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q-Deformed Fock Space and Statistical Properties of Quons

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Abstract. The q-deformed Fock space is contructed starting from a proposed q-deformed exterior algebra. This construction leads to the introduction of the oscillator algebra corresponding to quons. The quantum distribution of these particles is given. We discuss the Bose-Einstein condensation in D-dimensional and we analyse the sommerfeld expansion near low temperature. Finally, we give the coherence factor and the correlation function

1 Introduction

The concept of quantized universal envolopping algebras (quantum algebras) has been the object of numerous developments in mathematics and physics. Most of these quantum algebras can be realized in terms of q-deformed bosons [1]-[8] and different types of them has been defined through the algebra generated by $\{1, a, a^+, N\}$ and the structure function $\Phi(x)$, satisfying the relations:

$$\begin{aligned} &[a, N] = a, \qquad [a^+, N] = -a^+ \\ &a^+a = \Phi(N) = [N], \qquad aa^+ = \Phi(N+1) = [N+1]. \end{aligned} \tag{1.1}$$

where $\Phi(x)$ is an analytic function with $\Phi(0) = 0$ and N the number operator. The structure function is given by

$$\Phi(x) = \frac{q^x - 1}{q - 1} \tag{1.2}$$

We note that these particles are called quons (we shall deal here with this deformation scheme; that may be clarified after). The case of the deformed oscillator with q being a root of unity is qualitatively different from the case of q is real or complex. In the first case the Hilbert space is finite dimensional, while in the latter the Fock space is an infinite dimensional Hilbert space. If we consider q a k-th root of unity (k is a natural number and $k \geq 0$), this implies $n = 0, ..., k-1$ and quons interpolate between fermions $k = 2$ and bosons $k \to \infty$ (n is the number of particles on a given quantum state). The quons are supposed to obey a generalized Pauli exclusion principle according to which no more than $(k-1)$ particles can live in the same quantum state. Moroever the introduction of the q-deformed oscillator has been the object of investigation [10]-[21] of the statistical thermodynamical properties corresponding to these class of particles. In this context, we note that the phenomenon of Bose-Einstein condensation of deformed bosons has been studied by several authors[10, 13, 14, 16, 20, 21, 22]. Most of preceding works concern the deformed bosons with a real or complex deformation parameter and not offer an adequate intermediate formulation among Fermi-Dirac and Bose-Einstein statistics.

The present work is devoted to the construction of the deformed Fock space starting from the q-deformation of the exterior algebra on a given Hilbert space. This Fock space will be constructed for a generic q. The case of q is a root of unity is discussed. This mathematical structure allows as to compute, in a special way, the partition function. We will give also the quantum distribution corresponding to the introduced Fock space. Another important result consists on the study of the Bose-Einstein condensation for such particles considered as non relativistic or ultra-relativistic objects. Moroever, we analyse the low temperature behaviour (Sommerfeld formula). We end this work by giving the derivation of the coherence function of order two and the correlation function. We will attract the attention on the fact that the latters reflect some quantum effects arising from the deformation formalism.

2 Fock space from q-deformation of exterior algebra

Let us start by recalling the construction of Fock space from anti-symmetric and symmetric algebra corresponding to fermions and bosons, respectively. So, we consider a Hilbert space $\{H; (.,.)\}$ and its n-fold tensor power H^nwhich, in physical terms, is the n-particle space. This Hilbert space is viewed in this context as $L^2(R^D, d^D x)$.The direct sum

$$F(H) = \otimes_{n=0}^{\infty} H^n \tag{2.1}$$

is called the Fock space over $H(H^0 = C)$. The elements of $F(H)$ can be represented by sequences $\{\varphi = (\varphi^0, ..., \varphi^n)\varphi^n \in H\}$. We denote by D^n the set of decomposable vectors

$$D^n = \{f_1 \otimes f_2 \otimes ... \otimes f_n; f_i \in H\} \tag{2.2}$$

Recall that $f_i \in H; i = 1, ..., n$ are nothing but one differencial forms which correspond to the quantum state of i-th particle; then the exterior product and symmetric product are respectively given by:

$$f_i \wedge f_j = \frac{1}{\sqrt{2}}(f_i \otimes f_j - f_j \otimes f_i) \tag{2.3}$$

$$f_i \vee f_j = \frac{1}{\sqrt{2}}(f_i \otimes f_j + f_j \otimes f_i) \tag{2.4}$$

A simple and natural generalization of is:

$$f_i \tilde{\wedge} f_j = \frac{1}{\sqrt{2}}(f_i \otimes f_j + q f_j \otimes f_i) \tag{2.5}$$

where q an arbitrary complex number. The first remark to do is that the anti-symmetry and the symmetry of the the classical products given by Eqs (2.3), (2.4) are broken. By H^2_q we denote the vector space generated by the set $\{f_i \tilde{\wedge} f_j, i, j = 1, ..., n\}$. It is clear that when $q = 1$ or -1, we obtain H^2_+ or H^2_- corresponding to the well known classical limits (algebras generated by Eqs (2.3), (2.4)).
Before going to construct the higher order, we define the overlapping between the two operations $\otimes$ and Λ as follows:

$$(f_i \otimes f_j)\tilde{\wedge} f_k = \frac{1}{\sqrt{3}}(f_i \otimes f_j \otimes f_k + q f_i \otimes f_k \otimes f_j + q^2 f_k \otimes f_i \otimes f_j q f_j \otimes f_i) \tag{2.6}$$

$$f_k \tilde{\wedge}(f_i \otimes f_j) = \frac{1}{\sqrt{3}}(f_i \otimes f_j \otimes f_k + q f_j \otimes f_i \otimes f_k + q^2 f_j \otimes f_k \otimes f_i q f_j \otimes f_i) \tag{2.7}$$

We rewrite the above formulae in a compact simple form:

$$(f_1 \otimes f_2)\tilde{\wedge} f_3 = \frac{1}{\sqrt{3}}(E + qP_{23} + q^2 P_{23} P_{12})(f_1 \otimes f_2) \otimes f_3 \tag{2.8}$$

$$f_3 \tilde{\wedge}(f_2 \otimes f_3) = \frac{1}{\sqrt{3}}(E + qP_{12} + q^2 P_{23} P_{12})(f_1 \otimes f_2) \otimes f_3 \tag{2.9}$$

Now, we can write the deformed product of three one forms. A simple computation leads to the following expression:

$$(f_1 \tilde{\wedge} f_2)\tilde{\wedge} f_3 = Q_{(3)}(f_1 \otimes f_2 \otimes f_3) \tag{2.10}$$

where the operator $Q_{(3)}$ is given by

$$Q_{(3)} = \frac{1}{\sqrt{3!}}(E + qP_{12} + qP_{23} + q^2 P_{123} + q^2 P_{321} + q^3 P_{13}) \tag{2.11}$$

In the above equation, the operators "P's" are related to elements of the permutation group S_3. It is easy to verify that the new product Λ is associative. Thus, the space H^3_q is generated by the set $\{f_i \tilde{\wedge} f_j \tilde{\wedge} f_k; i, j, k = 1, ..., n\}$. This realization can be extended to an arbitrary order p. Indeed $H_q^{(3)}$ is generated by: $\{f_{i_1}\tilde{\wedge}f_{i_2}...\tilde{\wedge}f_{i_p}; i_1, ..., i_p = 1, ..., n\}$. The Fock space $F_q(H)$ is defined, by analogy with the classical limit, to be the direct sum of all H_q^p

$$F_q(H) = \otimes_{n=0}^{\infty} H_q^n \tag{2.12}$$

Following this mathematical construction, one can introduce the multi-particles wave functions. So, let us denote by D_q^n the set:

$$D_q^n = \{f_1\tilde{\wedge}f_2\tilde{\wedge}...\tilde{\wedge}f_n; f_i \in H\} \tag{2.13}$$

which clearly reduces to D_+^n and D_-^n when $q = 1$ and -1, respectively. D_+^n and D_-^n correspond to n-forms which are n-fold symmetric and totally anti-symmetric under the exchange of indices i and j $(i, j = 1, ..., n)$. Then, D_+^n and D_-^n are associated to bosonic and fermionic Fock space, respectively. At this point we have a sufficient background for building the wavefunctions. So, we consider a collection of n identical D-dimensional particles. Let x_i denote the coordinates of the i-th particle. The state of this system is characterized by the wave function

$$\Psi(x_1, x_2, ..., x_n) = (f_1\tilde{\wedge}f_2\tilde{\wedge}...\tilde{\wedge}f_n)(x_1, x_2, ..., x_n) \tag{2.14}$$

Here also, we can express Ψ in terms of the one particle functions $f_i(x_j)$ by help of the operator

$$Q_n = \frac{1}{\sqrt{n!}} \sum_{P \in S_n} q^{m(P)P} \tag{2.15}$$

where S_n is the permutation group and the integers $m(P)$ appearing in Eq (2.15) are obtained by computation of the minimal number of transpositions generated by the permutation P.

The operator Q_n reduces to the Symmetric operators S and the anti-symmetric one A, if one takes respectively the values 1 or -1 for the parameter q. We note finally that in the case where all f_i are identical to a function f (all particles exist on the same quantum state), the wave function expressed by Eq (2.14) reads as

$$\Psi(x_1, x_2, ..., x_n) = \frac{1}{\sqrt{n!}} \prod_{j=2}^{n} [j](f_1\tilde{\wedge}f_2\tilde{\wedge}...\tilde{\wedge}f_n)(x_1, x_2, ..., x_n) \tag{2.16}$$

where the deformed number $[j]$ is given by 2. In analogy with the classical case (undeformed Fock space), we introduce the creation and annihilation operators characterized through their action:

$$\begin{aligned} a^+ : D_q^n \to D_q^{n+1} // a^+ f\tilde{\wedge}f\tilde{\wedge}...\tilde{\wedge}f &= f\tilde{\wedge}f...\tilde{\wedge}f && (2.17) \\ a : D_q^n \to D_q^{n-1} // af\wedge f\wedge...\wedge f &= [n]f\tilde{\wedge}...\tilde{\wedge}f && (2.18) \end{aligned}$$

with $aD^0 = 0$.

Using Eqs (2.17), (2.18), one easily checks that a and a^+ satisfy the commutation relations:

$$\begin{aligned} &[a, N] = a, \qquad [a^+, N] = -a^+ \\ &aa^+ - qa^+a = 1 \end{aligned} \tag{2.19}$$

So, starting from the q-deformation of the exterior algebra, we have proved that this Fock space is associated to quons satisfying the relations Eqs (2.19). Another interesting point is the case where q is k-th root of unity. It is clear from Eq (2.14) that when $q = e^{\frac{2\pi i}{k}}$, the wave function describing states of k particles or more vanishes. The Fock space are truncated and restricted to states $|n>$ with $n \leq k-1$. That is in agreement with the generalized Pauli principle discussed in the introduction.

3 Partition function and quantum distribution

The Hamiltonian for an ideal q-gas is given by:

$$H = \sum_{\lambda} H_{\lambda}, \qquad H_{\lambda} = (E_{\lambda} - \mu)N_{\lambda} \tag{3.1}$$

In Eq (3.1) ,μ is the chemical potentiel while E_{λ} and N_{λ} are kinetic energy of a q-bosons (quons) and the number operator for q-bosons, in the $\lambda-$mode, respectively. then the q-deformed analogue of the bose factor for the $\lambda-$mode is:

$$(f_{\lambda})_q = \frac{1}{Z} tr(e^{-\beta H} a_{\lambda}^{+} a_{\lambda}) \tag{3.2}$$

where $Z = tr\ e^{-\beta H})$ is the partition function and $\beta = (k_B T)^{-1}$ the reciprocal temperature (k_B is the Boltzman constant). At this step, it is important to distinguish among q complex $||q|| \neq 1$ and $||q|| = 1$. In the first case, the q-bosons are not new particles. However, in the second case, the associate Fock space is finite dimensional. then for $q \in C$, the partition function:

$$Z = \prod_{\lambda} \frac{1}{1 - e^{-\eta}}, \qquad \eta = \beta(E_{\lambda} - \mu) \tag{3.3}$$

is independent of the deformation parameter q. The bose factor $(f_{\lambda})_q$ reads as:

$$(f_{\lambda})_q = (1 - e^{-\eta}) tr \left(e^{-\eta N_{\lambda}} [N_{\lambda}] \right) \tag{3.4}$$

It is immediate to verify that this quantity converges for $||q|| \langle e^{-\beta\mu}$. As a result we obtain:

$$(f_{\lambda})_q = \frac{1}{e^{\eta} - q} \tag{3.5}$$

which, in the boundary situations where $q = 1$ or -1; we recover the ordinary Bose and Fermi distributions, respectively.
For $q = e^{(\frac{2\pi i}{k})}$, the partition function is calculated by taking into account of the generalized Pauli exclusion principle. We have then:

$$Z = \prod_{\lambda} Z_{\lambda}, \qquad Z_{\lambda} = \sum_{n_{\lambda}=0}^{k-1} e^{-\eta n_{\lambda}} = \frac{1 - e - \eta k}{1 - e - \eta} \tag{3.6}$$

The expression Eq (3.6) extends the well-known Fermi and Bose function partition for $k = 2$ and $k \to \infty$, respectively. Having determined the partition function, we can derived the occupation number

$$(f_\lambda)_q = \frac{1}{Z} \sum_{n_\lambda=0}^{k-1} [n_\lambda] e^{-\eta n_\lambda} \tag{3.7}$$

The above equation leads to:

$$(f_\lambda)_q = \frac{1}{e^\eta - q} \tag{3.8}$$

This is the distribution of the quantum gas obeying the relation commutation Eq (2.19) and generalized exclusion Pauli Principle. Here also, Taking $k = 2$ and $k \to \infty$, one obtains, respectively, the Fermi and Bose distribution. It clearly shows that for $q \in C$ or $q = e^{\frac{2\pi i}{k}}$, our q-gas is described by the same quantum distribution $(f_\lambda)_q$ playing a central role in the derivation of the thermodynamical functions. The distribution $(f_\lambda)_q$ can be developped in an integer series. We obtain the expression:

$$(f_\lambda)_q = \sum_{j=0}^{\infty} e^{-\eta(j+1)} q^j \tag{3.9}$$

which reduces (when $q = +1$ and $q = -1$, respectively) to the expansions $\sum_{j=1}^{\infty} e^{-\eta j}$ for ordinary bosons and $\sum_{j=1}^{\infty} e^{-\eta j} (-)^{j+1}$ for ordinary fermions.

4 Bose-Einstein Condensation

As in the usual approach, we enclose the system in a large D-dimensional volume V(D) and the energy spectrum of quons is considered as a continuum. Thus, $(f_\lambda)_q$ is replaced by the factor $f(\epsilon)$ with $\epsilon = \gamma^{-1} p^\alpha$, where $\alpha = 1$ or 2 correspond to the ultra-relativistic or non-relativistic q-gas respectively with $\gamma^{-1} = 1$ or$\frac{1}{2m}$.
The density $\rho(D) = N(D)/V(D)$ of $N(D)$ quons enclosed in the volume$V(D)$ is given by:

$$\rho(D) = N_0(D) J_{D/_{\alpha^{-1}}} \tag{4.1}$$

where

$$N_0(D) = g \frac{V(D) \pi^{D/2} D \gamma^{D/\alpha}}{\alpha (2\pi\bar{h})^D \Gamma(D/2 + 1)} \tag{4.2}$$

In Eq (4.1), $J_{D/_{\alpha^{-1}}}$ is an integral of type

$$J_s = \int_0^\infty \epsilon^s f(\epsilon) d\epsilon \tag{4.3}$$

which, by using the development (3.9), gets

$$J_s = \Gamma(s+1)(k_B T)^{s+1} \sigma(s+1)_q \tag{4.4}$$

where

$$\sigma(s+1)_q = \sum_{j=0}^{\infty} \frac{e^{-\beta\mu(j+1)}}{(j+1)^{s+1}} q^j, \qquad s > -1 \tag{4.5}$$

In Eq (4.4), Γ is the Euler integral of the second type. In (4.2), g is the degree of spin degeneracy.

Analogously to the case of ordinary bosons, we examine the condensation of a system of quons (relativistic or ultra-relayivistic) in D-dimension by taking $\mu = 0$. Then, the bose temperature below for which we obtain a Bose condensation phenomenon is given by;

$$T_B(D) = \frac{1}{k_B} \left(\frac{\rho(D)}{N_0(D)} \frac{1}{\Gamma(D/\alpha)\sigma_0(D/\alpha)} \right)^{\alpha/D} \tag{4.6}$$

where

$$\sigma_0(D/\alpha) = \sum_{j=0}^{\infty} \frac{q^j}{(j+1)^{D/\alpha}} \tag{4.7}$$

The Bose-Einstein condensation is present in our deformed system if the series (4.7) is convergent.

It is well known that in the classical case (i.e $q \to 1$), the Bose-Einstein condensation takes place only when $D/\alpha > 1$. However, in the deformed case the Bose-Einstein condensation occurs for $q \in C$ when $|q| < 1$ and for all quons corresponding to $q = e^{(\frac{2\pi i}{k})}$ independently of the ratio D/α

5 Generalized Sommerfeld expansion

In the this section, we study some low temperature properties of statistical system of quons. Especially, we derive the generalized Sommerfeld expansion of the deformed quantum distribution near the zero temperature. This expansion is very useful in the analysis of the low temperature behaviour of thermodynamic functions.

In order to develop the quantum distribution of quons at zero absolute, we have to consider integrals of types

$$I = \int_0^{\infty} \frac{g(\epsilon)}{e^{\beta(\epsilon-\mu)} - q} d\epsilon \tag{5.1}$$

where $g(\epsilon)$ is a test function. Here we would like to note, near the zero absolute, that the chemical potential μ is larger than zero.

The change of variables $z = \beta(\epsilon - \mu)$ in (5.1) leads to:

$$I = \beta^{-1} \int_0^{\beta\mu} \frac{g(\mu - \beta^{-1}z)}{e^{-z} - q} dz + \beta^{-1} \int_0^{\infty} \frac{g(\mu + \beta^{-1}z)}{e^{z} - q} dz \tag{5.2}$$

and by using the following identity:

$$\frac{1}{e^{-z} - q} = -q^{-1} - \frac{q^{-2}}{e^{z} - q^{-1}} \tag{5.3}$$

the Eq (5.2) becomes then:

$$I=-q^{-1}\beta^{-1}\int_0^{\beta\mu}g(\mu-\beta^{-1}z)dz-q^{-2}\beta^{-1}\int_0^{\beta\mu}\frac{g(\mu-\beta^{-1}z)}{e^z-q^{-1}}dz+\beta^{-1}\int_0^{\infty}\frac{g(\mu+\beta^{-1}z)}{e^z-q}dz \quad (5.4)$$

Finally by setting $\beta\mu\to\infty$ in the second integral of the above equation, we have

$$I=-q^{-1}\int_0^{\mu}g(\epsilon)d\mu+\sum_{p=0}^{\infty}\frac{g^{(p)}(\mu)}{p!}(k_BT)^{p+1}\Gamma(p+1)s(p+1)_q \quad (5.5)$$

where

$$s(p+1)_q=\sum_{j=0}^{\infty}\frac{q^{j+1}+(-)^{p+1}q^{-(j+1)}}{q(j+1)^{p+1}} \quad (5.6)$$

In Eq (5.6), $g(p)$ is the p-th derivative of the test function $g(\epsilon)$ with the respect to the energy ϵ. We note that:

$$\int_0^{\infty}\frac{g(\epsilon)}{e^{\beta(\epsilon-\mu)}-q}d\epsilon=\int_0^{\infty}f(\epsilon)g(\epsilon)d\epsilon \quad (5.7)$$

So, we obtain the expansion:

$$f(\epsilon)=-q^{-1}H(\mu-\epsilon)+\sum_{p=0}\infty\frac{\delta(p)(\mu-\epsilon)}{p!}(k_BT)^{p+1}\Gamma(p+1)s(p+1)_q \quad (5.8)$$

where $H(x)$ is the Heaviside function and $\delta(p)$ is the p-th derivative of the Dirac function δ. This generalizes the Sommerfeld expansion of the Fermi distribution near to the zero absolute. We note that the above expansion is not invariant under the change T into -T: odd integer powers of the temperature occurs in Eq(5.8) . In the classical case $q=-1$, the expansion given by Eq (5.8) should reduce to the well known Sommerfeld development for Fermi gas near T=O:

$$f(\epsilon)=H(\mu-\epsilon)+\sum_{n>0}\frac{(2\pi k_BT)^{2n}}{(2n-1)!2n}B_n\frac{2^{2n-1}-1}{2^{2n-1}}\delta^{2n-1}(\mu-\epsilon) \quad (5.9)$$

The $B_n's$ are the Bernouilli numbers.
In Eq (5.9), only the even integer powers of the temperature occurs contrarely to the deformed case.

6 The coherence and correlation functions

Firstly, we would like to recall that in the case of non deformed bosons, the coherence function $g^{(2)}$ of order two associated to the radiation field takes values $g^{(2)}=0$ for radiation field considered as fermions, $g^{(2)}=1$ for a coherent monomode radiation (calculated by employing coherent states) and $g^{(2)}=2$ for a chaotic monomode radiation. The deformation of the coherence function has been recently investigated [15]-[22]. here we want to discuss

from the results obtained in the above sections the implication of q-deformation on $g^{(2)}$.
The radiation field is described by the Hamiltonian:

$$H = \sum_{\lambda} \hbar\omega_\lambda N_\lambda \tag{6.1}$$

The quantum distribution corresponding to λ mode is given by Eqs(3.5), (3.8) where η is taken to be equal to $\hbar\omega_\lambda$.
The coherence function $g^{(2)}$ is defined by:

$$g^{(2)} = \frac{< a^+a^+aa >}{< a^+a >^2} \tag{6.2}$$

where a and a^+ are the annihilation and creation operator for the $\lambda-$th mode. The notation $< X >$ in Eq (6.2) means the mean statistical value $Z^{-1}tr(e^{-\beta H})X$ for an operator X defined on the Fock space. Using the commutation relations, Eqs (2.19), (6.2) can be developed as:

$$g^{(2)} = \frac{< (a^+a)^2 >}{< a^+a >^2} - q^{-1} < a^+a >^{-1} \tag{6.3}$$

A direct calculation of different values in (6.3) yields:

$$g^{(2)} = (1+q)\frac{e^\eta - q}{e^\eta - q^2} \tag{6.4}$$

with convergence condition $|q| < e^{\eta/2}$ (this result is valid also for $q \in C$ and $|q| = 1$).
The deformed $g^{(2)}$ depends on the parameter q and presents a dependance of the energy and temperature. When $q = -1$, we recover the fermionic behaviour (i.e $g^{(2)} = 0$), for $q = 0$, we have $g^{(2)} = 1$ (coherent phase) and for $q = 1$, we have $g^{(2)} = 2$ corresponding to the chaotic monomode radiation.
On other hand, by using an analogue procedure as in the non deformed case [23] and the relation commutations given by (2.19), we can prove that the correlation between two quons separated by the distance r is:

$$V(r) = qA(r) \tag{6.5}$$

where$A(r)$ is a positive function depending on q. In the classical limit $V(r) = \mp qA(r)$. The situation $V(r) > 0$ corresponds to bosons and traduce a strong correlation between particles. The condition $V(r) < 0$ implies a weak correlation between fermions in agreement with the exclusion Pauli principle. We point out that the correlation $V(r)$ vanishes for a classical gas. In general (deformation case), the quantity $V(r)$ has the same sign as the parameter q. So, when $-1 < q < 0$, there exhibits a repulsive quantum interaction among particles. The situation where $g^{(2)} < 1$, which may be interesting for describing antibunching effects of the light field (arising from the "non classical" nature of radiation field [24], can be understood in our formulation with $-1 < q < 0$. in this case the photons of field radiation have a some quasi-fermionic behaviour. For q=0, the correlation $V(r) = 0$ and the correlation function of order two $g^{(2)}$ takes the value 1. It is interesting to note that when $g^{(2)} = 1$, the radiation field is completely coherent and $V(r) = 0$ corresponds to the absence of quantum effects in the system.
Finally, for $0 < q < 1$, the radiation field can be represented by quons with a bosonic behaviour and becomes chaotic for q=1 (the function $V(r)$ is positive and we have an important quantum interaction among photons).

7 Conclusion

In this work we have proposed a consistent deformation of the Fock space . This deformation has been based on a development of a rigourous framework allowing the introduction of a deformed wedge product. The latter constitutes the main goals of the generalization of the universal strucures of any fock space defined by:

$$F(H) = \otimes_{n=0}^{\infty} H^n$$

To construct the bosonic and fermionic statistical operators, one has to apply the symmetric and anti-symmetric operator on $F(H)$, respectively. The deformed Wedge product introduced in this work allows the unification of these two statistics. In fact for a particular values of the deformation parameter q one can recover the classical ones. Moroever, by using this mathematical construction of the q-deformed Fock space leading to the description of quons and thus we discussed their statistical properties. In fact, the Bose-Einstein condensation has been discussed in D-dimension. We have also derived the generalized Sommerfeld expansion of the deformed quantum distribution. Another result in this paper concerns the correlation function $g^{(2)}$ for a deformed gas of photons. The values of this function become continuous in the deformation case.

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