0(\vec{r}, t) = \exp[i\vec{\tau} \cdot \vec{r_0} F(r; \epsilon(t))]. \tag{2.5}$$

The static part of the Lagrangian (2.3), i.e.

$$L = \frac{F_\pi^2}{16} \int d^3x \, Tr(\vec{\nabla} U \cdot \vec{\nabla} U^+) = -M, \tag{2.6}$$

is equal to minus the energy M given by (2.1). The kinetic part of the Lagrangian is obtained using (2.4) with (2.5) and it is equal to

$$L = \frac{F_\pi^2}{16} \int d^3x \, Tr(\partial_0 U \partial_0 U^+) = bx^2 Tr(\partial_0 A \partial_0 A^+) + c[\dot{x}(t)]^2, \tag{2.7}$$

where

$$b = \frac{2\pi}{3} F_\pi^2 \int_1^\infty dy \, y^2 \sin^2 F \ , \quad c = \frac{2\pi}{9} F_\pi^2 \int_1^\infty dy \, y^2 (\frac{dF}{dy})^2 y^2, \tag{2.8}$$

with $x(t) = [\epsilon(t)]^{3/2}$ and $y = r/\epsilon$. On the other hand the static energy functional (2.1) can be rewritten as

$$M = ax^{2/3} \ , \quad a = \frac{\pi}{2} F_\pi^2 \int_1^\infty dy [y^2 (\frac{dF}{dy})^2 + 2\sin^2 F]. \tag{2.9}$$

Thus the total Lagrangian of the rotating soliton is given by

$$L = c\dot{x}^2 - ax^{2/3} + 2bx^2\dot{\alpha}_\nu\dot{\alpha}^\nu \ , \tag{2.10}$$

where $Tr(\partial_0 A \partial_0 A^+) = 2\dot{\alpha}_\nu\dot{\alpha}^\nu$ and α_ν ($\nu = 0, 1, 2, 3$) are the collective coordinates defined as in [8] . In the limit of a time-independent cutoff ($\dot{x} \longrightarrow 0$) we can write

$$H = \frac{\partial L}{\partial \dot{\alpha}^\nu}\dot{\alpha}^\nu - L = ax^{2/3} + 2bx^2\dot{\alpha}_\nu\dot{\alpha}^\nu = ax^{2/3} + \frac{1}{2bx^2}J(J+1) \ , \tag{2.11}$$

where $\vec{J}^2 = J(J+1)$ is the eigenvalue of the square of the soliton laboratory angular momentum. A minimum of (2.11) with respect to the parameter x is reached at

$$x = [\frac{2}{3}\frac{ab}{J(J+1)}]^{-3/8} \quad \Rightarrow \quad \epsilon^{-1} = [\frac{2}{3}\frac{ab}{J(J+1)}]^{1/4} \ . \tag{2.12}$$

The energy obtained by substituting (2.12) into (2.11) is given by

$$E = \frac{4}{3}[\frac{3}{2}\frac{a^3}{b}J(J+1)]^{1/4}. \tag{2.13}$$

This result is identical to the result obtained by Mignaco and Wulck which is easily seen if we rescale the integrals a and b in such a way that $a \longrightarrow \frac{\pi}{4}F_\pi^2 a$, $b \longrightarrow \frac{\pi}{4}F_\pi^2 b$ and introduce $f_\pi = 2^{-2/3}F_\pi$. However in the present approach, as shown in [6], there is a profile function $F = F(y)$ with proper soliton boundary conditions $F(1) = -\pi$ and $F(\infty) = 0$ and the integrals a, b and c in (2.8-2.9) exist and are shown in [6] to be $a = 0.78$ GeV^2, $b = 0.91$ GeV^2, $c = 1.46$ GeV^2 for $F_\pi = 186$ MeV.

Using (2.13) we obtain the same prediction for the mass ratio of the lowest states as Mignaco and Wulck [3] which agrees rather well with the empirical mass radio for the Δ-resonance and the nucleon. Furthermore using the calculated values for the integrals a and b we obtain the nucleon mass M(N) = 1167 MeV which is about 25% higher than the empirical value of 939 MeV. However if we choose the pion decay constant equal to $F_\pi = 150$ MeV we obtain $a = 0.507$ GeV^2 and $b = 0.592$ GeV^2 giving the exact agreement with the empirical nucleon mass.

Finally it is of interest to know how large the constant cutoffs are for the above values of the pion-decay constant in order the check if they are in the physically acceptable ball park. Using (2.12) it is easily shown that for the nucleons ($J = \frac{1}{2}$) the cutoffs are equal to

$$\epsilon = \begin{cases} 0.22 \text{ fm} \ , & \text{for } F_\pi = 186 \text{ MeV} \\ 0.27 \text{ fm} \ , & \text{for } F_\pi = 150 \text{ MeV} \end{cases} \ . \tag{2.14}$$

Clearly, the cutoffs have to be smaller than the nucleon size (0,72 fm), and from (2.14) we see that it is the case. It should, however, be noted that the simple Skyrme model discussed here is at variance with some physical constraints since the isoscalar charge radius ($\sim$ 0.8 fm) is identical to the baryon charge radius ($\sim$ 0.5 fm).

3 Eta photoproduction in the Constant-Cutoff Approach

In the present paper we consider the η photoproduction reaction from the nucleon

$$\gamma + N \rightarrow \eta + N \tag{3.1}$$

The reaction (3.1) is supposed to be strongly dominated by the Born terms through the negative-parity nucleon resonance $N(1535)^*$, in the following denoted simply as N^*, and its investigation can provide new insight into the structure of such a state. In the present paper N^* is regarded as a bound state of the η-meson and the Skyrme soliton, analogous to the bound state of the kaon and the Skyrme soliton in the case of Hyperons [7].

The study of the reaction (3.1) in the framework of the complete Skyrme model has been performed in [9], and in the present paper we will compare the results obtained here with those obtained in [9].

From (1.1) adding a mass term [7],we may write the Lagrangian of our model as follows

$$L = \frac{F_\pi^2}{16}\ Tr\ \partial_\mu U \partial^\mu U^+ + \frac{F_\pi^2}{48}(m_\pi^2 + 2m_K^2)Tr(U + U^+ - 2) + \frac{\sqrt{3}}{24}F_\pi^2(m_\pi^2 - m_K^2)Tr\lambda_8(U + U^+) \tag{3.2}$$

The mass of the η is then obtained using the Gell-Mann-Okubo relation, $3{m_\eta}^2 = 4{m_K}^2 - {m_\pi}^2$, giving $m_\eta = 567MeV$ being somewhat higher than the experimental value of $m_\eta = 548.8MeV$.

Following [9] we introduce the η-field as the eighth component of the SU(3)-octet

$$U(x) = u_\pi(x)\ exp(2i\lambda_8\eta/F_\pi) \tag{3.3}$$

where u_π is the SU(2) pion part and λ_8 is the eighth SU(3) matrix. Using the static hedgehog ansatz 1.4 and expanding η to the second order, we obtain

$$L = M_H + \frac{1}{2}\int d^4x[(\frac{\partial\eta}{\partial t})^2 - (\frac{\partial\eta}{\partial r})^2 + \frac{1}{3}(m_\pi^2(2-\cos F) - 4m_K^2)\eta^2] \tag{3.4}$$

where M_H is the classical hedgehog mass. Writing now the S-mode, where the bound state corresponding to N^* occurs, as $\eta(t,r) = e^{-i\omega t}\eta(r)$, the differential equation for $\eta(r)$ becomes

$$\frac{\partial^2\eta}{\partial r^2} + \frac{2}{r}\frac{\partial\eta}{\partial r} + [\omega^2 + \frac{1}{3}(m_\pi^2(2-\cos F) - 4m_K^2)]\eta = 0 \tag{3.5}$$

The bound state $\eta_0(r)$ in (3.5) is normalized according to [7]

$$8\pi\omega_0 \int r^2 dr \eta_0^2(r) = 1 \tag{3.6}$$

where $\omega_0 < m_\eta$ is the bound-state energy. After the collective coordinate quantization of the SU(2) rotational modes, the η-Skyrmion bound state has spin and isospin $J = I = \frac{1}{2}$. The parity of this state is negative and we can therefore identify it with the lowest negative-parity resonance N^*. A more detailed discussion of the validity of this assumption can be found in [9].

In the study of the process (3.1) we calculate matrix elements of the electromagnetic (EM) interaction between the initial and final states, i.e.

$$2\pi\delta(E_f - E_i)M = -\mathrm{i} < \gamma(\mathrm{k})\mathrm{N}(\mathrm{p}_1)|\mathrm{J}_\mu \mathrm{A}^\mu|\eta(\mathrm{q})\mathrm{N}(\mathrm{p}_2) > \tag{3.7}$$

where the labeling of the momenta of the four particles in the reaction (3.1) closely follows the labeling in [9]. The current J_μ is found by inserting the ansatz (3.3) into the expression for the EM current, which for the case of the constant-cutoff approach can be found in [7], and keeping only the terms linear in the η-field. It is easily shown [7] that only the isovector ($Q = \tau_3$) term contributes as in the case of the complete Skyrme model [9], and we use the expression

$$J^3_m = ie\frac{N_C}{48\sqrt{3}\pi^2 F_\pi}\Big(\frac{\partial\eta}{\partial t}\varepsilon_{mnp}\, Tr\,[\tau_3(u^+_\pi\partial_n u_\pi u^+_\pi\partial_p u_\pi + \partial_n u_\pi u^+_\pi\partial_p u_\pi u^+_\pi)]$$
$$- \varepsilon_{mnp}\partial_n\eta\, Tr\,\{\tau_3([u^+_\pi\partial_p u_\pi, u^+_\pi\partial_0 u_\pi] + [\partial_p u_\pi u^+_\pi, \partial_0 u_\pi u^+_\pi]\}\Big) \tag{3.8}$$

Using (3.8) and taking only the S-mode part of $\eta(x)$ for treshold production, the photoproduction amplitude is obtained in the following form

$$M = \mathrm{i} < \gamma|\vec{\varepsilon}\cdot\vec{\mathrm{J}}|\eta >=$$
$$= \frac{2eN_C\omega_0}{27\sqrt{3}\pi F_\pi}\int_\epsilon^\infty r^2dr\{-[\tfrac{1}{r}\sin 2F\tfrac{dF}{dr}j_0(kr) + (\tfrac{1}{r^2}\sin^2 F - \tfrac{1}{2r}\sin 2F\tfrac{dF}{dr})j_2(kr)]\eta +$$
$$+ \frac{1}{\omega_0\Omega}[j_0(kr) + j_2(kr)]\frac{2}{r}\sin^2 F\frac{d\eta}{dr}\} \tag{3.9}$$

where $\vec{\varepsilon}$ is the polarization vector of the incoming photon and $j_0(kr)$ and $j_2(kr)$ are the spherical Bessel functions of zeroth and second order, respectively. In (3.9) Ω is the moment of inertia of the Skyrmion given by $\Omega = b\epsilon^3$, with ϵ being the constant cutoff defined by (2.12). In order to discuss the choice to retain only the S-mode part of $\eta(x)$, it should be noted that there is a similarity between the neutral pion photoproduction and the eta photoproduction in both the complete Skyrme model [9] and the present approach. Both amplitudes are basically determined by the Wess-Zumino term and are of the same order of magnitude in absolute value (but with the opposite sign). The dominance of the Wess-Zumino term is one of the reasons why the results of the constant-cutoff approach are generally in agreement with the results of the complete Skyrme model, and why the effects of the removal of the Skyrme stabilizing terms in the present approach are not so essential [7]. However in the constant-cutoff approach there is no dependency of the various quantities on the choice of the Skyrme stabilizing term as in the complete Skyrme model [9], and the bound state structure is such that the effect of S-wave bound state becoming a virtual state for some choices of stabilizing-term parameters is not present. It can thus be shown that, in the present bound state structure, the effects of P-modes and their taking over as in the case of the neutral pion photoproduction are smaller than in the complete Skyrme model [9], where it was already shown that only the S-wave bound state need to be retained for the proper treatment of the eta photoproduction.

Using the convention for kinematics as in [9], we obtain the cross section and the S-mode multipole amplitude A as follows

$$\frac{d\sigma}{d\Omega} = \frac{q}{k}|A|^2 \tag{3.10}$$

$$A = \int_\epsilon^\infty r^2 dr\{-[\tfrac{1}{r}\sin 2F\tfrac{dF}{dr}j_0(kr) + (\tfrac{1}{r^2}\sin^2 F - \tfrac{1}{2r}\sin 2F\tfrac{dF}{dr})j_2(kr)]\eta + + \frac{1}{\omega_0\Omega}[j_0(kr) + j_2(kr)]\frac{2}{r}\sin^2 F\frac{d\eta}{dr}\} \tag{3.11}$$

Introducing here the dimensionless coordinate $y = \frac{r}{\epsilon}$ and the dimensionless momentum parameter $\kappa = k\epsilon$, we may rewrite (3.11) as follows

$$A = \epsilon\int_1^\infty y^2 dy\{-[\tfrac{1}{y}\sin 2F\tfrac{dF}{dy}j_0(\kappa y) + (\tfrac{1}{y^2}\sin^2 F - \tfrac{1}{2y}\sin 2F\tfrac{dF}{dy})j_2(\kappa y)]\eta + + \frac{1}{\omega_0 b\epsilon^3}[j_0(\kappa y) + j_2(\kappa y)]\frac{2}{y}\sin^2 F\frac{d\eta}{dy}\} \tag{3.12}$$

We will use the expression (3.12) together with (2.12) to calculate the η-photoproduction amplitude at the treshold in the Constant-cutoff approach to the Skyrme model.

Following [9] it can be shown how the Born term for the N^* resonance can be extracted from (3.11) or (3.12). The procedure is basically the same as in [9] and the end result in our notation is following

$$M = \mathrm{i} < \mathrm{k}|\vec{\varepsilon}\cdot\vec{\mathrm{J}}|\eta_0 > (\frac{1}{\omega-\omega_0} - \frac{1}{\omega+\omega_0}) < \eta_0|\frac{1}{3}[4\mathrm{m}_\mathrm{K}^2 + 3\mathrm{m}_\eta^2 - \mathrm{m}_\pi^2(2-\cos\mathrm{F})]|\mathrm{q}> \tag{3.13}$$

where $exp(ikx) \equiv |k>$ and $exp(iqx) \equiv |q>$ are plane waves of the corresponding free states.

4 Numerical results

The present results for the full multipole anplitude A given by (3.12) and the Born contribution obtained from (3.13) in units of $10^{-3}/m_\pi$, are sumarized in Table 1. Thus we see that the present results are in qualitative agreement with the results quoted in [9] and they are somewhat closer to the results quoted in [11] even though they are also generally too low compared to the results of [11].

Table 1: Multiple amplitude A in units of $10^{-3}/m_\pi$

Multipole amplitude	Present results (3.12) , (3.13)	Ref. [9] Set A	Set B	Ref. [11]
Full amplitude	2.27	0.64	1.72	9.5
Born term	2.74	0.89	2.17	12.4

In order to explain why the present predictions for the eta-photoproduction amplitude, similarly to those reported in the case of the complete Skyrme model [9], are too small compared to those reported in [11], we also present the results for the EM transition amplitude from the nucleon to the N^*-resonance $A_{1/2}$, given in units of $10^{-3}\ GeV^{-1/2}$, and the ηNN^* coupling constant $g_{\eta NN^*}$. In the constant-cutoff approach they are given by

$$A_{1/2} = (\frac{M^* + M}{2M(M^* - M)})^{1/2} < k|\vec{\varepsilon}\cdot\vec{J}|\eta_0 > \tag{4.1}$$

$$g_{\eta NN^*} = - < \eta_0|\frac{1}{3}[4m_K^2 + 3m_\eta^2 - m_\pi^2(2 - \cos F)]|q > \tag{4.2}$$

and the numerical values obtained using the constant-cutoff approach are $A_{1/2} = 17.7$ and $g_{\eta NN^*} = 3.6$ which are in qualitative agreement with the results quoted in [9], i.e. $A_{1/2} = 5.2$ and $g_{\eta NN^*} = 4.0$ for parameter set A and $A_{1/2} = 10.4$ and $g_{\eta NN^*} = 4.9$ for parameter set B. However present result for the EM transition amplitude is still much too low, as in the case of the complete Skyrme model [9], compared to the empirical results found in [12], i.e. $A^p_{1/2} = 74 \pm 11$ $A^n_{1/2} = -72 \pm 25$ in units of $10^{-3}\ GeV^{-1/2}$ as above. On the other hand the result for the coupling constant $g_{\eta NN^*}$ is almost the double of the empirical value of $g_{\eta NN^*} \sim 2$, obtained using the branching ratio $\Gamma_{N^*\to\eta N}/\Gamma_{N^*\to all}$. The present numerical values for $A_{1/2}$ and $g_{\eta NN^*}$ using (3.9, 3.11,4.1 and 4.2) are consistent with our predictions for the eta-photoproduction amplitude. However, as argued in [9], in the neutral pion case, due to the Kroll-Ruderman theorem, the charged pion production is much larger than the neutral one. If N^* then spends part of the time as a pion-nucleon composite in the S-state, it is possible that the transition amplitude and thereby the eta-photoproduction amplitude would become closer to the results in [11]. Furthermore it should be noted that the fact that the helicity amplitudes are rather small might give rise to the question of the S-wave bound being an eta-nucleon bound state. Further study of these issues, both in the complete Skyrme model [9] and constant-cutoff approach, used in the present paper, would be interesting.

Further it should be noted that although the general qualititative picture of the constant-cutoff results is similar to that of the results obtained using the complete Skyrme model

[9], the results obtained here are slightly closer to the available empirical data. However, as pointed out in [9], the further refinements of both the complete Skyrme model [9] and the constant-cutoff approach used in the present paper is neccessary to obtain the more realistic predictions of the Eta photoproduction data.

5 Conclusions

We have shown the possibility of using the Skyrme model for the study of Eta photoproduction from the nucleon at treshold regions, where N(1535) Born terms are separated from the total amplitude by identifying the resonance with an η bound state to the soliton, without the use of the Skyrme stabilizing term proportional to e^{-2}, which makes the practical calculations very complicated and introduces the problem of the choice of the stabilizing term.

For such a simple model with only one arbitrary dimensional constant F_π which is chosen equal to its empirical value $F_\pi = 186MeV$, we find that the results obtained are of the same order of magnitude as those obtained using the complete Skyrme model [9] and are somewhat closer to the results estimated in a phenomenologocal model or the available empirical data.

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