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The Introduction of an Old-Age Pension in a Neo-Classical Economy [1]

I. Introduction

Before the introduction of a social security system in our society older or disabled people depended for their living on their saving balances, on their family or on welfare. As far as people were used to save for the contingencies of life primarily the question arises whether the introduction and growth of a social security system has lead to reduced personal saving, which induces a capital shortfall and inhibits growth in productivity. This question has extensively been dealt with in recent literature for the United States, Canada and various European countries. A useful survey is given by Kessler et al. (1981). It appears from them that no unambiguous empirical support can be found for the hypothesis of a negative effect of social security on personal saving.
On the other hand a public pension plan can be an important source of saving. Public pension benefits may be financed out of current contributions, accumulated capital created with contributions made in the past or a combination of the two. The capital reserve method produces an increase in investment and capital formation. The pay-as-you-go method simply transfers money income from workers to pensioners. Though funding is not necessary for the proper functioning of a pension scheme, it is nevertheless of crucial importance since it contributes to the national capital stock. As far as we know the attention paid to pensions and their financing in economic theory is very slight. An interesting contribution was made by Van Praag & Poeth (1975), who studied the working of an old-age pension in a Harrod-Domar economy. Recently a thorough exposition on pension financing in a neo-classical setting was given by Müller (1980) in this journal. The study dealt with welfare effects of a pension scheme; particularly the dependence of consumer's utility on the pay-as-you-go contribution rate was considered. Worth mentioning is also the article in this journal by Gredig and Müller (1981).
The present study introduces an old-age pension scheme in the basic neo-classical growth model and the role a pension plan can play to let the economy

[1] The author is indebted to C. Withagen and the referee for helpful comment.

move along the golden rule growth path is made clear. Section II gives a general mathematical setting for studying a pension system based on the theory of stable populations subject to fixed rates of fertility and mortality (see e.g. Keyfitz [1977]). Section III presents the neo-classical model with the embedded national pension scheme. In section IV the question dealt with is which part of a given public pension scheme should be financed by the investment fund method and which part by the pay-as-you-go method in order to maximize per capita consumption over time or, in other words, to let the economy move along the golden rule growth path.

II. Demography, pensions and capital accumulation

In this section the mathematics of the capital reserve method are given. Under the capital reserve or investment fund method the active population saves to make sure that funds are available to meet future pension liabilities. To determine per capita pension capital accumulation it is useful to divide the population into two parts, the active population or the labour force and the non-active or the retired. We assume that the working population enters the labour force at age 25 and leaves it at age 65. Let $w(t)$ be the wage rate at time t, which due to technical progress grows at an exogenously given rate g. Contributions (only paid by workers) amount to a fraction δ of worker's income and pension payments per retired are a fraction θ of worker's income. The population is given as growing at a constant rate n and the constant interest rate is denoted by r.

We consider at time t all people born in year α $(25+\alpha \leqq t < 65+\alpha)$. People are assumed to be born at the beginning of the year. Payment of wages, contributions and pension benefits is also assumed to take place at the beginning of the year. These people started work at time $25+\alpha$ and at time t, so far as alive, they all belong to the labour force. The sum of wages paid at time $25+\alpha$ amounted to,

$$w(0)\, e^{g(\alpha+25)}\, N(0)\, e^{n\alpha}\, p(25). \tag{1}$$

The number of births in year 0 is given by $N(0)$ and $p(x)$, where x is age, represents the number living column of the life table with radix unity. At time t the sum of contributions paid at time $25+\alpha$ has increased to,

$$\delta w(0)\, e^{g(\alpha+25)}\, N(0)\, e^{n\alpha}\, p(25)\, e^{r(t-\alpha-25)}. \tag{2}$$

Repeating the argument for the points of time $\alpha+26$, $\alpha+27$, ..., t we find total pension capital accumulation at time t of all workers born in year α,

$$K^W(t, \alpha) = \delta w(0) N(0)\, e^{(n+g-r)\alpha}\, e^{rt} \int_{25}^{t-\alpha} e^{(g-r)u}\, p(u) du \tag{3}$$

It follows that total pension capital accumulation at time t of all those who work amounts to

$$K^W(t) = \int_{t-65}^{t-25} \delta w(0) N(0)\, e^{(n+g-r)\alpha}\, e^{rt} \int_{25}^{t-\alpha} e^{(g-r)u}\, p(u)\, du\alpha \tag{4}$$

$$= \frac{\delta w(t) N(t)}{r-g-n} \left[e^{(r-g-n)65} \int_{25}^{65} e^{(g-r)u}\, p(u) du - \int_{25}^{65} e^{-nu}\, p(u) du \right]; \quad r \neq g+n \tag{5}$$

Next we consider at time t all people born in year α ($t > 65+\alpha$), who are now 65 years of age or older. Pension capital formation of these people at time $65+\alpha$ is given by,

$$K^W(\alpha+65, \alpha) - \theta w(0) e^{g(\alpha+65)}\, N(0) e^{n\alpha}\, p(65). \tag{6}$$

Repeating the argument for the points of time $\alpha+66$, $\alpha+67$, ..., t we find total pension capital accumulation at time t of all retired people born in year α,

$$K^R(t, \alpha) = -\theta w(0) N(0) e^{(n+g-r)\alpha}\, e^{rt} \int_{65}^{t-\alpha} e^{(g-r)u}\, p(u) du + e^{r(t-\alpha-65)}\, K^W(\alpha+65, \alpha). \tag{7}$$

It follows that total pension capital formation at time t of all those who are retired add up to,

$$K^R(t) = -\theta w(0) N(0) \int_{t-\omega}^{t-65} \left(e^{(n+g-r)\alpha} e^{rt} \int_{65}^{t-\alpha} e^{(g-r)u}\, p(u) du + e^{r(t-\alpha-65)}\, K^W(\alpha+65, \alpha) \right) d\alpha \tag{8}$$

$$= \frac{\theta w(t) N(t)}{r-g-n} \left[-e^{(r-g-n)65} \int_{65}^{\omega} e^{(g-r)u}\, p(u) du + \int_{65}^{\omega} e^{-nu}\, p(u) du \right]; \quad r \neq g+n \tag{9}$$

where ω represents the ultimate age. From $K^R(\alpha+\omega, \alpha) = 0$ follows the contribution rate δ,

$$\delta = \frac{\theta \int_{65}^{\omega} e^{(g-r)u}\, p(u) du}{\int_{25}^{65} e^{(g-r)u}\, p(u) du}. \tag{10}$$

Define,

$$\zeta = \frac{\theta \int_{65}^{\omega} e^{-nu}\, p(u)\,du}{\int_{25}^{65} e^{-nu}\, p(u)\,du} \tag{11}$$

Note that ζ can be interpreted as the contribution rate under the pay-as-you-go system.

On adding (5) and (9) and substituting (10) and (11) in the resulting expression, we find total pension capital accumulation, $K(t, 2)$[2],

$$K(t,2) = \frac{w(t)N(t)}{(r-g-n)}\left[(\zeta-\delta)\int_{25}^{65} e^{-nu}\, p(u)\,du\right]; \quad r \neq g+n. \tag{12}$$

When $r = g+n$ total pension capital accumulation is found to be

$$K(t, 2) = w(t)N(t)\left[\theta \int_{65}^{\omega} u\, e^{(g-r)u}\, p(u)\,du - \delta \int_{25}^{65} u\, e^{(g-r)u}\, p(u)\,du\right]. \tag{12'}$$

III. An old-age pension in a neo-classical economy

In this section we want to embody an old-age pension plan in a simple macro-economic framework. The basic neo-classical assumption is that there is continuous substitution between capital, K, and labour, L. A well-behaved Cobb-Douglas production function is chosen. The neo-classical model is characterized by three equilibrium conditions in respect of full capacity, full employment and equality of saving and investment. It will be clear that e.g. due to considerable unemployment rates the neo-classical model can nowadays hardly be accepted for most western countries.

In the present study personal saving in year t comes from the public and from the pension fund, denoted by $S(t, 1)$ and $S(t, 2)$ respectively. Enterprises do not save and government is absent. We postulate a linear saving function of the households. Their saving depends on disposable income and the saving propensity σ, yielding the following private saving function,

$$S(t, 1) = \sigma\left[Y(t) - S(t, 2)\right] \tag{13}$$

where $Y(t)$ is national product.

[2] This result is also found by Bourgeois-Pichat (1978), who presented a different argumentation.

Feldstein (1974) has argued that the introduction and growth of a pension system may depress direct saving of the households. Individuals view current contributions or anticipated pension benefits as a substitute for their own pre-retirement saving and thus are less motivated to save during their working years[3]. In our case therefore,

$$\sigma = \sigma(\kappa, Z); \quad \frac{\partial \sigma}{\partial \kappa} \leqq 0, \; 0 \leqq \sigma < 1, \tag{14}$$

where κ represents the total pension contribution rate as a fraction of worker's income and Z denotes a complex of other factors determining saving. They are not considered here. The amount of pension saving can be derived from eq. (12) or (12′).

There is labour-augmenting technical progress at a given Harrod-neutral rate g. As a consequence the labour force can be measured in efficiency units $\bar{L}(t) = L(t)e^{gt}$.

It should be observed that the pension plan under study is not entirely funded, but is partly based on the investment fund method and partly on the pay-as-yougo method. The rationale of the mixture is given below.

The neo-classical model with an embodied pension plan can be reduced to the following set of equations. Discussion follows below.

$$Y(t) = K^{\mu}(t)\,\bar{L}^{1-\mu}(t) \tag{15}$$

$$DK(t) = S(t, 1) + S(t, 2); \quad DK(t) = \dot{K}(t), \; K(0)\,\text{given} \tag{16}$$

$$\bar{L}(t) = L(0)e^{(g+n)t}; \; L(0)\,\text{given} \tag{17}$$

$$S(t, 1) = \sigma\left[Y(t) - S(t, 2)\right] \tag{13}$$

$$\sigma = \sigma(\kappa, Z); \quad \frac{\partial \sigma}{\partial \kappa} \leqq 0, \; 0 \leqq \sigma < 1 \tag{14}$$

$$\kappa = \delta + \xi; \quad 0 \leqq \delta, \xi, \kappa \leqq 1 \tag{18}$$

$$S(t, 2) = D\left[w(t)N(t)\,\frac{\zeta - \delta}{r - g - n} \int_{25}^{65} e^{-nu}\,p(u)\,du \right]; \quad r \neq g + n \tag{19}$$

[3] However, the empirical evidence so far is inconclusive.

or

$$S(t,2)=D\left[w(t)N(t)\left\{\theta\int_{65}^{\omega} u\, e^{(g-r)u}\, p(u)\,du-\delta\int_{25}^{65} u\, e^{(g-r)u}\, p(u)\,du\right\}\right];\quad r=g+n \tag{19'}$$

$$w(t)=(1-\mu)\frac{Y(t)}{L(t)} \tag{20}$$

$$r=\mu\,\frac{Y(t)}{K(t)} \tag{21}$$

$$\delta=\frac{\theta\int_{65}^{\omega} e^{(g-r)u}\, p(u)\,du}{\int_{25}^{65} e^{(g-r)u}\, p(u)\,du};\quad 0\leqq\delta\leqq 1 \tag{22}$$

$$\zeta=\frac{\theta\int_{65}^{\omega} e^{-nu}\, p(u)\,du}{\int_{25}^{65} e^{-nu}\, p(u)\,du};\quad 0\leqq\zeta\leqq 1 \tag{23}$$

$$\xi=\frac{\eta\int_{65}^{\omega} e^{-nu}\, p(u)\,du}{\int_{25}^{65} e^{-nu}\, p(u)\,du};\quad 0\leqq\xi\leqq 1 \tag{24}$$

$$\eta=\gamma-\theta;\quad 0\leqq\eta,\ \gamma,\ \theta\leqq 1. \tag{25}$$

The equations (15), (16) and (17) are the three equilibrium conditions in respect of full capacity, the equality of saving and investment and full employment (see e.g. Allen [1970], Wan [1971]). The saving function of the households and of the pension fund are found in (13) and (19) or (19′). Note that the equations (19) and (19′) are the differential form of the equations (12) and (12′) respectively. The saving function of the households used in this study is very simple. It can clearly be modified in several ways, e.g. different proportions saved out of wage income and profits, out of income of the working population and income of the retired and saving out of capital gain may be considered. Clearly, due attention should be given to all the factors at work. This is not pursued here.

The equations (20) and (21) assert that the profit rate and the wage rate, r and w respectively, at any time t are equal to the corresponding marginal products.

This is true when it can be assumed that production is based on profit maximization under perfect competition on both product and input markets. The pension plan considered in this paper is partly based on the capital reserve method and partly on the pay-as-you-go method. The corresponding contribution rates as a fraction of worker's income, δ and ξ respectively, are found in (22) and (24). Their sum κ appears in (18). The resulting pensions per retired from these two sources amount to fractions θ and η (eq. (25)) of current worker's income. Total pension income as a fraction of worker's income, γ is exogenously determined by pension regulations, forming part of labour-conditions. The parameter ζ (eq. (23)) is an auxiliarly variable.
For simplicity's sake we are only considering steady state solutions of the model[4]. It is a well-known characteristic of the equilibrium state of the neoclassical economy that both K and $\bar{L}$ are growing at the rate $g+n$. Since the production function used (eq. (15)) is linear and homogeneous, national product, Y, also grows at the rate $g+n$. The steady state paths follow from the model, provided that there are "right" initial values, viz.,

$$K(0)=\left(\frac{s}{g+n}\right)^{\frac{1}{1-\mu}} L(0) \text{ and } Y(0)=\left(\frac{s}{g+n}\right)^{\frac{\mu}{1-\mu}} L(0)$$

where $L(0)$ is given and s represents the fraction of total income saved.
The total saving rate depends on the saving propensity of the households, σ, and a parameter π, defined as

$$\pi=\frac{K(t, 2)}{K(t)}; \quad 0 \leqq \pi \leqq 1 \tag{26}$$

The national capital stock, like national saving, is composed of two parts, viz. $K(t, 1)$, the amount of capital owned by the public and $K(t, 2)$, the amount of

[4] Considering disequilibrium situations we should need to construct a model in which one of the equilibrium conditions lapses and is replaced by some error-adjustment mechanism. This matter is complicated mathematically and it is not pursued here.

capital owned by the pension fund. Following Van Praag & Poeth (1975) π is called the socialization ratio. From the equations (13) and (26) it follows that,

$$s=\frac{\sigma}{1-(1-\sigma)\pi} \quad \text{if } 0<\sigma\leqq 1,\ 0\leqq\pi<1;\ 0\leqq s\leqq 1 \tag{27}$$

$$s=\frac{S(t,2)}{Y(t)} \quad \text{if } \sigma=0,\ \pi=1;\ 0\leqq s\leqq 1. \tag{28}$$

It is also a well-known characteristic of the equilibrium state of the neo-classical model that the fraction of income saved affects the level at which the economy grows (the initial conditions), but that is does not affect the rate at which it grows. This circumstance gives rise to a crucial result of the neo-classical model that at a particular saving rate, viz. $s=\mu$ per capita consumption is maximized for all time. The condition $s=\mu$ is referred to as the golden rule of accumulation. Moreover $r=g+n$ holds.

IV. Some characteristics of the pension model

This paragraph pays attention to the question of the role a pension fund can play in order to let the economy move along the golden rule growth path[5]. The second objective is to guarantee pensioners an income level of a fraction γ of worker's income.

In view of these objectives two questions should be answered. First, which part of necessary saving will come from the public and which part from the pension fund? The second question originates from the first one and deals with the part of the national pension scheme that will be financed by the investment fund method and the part that will be financed by the pay-as-you-go method. If the optimal golden age path prevails the golden rule condition $s=\mu$ implies that,

$$\pi=\frac{1-\sigma/\mu}{1-\sigma}; \quad 0<\sigma\leqq\mu \tag{29}$$

[5] When $\frac{K(0)}{Y(0)}<\frac{\mu}{n+g}$ the question arises whether the present generation is willing at least temporary to increase saving in order to restore the initial condition. This question is not pursued here.

As a result necessary pension saving amounts to,

$$\begin{aligned} S(t,2) &= \pi\,\mu\,\bar{Y}(t) \\ &= \frac{\mu-\sigma}{1-\sigma}\,\bar{Y}(t) \end{aligned} \tag{30}$$

where $\bar{Y}(t)$ is optimal golden age income.

It might happen that the volume of total saving and thus of pension saving required for optimal growth is too small to guarantee incurred pension liabilities. Therefore, we introduced in the equations (24) and (25) a supplementary pension provision financed by the pay-as-you-go system with contribution rate ξ, thereby increasing pensioner's income with a fraction η of worker's income to the statutory fraction γ. If the socialization ratio π is determined by law or institution the required value of σ under optimal growth conditions follows from (29). It is questionable, however, whether a positive contribution rate κ can be found producing the required value of σ. More important, it would be sheer luck when this contribution rate would lead to the desired pension benefit ratio γ. It can be concluded that with a given value of π optimal growth can only be realized when the private saving propensity can be fixed arbitrarily, which does not seem very likely. The system, thus, is too rigid. Therefore, from here the ratio π will be considered as an endogenous variable.

From the model equations and the equations (29) and (30) follow simultaneously the optimal income path, the personal saving propensity σ, the socialization ratio π, the volumes of personal saving and pension saving and thereby that part of pension income that can be financed by the capital reserve system and the contribution rate δ. Thereupon that part of pensioner's income to be financed by the pay-as-you-go method and the contribution rate ξ are found. It is interesting to note a well-known result of the $r=g+n$ situation, that from the contributor's point of view there is no preference for the capital reserve system, the pay-as-you-go system or any combination of the two (see e.g. Pesando & Rea [1977]). Hence, in the optimal neo-classical growth situation total contributions recieved are equal to the amount of pension payments. At the same time there may be or there may not be pension capital accumulation. As far as the volume of private saving, however, falls short of the amount of saving necessary for golden rule growth, the pension system will be at least partly based on funding. The amount of pension saving, then, appears to be equal to the interest revenues of the investment fund,

$$S(t,2) = r\,\pi\,K(t). \tag{31}$$

The particular case $\sigma = 0$ implies $\pi = 1$. All necessary saving comes from the pension fund or, in other words, society consumes all its labour income and pension fund saves and invests all its capital revenues. It might happen that the volume of saving required for optimal growth exceeds the amount of capital necessary to guarantee incurred pension liabilities under the pure capital reserve system. It is presumedly beyond the objectives of a pension plan to accumulate capital in excess of the present value of expected future pension payments minus the present value of expected future contributions. Clearly in this case $\gamma < \theta$ or $\eta < 0$, which is not allowed, or, in other words, there is a minimal value of γ. The optimal growth path can not be arrived at due to a shortage of saving.

V. Summary and conclusions

In this paper the consequences of the introduction of an old-age pension plan in a growing economy are considered. The new elements demography and pension saving are integrated in a neo-classical setting. Attention is given to the role a national pension plan can play in order to let the economy move along the optimal growth path. From the model follow the private saving propensity, the volumes of private saving and of pension saving and thereby the ratio of pension capital to the total capital stock. Then we know the part of pension income that will be financed by the capital reserve system. The supplementary pension provision is financed by the pay-as-you-go system.

The neo-classical model used here is not an adequate representation of the most sophisticated and mathematically advanced neo-classical theories of growth. The model can be presented far more rigorously and much more complicated models can be constructed which retain the neo-classical flavour. A more thorough-going approach e.g. would attempt to derive the saving behaviour of the community from the intertemporal preferences of the individuals and the incomes which they expect to receive during their lifetimes. Furthermore, more complex demographic applications are feasible, for example cases of covered groups which have not yet reached maturity can be considered. It can be concluded, therefore, that there is much room for further theoretical and empirical research.

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Summary

The present study deals with an old-age pension scheme embedded in a neo-classical growth model. A stable underlying population is assumed. First a general mathematical setting of a pension system based on funding is given. Then the question of the role a public pension scheme can play to let the economy move along the golden rule growth path is dealt with.

Zusammenfassung

Die vorliegende Arbeit behandelt Altersvorsorgepläne in einem neoklassischen Wachstumsmodell. Zunächst wird eine allgemeine mathematische Darstellung eines auf Kapitaldeckung basierenden Vorsorgesystems gegeben, wobei eine stabile Bevölkerung angenommen wird. Sodann wird die Frage untersucht, welche Rolle ein solches Vorsorgesystem spielen kann, damit im Wachstumsmodell die Goldene Regel erfüllt wird.

Resumé

L'article traite des systèmes de pensions de retraite placés dans un modèle néo-classique de croissance. Les considérations présupposent une population stable. L'étude indique tout d'abord la forme mathématique que prend un système de pensions basé sur la capitalisation. Elle aborde ensuite la question du rôle que peut jouer un système de pensions généralisé en vue d'amener l'économie vers une croissance optimale selon la règle d'or.