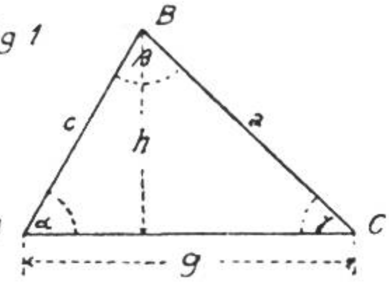
Geometrie.

Formeln zur Jnhaltsberechnung von Flächen und Körpern

Fig 1



Dreieck.

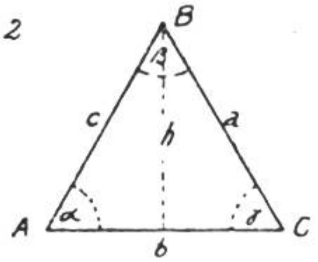
Grundlinie = g; Höhe = h; Fläche = F

$$F = \frac{g \times h}{2} = \frac{g}{2} \cdot h = \frac{h}{2} \cdot g\,;$$

$$g = \frac{2F}{h}\,; \quad h = \frac{2F}{g}$$

$$\sphericalangle\alpha + \sphericalangle\beta + \sphericalangle\gamma = 180^\circ = 2R.$$

Fig 2



Gleichseitiges Dreieck.

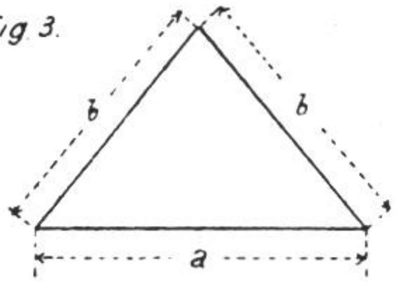
Seiten = $a = b = c$. $\angle\alpha = \beta = \gamma = 60^\circ$

$$F = \frac{a^2}{4} \cdot \sqrt{3} = 0{,}433\, a^2$$

(genauer $0{,}4330127\, a^2$)

$$h = \sqrt{a^2 - \frac{b^2}{4}} = \sqrt{a^2 - \frac{c^2}{4}}\,;$$

Fig 3.

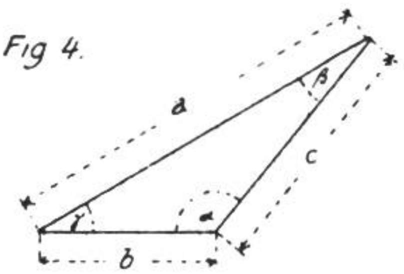


Gleichschenkliges Dreieck.

Grundlinie = a; gleiche Seiten = b

$$F = \frac{a}{4}\sqrt{(2b+a)\cdot(2b-a)} = \frac{a}{2}\sqrt{\left(b+\frac{a}{2}\right)\left(b-\frac{a}{2}\right)}$$

Fig 4.

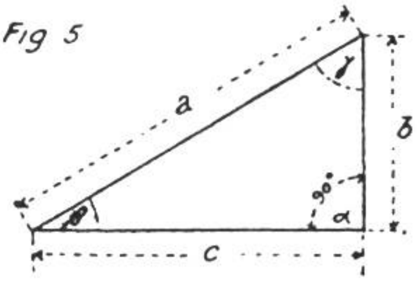


Ungleichseitiges Dreieck.

Seiten a, b und c, $s = \frac{a+b+c}{2}$;

$$F = \sqrt{s(s-a)(s-b)(s-c)}$$

Fig 5



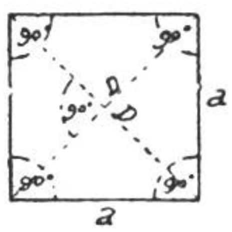
Rechtwinkliges Dreieck. $\sphericalangle\alpha = 90^\circ$

Hypotenuse = a; Katheten = b und c,

$$F = \frac{b \cdot c}{2}\,; \quad a^2 = b^2 + c^2\,; \quad a = \sqrt{b^2 + c^2}\,;$$

$$b = \sqrt{a^2 - c^2}\,; \quad c = \sqrt{a^2 - b^2}$$

Fig. 6



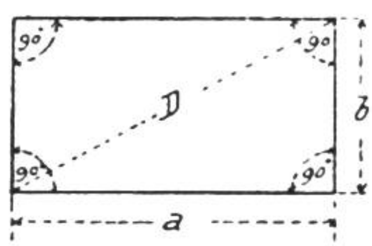
Quadrat

Seite = a; Diagonale = D;

$$F = a \times a = a^2 \qquad a = \sqrt{a^2}$$

$$D = \sqrt{a^2 + a^2} = \sqrt{2a^2} = a\sqrt{2} = a \cdot 1{,}4142$$

Fig. 7



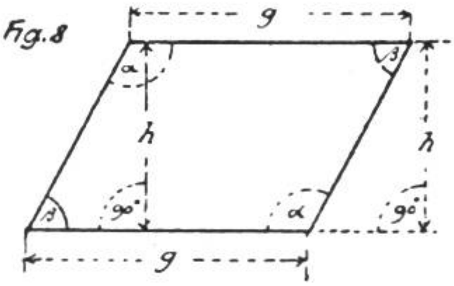
Rechteck.

Seiten a und b, Diagonale D;

$$F = a\,b;\quad a = \frac{F}{b};\quad b = \frac{F}{a};$$

$$D = \sqrt{a^2 + b^2}$$

Fig. 8

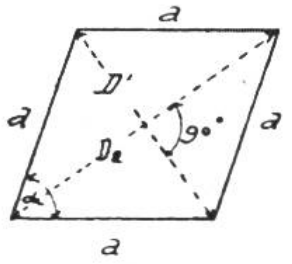


Parallelogramm.

Grundlinie = g Höhe (rechtwinklig auf Grundlinie) = h

$$F = g \cdot h;\quad g = \frac{F}{h},\quad h = \frac{F}{g};$$

Fig. 9

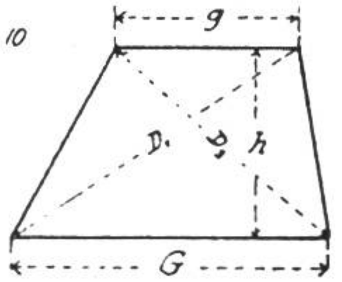


Rhombus.

Gleiche Seiten a; Diagonalen D_1 u. D_2

$$F = a^2 \sin\alpha;\quad F = \frac{D_1 \cdot D_2}{2};$$

Fig 10

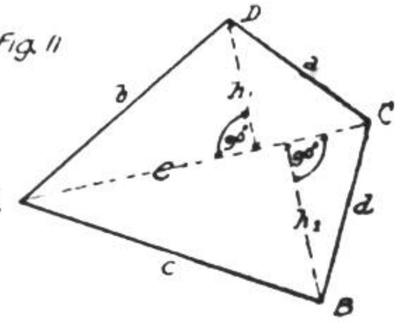


Trapez.

Parallelseiten = G und g, Höhe = h
Diagonalen = D_1 und D_2

$$F = \frac{G + g}{2} \cdot h;$$

Fig. 11

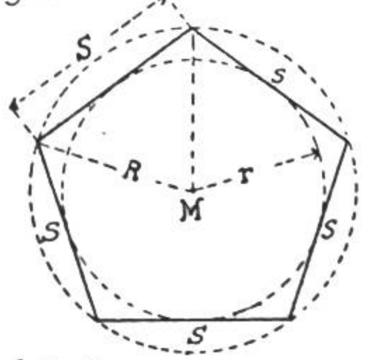


Trapezoid.

Diagonale $\overline{AC}$ und rechtwinklig darauf die Höhen h_1 und h_2

$$F = \frac{\overline{AC}}{2} \cdot (h_1 + h_2) = \frac{e}{2} \cdot (h_1 + h_2).$$

Fig 12



Seite = S
Radius des umschriebenen Kreises = R
Radius des eingeschriebenen Kreises = r

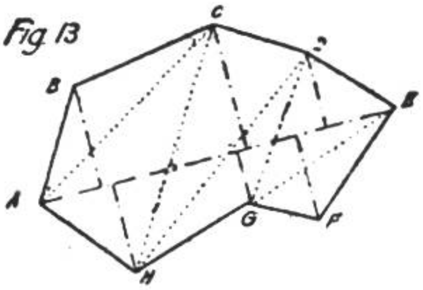
Reguläre Vielecke (Polygon)

Polygon	R	r	S	F
Dreieck	0.577 S	0.289 S	1.732 R od. 3.463 r	0.433 S² od. 1.299 R²
Quadrat	0.707 S	0.500 S	1.414 R " 2.000 r	1.000 S² " 2.000 R²
Fünfeck	0.851 S	0.695 S	1.176 R " 1.453 r	1.721 S² " 2.378 R²
Sechseck	1.000 S	0.866 S	1.000 R " 1.155 r	2.598 S² " 2.598 R²
Siebeneck	1.152 S	1.038 S	0.868 R " 0.963 r	3.364 S² " 2.736 R²
Achteck	1.307 S	1.208 S	0.765 R " 0.828 r	4.828 S² " 2.828 R²
Neuneck	1.462 S	1.374 S	0.684 R " 0.728 r	6.182 S² " 2.892 R²
Zehneck	1.618 S	1.540 S	0.618 R " 0.649 r	7.694 S² " 2.939 R²
Elfeck	1.775 S	1.704 S	0.563 R " 0.587 r	9.366 S² " 2.973 R²
Zwölfeck	1.932 S	1.866 S	0.518 R " 0.536 r	11.190 S² " 3.000 R²

E. Pöchos

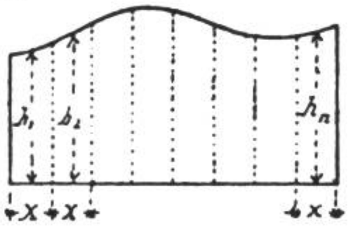
Unregelmässige Vielecke od Flächen:

Fig 13



Fläche kann bestimmt werden durch Zerlegung des Vielecks in Dreiecke mittelst Diagonalen und Summirung der ermittelten Dreiecksflächen, oder durch Einteilung in Trapeze u. Dreiecke vermittelst der Koordinaten der Eckpunkte auf eine rechtwinklig gewählte Axe. Fig 13

Fig. 14

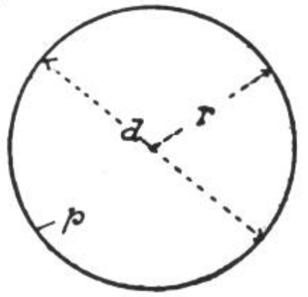


Durch Zerlegung in parallele Streifen gleicher Breite Fig 14

$$F = x.(h_1 + h_2 + \ldots \ldots h_n)$$

Kreis:

Fig. 15

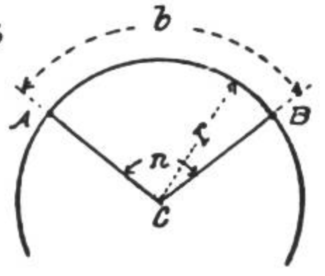


Durchmesser = d; Radius = r
Umfang = p; Jnhalt = F.

$$p = d\,\pi = d.\,3.14159$$
$$= 2r\pi,$$
$$F = \frac{d^2\pi}{4} = 0.785\,d^2 = r^2\pi$$
$$r = \sqrt{\frac{F}{\pi}} = 0.564\sqrt{F};$$
$$d = 2\sqrt{\frac{F}{\pi}} = 1.128\sqrt{F}$$

Kreissektor: (A.B.C) Fig. 16.

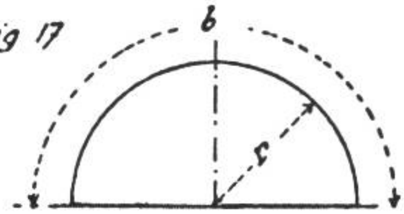
Fig 16



Radius = r; Bogen = b;
Zentriwinkel = n;

$$F = \frac{r^2\pi.n}{360} = \frac{b.r}{2}.$$
$$r = \sqrt{\frac{F}{\pi}.\frac{360}{n}} = \frac{b}{\pi}.\frac{180}{n};$$
$$n = 360.\frac{F}{r^2\pi} = \frac{b}{r\pi}.180$$
$$b = 2r\pi\frac{n}{360} = r\pi.\frac{n}{180}.$$

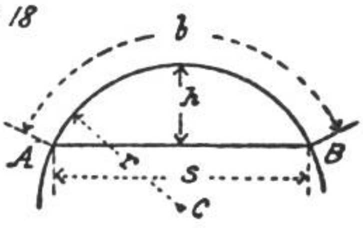
Fig 17



Halbkreisbogen = $b = \pi.r$.
Halbkreisfläche = $F = \frac{\pi.r^2}{2}$.
Viertelkreisbogen = $b = \frac{\pi.r}{2}$
Viertelkreisfläche = $F = \frac{\pi.r^2}{4}$.

Kreisabschnitt:

Fig 18

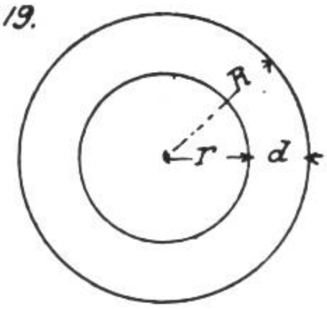


Sehne = s Höhe = h. $F = \frac{2}{3}\,s.h$.

genau. $F = \frac{r^2\pi n}{360} - \frac{1}{2}s\sqrt{r^2 - \frac{1}{4}s^2} = \frac{br - s(r-h)}{2}$

$$s = 2\sqrt{h(2r-h)} \qquad r = \frac{s^2}{8h} - \frac{h}{2}.$$

Fig 19.



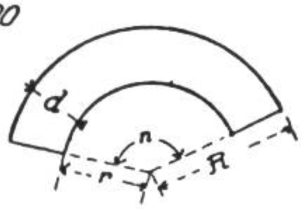
Kreisring:

Äusserer Radius = R;
Innerer Radius = r

$$F = R^2\pi - r^2\pi.$$
$$= \pi(R+r)(R-r).$$

wenn d = radiale Breite des Kreisrings,
so ist $F = \pi(2r+d)d$.

Fig. 20

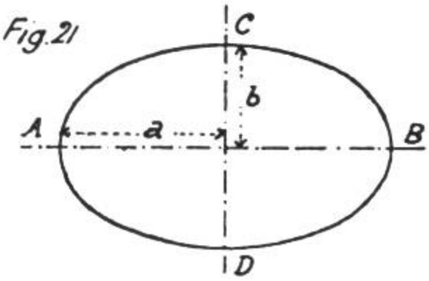


Kreisringstück: (Konzentrisch)

Äusserer Radius = R
Innerer Radius = r
Zentriwinkel = n.

$$F = (R^2\pi - r^2\pi)\frac{n}{360} = (R^2 - r^2)\cdot\frac{\pi . n}{360}$$
$$= (R+r)d\,\frac{\pi . n}{360} = (R+r)d . n . 0.0087.$$

Fig. 21

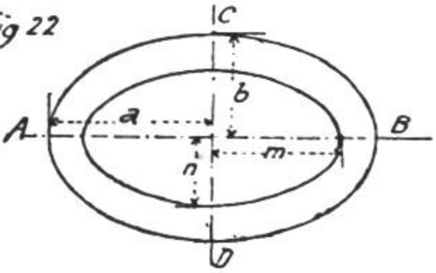


Ellipse:

Halbe Achsen der Ellipse = a und b

$$F = a.b.\pi;$$

Fig 22

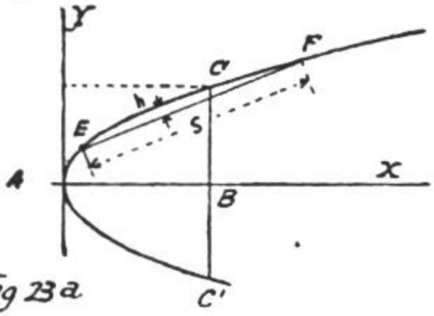


Elliptischer Ring:

Halbe Achsen der äussern Ellipse a, b,
Halbe Achsen der innern Ellipse m, n

$$F = \pi(ab - mn).$$

Fig 23.



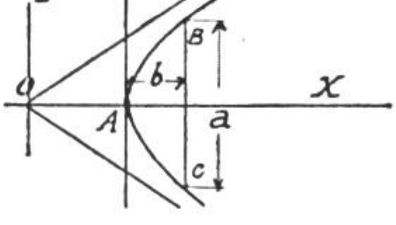
Parabelsegment ECF:

$$F = \frac{2}{3}\,s.h; \qquad s = EF.$$

Parabelfläche CAC':

$$F = \frac{2}{3}\,CC'.AB.$$

Fig 23a

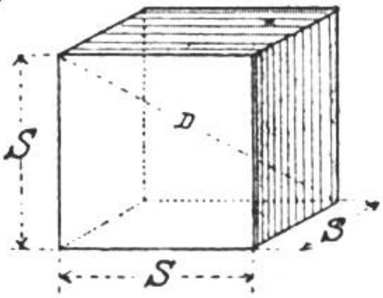


Hyperbelsegment ABC

Sehne = a; Höhe = b

$$F\text{(annähernd)} = \frac{3}{5}\,b.a.$$

Fig 24



Würfel:

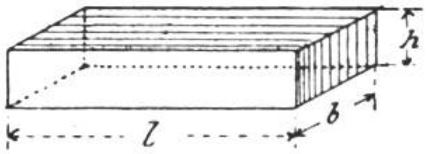
Seite = S; Inhalt = K; Oberfläche = O;

$$K = S^3; \quad O = 6S^2;$$

$$S = \sqrt[3]{K};$$

$$\text{Diagonale} = D = \sqrt{3S^2} = S\sqrt{3} =$$
$$= S \cdot 1.732050$$

Fig 25

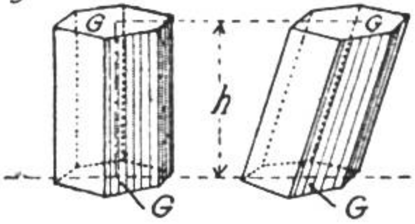


Parallelflach:

Länge = l, Breite = b, Höhe = h;

$$K = l \cdot b \cdot h.$$

Fig. 26



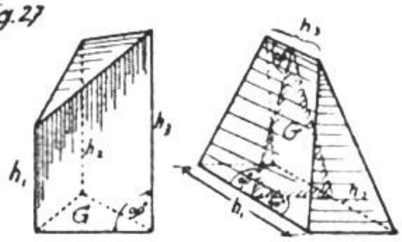
Prisma:

Grundfläche = G; Höhe = h;

$$K = G \cdot h;$$

Oberfläche O = Umfang der Grundfläche $U \times h + 2G$.

Fig. 27



Schiefabgeschnittenes Prisma

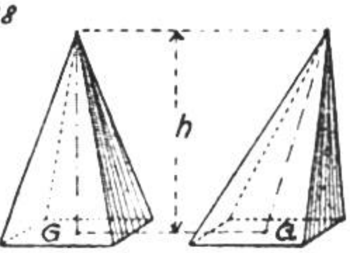
Flächeninhalt des senkrechten Querschnittes = G.

Länge der Kanten = $h, h_2, h_3 \ldots h_n$

$$K = G \times \frac{h_1 + h_2 + h_3 + \ldots h_n}{n}$$

Bei mehr als 3 Kanten ist das Prisma regelmässig oder hat dasselbe wenigstens Axensymetrie.

Fig. 28



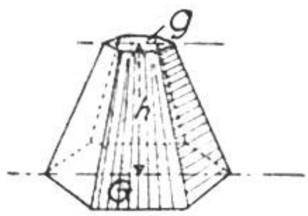
Pyramide:

Grundfläche = G; Höhe = h;

$$K = \frac{G \cdot h}{3}; \quad h = \frac{3K}{G}.$$

$$G = \frac{3K}{h};$$

Fig 29

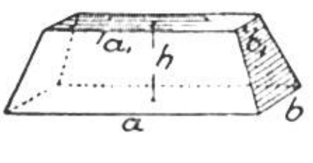


Abgestumpfte Pyramide:

Inhalt der beiden Grundflächen = G u. g.

$$K = \frac{h}{3} \cdot \left(G + g + \sqrt{G \cdot g}\right)$$

Fig 30

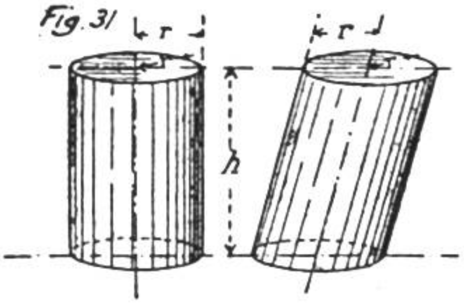


Obelisk, Wall, regelmässig aufgeschütteter Haufen.

$$K = \frac{1}{6} h \left[(2a + a_1) b + (2a_1 + a) b_1\right]$$

E. Pochon

Fig. 31



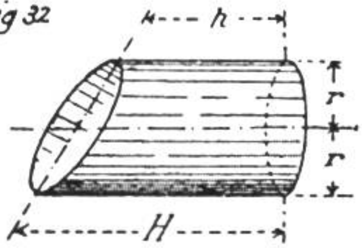
Cylinder, (Walze)

$K = r^2\pi . h \qquad \text{Mantel} = 2r\pi . h$

$r = \sqrt{\frac{K}{\pi . h}} ; \qquad h = \frac{K}{r^2\pi} ;$

Mantel = $2 r \pi . h$.

Fig 32



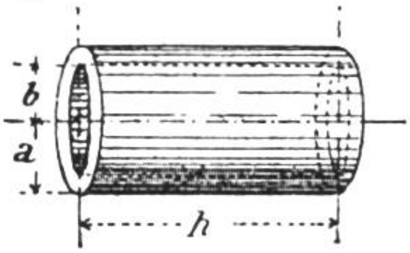
Schiefgeschnittener Cylinder

Grösste Höhe = H; Kleinste Höhe = h,

$K = r^2\pi . \frac{H+h}{2}$

Mantel = $\pi r (H+h)$

Fig 33



Hohlcylinder:

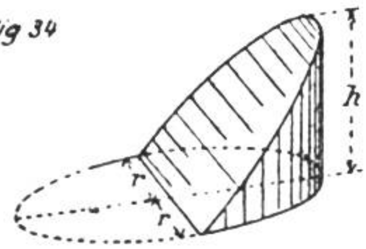
Innerer Halbmesser = b

äusserer Halbmesser = a, Länge = h,

$K = \pi . h (a+b) . (a-b) =$

$K = \pi . h (a^2 - b^2)$

Fig 34



Cylinderhuf:

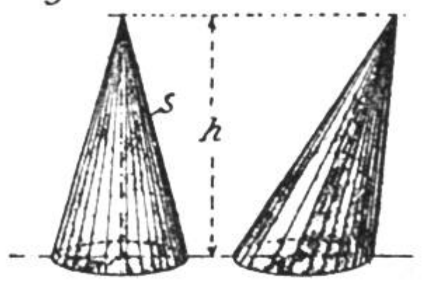
Radius der Grundflache = r

Höhe des Hufes = h

$K = \frac{2}{3} r^2 h$

$M = 2 r . h$.

Fig 35



Kegel.

Halbmesser der Grundfläche = r

Höhe = h, Seite = S

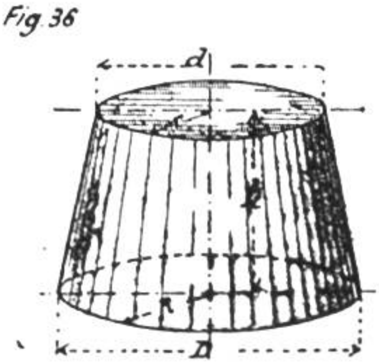
$S = \sqrt{r^2 + h^2}$

Mantel $M = r\pi . S$

Ganze Oberfläche $O = \pi r (r + \sqrt{r^2 + h^2}) =$

$= \pi r (S + r)$.

Fig 36



$K = \frac{1}{3} r^2 \pi . h$

$r = \sqrt{\frac{3K}{h\pi}} , \qquad h = \frac{3K}{r^2\pi}$

Abgestumpfter Kegel

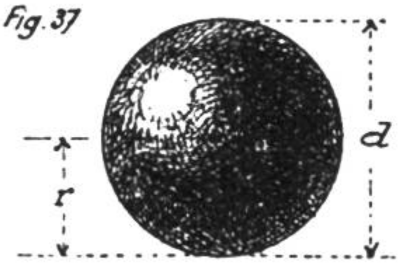
Halbmesser der Grundflächen R und r

Durchmesser der Grundflächen = D und d

$K = (R^2 + r^2 + Rr) \frac{h\pi}{3} = \frac{D^2 + d^2 + dD}{12} . h\pi$.

Mantel = $\pi S (R + r)$

Fig. 37



Kugel:

Radius = r, Durchmesser = d

Oberfläche $O = 4r^2\pi = d^2\pi$

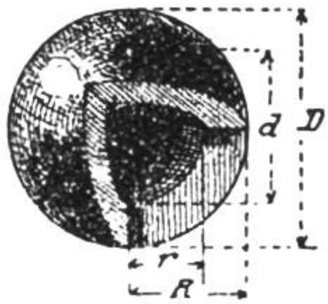
$= 12.566\ r^2$

Jnhalt $K = \frac{4}{3}r^3\pi = \frac{d^3\pi}{6} = \frac{O \cdot r}{3}$

$K = 4.189\ r^3 = 0.5236\ d^3$.

Radius $r = \frac{1}{2}\sqrt{\frac{O}{\pi}} = \sqrt[3]{\frac{3K}{4\pi}}$

Fig. 38



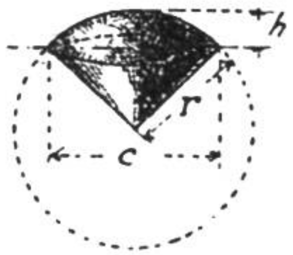
Hohlkugel:

Äusserer Radius = R innerer = r.

" Durchmesser = D " " = d.

$K = \frac{4\pi}{3}(R^3 - r^3) = \frac{\pi}{6}(D^3 - d^3)$.

Fig. 39



Kugelsektor:

Radius der Kugel = r

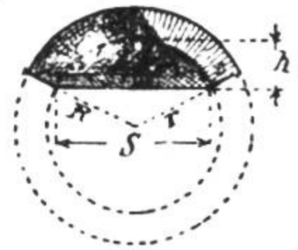
Begrenzende Kalotte: Höhe = h

" " Durchm = c

Oberfläche $= \frac{\pi r}{2}(4h + c)$

$K = \frac{2}{3} r^2 \pi \cdot h = 2.0944\ r^2 h$.

Fig. 40



Hohlkugelsektor:

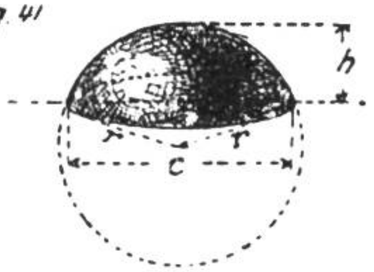
Äusserer Radius = R. iñerer = r

Wanddicke = S; $R = r + S$.

$r = \frac{S^2 + 4h^2}{8h}$

Jnhalt $K = 2.094 \frac{h}{r}(R^3 - r^3)$.

Fig. 41



Kugelsegment:

Radius der Kugel = r.

Kalottendurchmesser = c Höhe = h

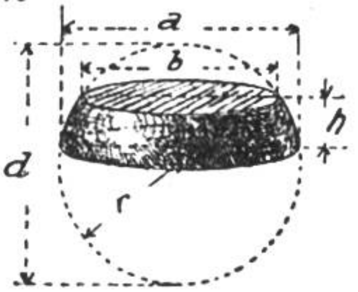
Gewölbte Oberfläche = O

$O = 2r\pi h = \pi\left(\frac{c^2}{4} + h^2\right)$.

$K = \frac{1}{6}\pi h(3c^2 + h^2) = \frac{1}{3}\pi h^2(3r - h)$

$= \pi h^2\left(r - \frac{1}{3}h\right) = \pi h\left(\frac{c^2}{8} + \frac{h^2}{6}\right)$.

Fig. 42



Kugelzone.

Höhe der Zone = h

Halbmesser der Kugel = r

Halbmesser der Endflächen a und b

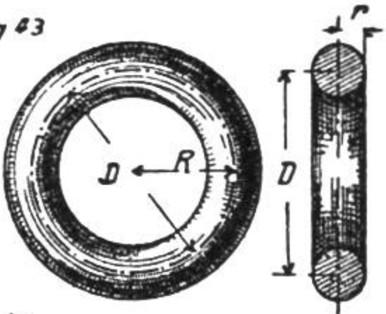
Mantel $= 2r\pi h$.

Oberfläche $= M + a^2\pi + b^2\pi$.

Jnhalt $K = \frac{1}{6}\pi h(3a^2 + 3b^2 + h^2)$.

E. Pochon

Fig 43



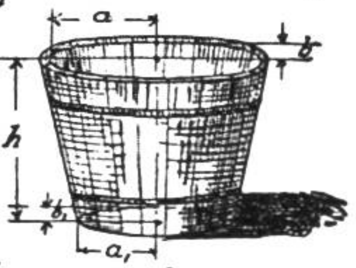
Cylindrischer Ring.

Radius des kreisförm. Querschnittes $= r$
Durchmesser des Ringes $= D$
Radius des Ringes $= R$. siehe Figur.

$K = 2\pi^2 R r^2 = 2.467\ D d^2$

Oberfläche $O = 4\pi^2 R r = 9.87\ D d$.

Fig. 44

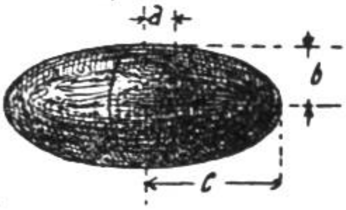


Kübel

Die Endflächen sind Ellipsen mit den Halbachsen a, b und a_1, b_1.
Höhe zwischen den Endflächen $= h$

Jnhalt $K = \frac{1}{6}\pi h\left[2(ab + a_1 b_1) + a b_1 + a_1 b\right]$

Fig 45

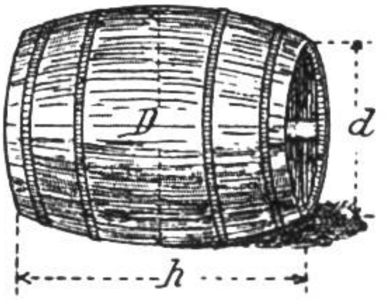


Ellipsoid

Bezeichnung der 3 Halbachsen $= a, b, c$.

Jnhalt $K = \frac{4}{3}\pi \cdot a \cdot b \cdot c$.

Fig 46



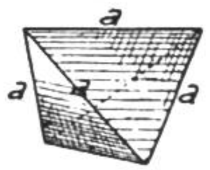
Fass

Spunddurchmesser $= D$
Bodendurchmesser $= d$
Höhe $= h$ (resp Länge)

Jnhalt $K = 1.0453\, h\left(0.4 D^2 + 0.2 D d + 0.15 d^2\right)$

Reguläre Polyeder:

Fig. 47

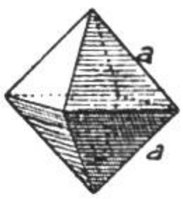


Tetraeder: (4 gleichseitige Dreiecksflächen

Länge der Kante $= a$; $O = a^2\sqrt{3} = 1.732\, a^2$.

$K = \frac{1}{12} a^3 \sqrt{2} = 0.11785\, a^2$.

Fig. 48

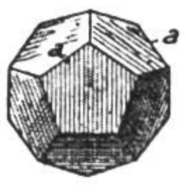


Oktaeder. (8 gleichseit. Dreiecksflächen

Kante $= a$, $O = 2a^2\sqrt{3} = 3.464116\, a^2$.

$K = \frac{a^3}{3}\sqrt{2} = 0.4714045\, a^3$.

Fig. 49

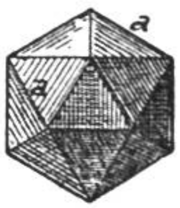


Dodekaeder (12 regelmässige Fünfecke)

Kante $= a$; $O = 3a^2\sqrt{25 + 10\sqrt{5}}$
$= 20.645729\, a^2$.

$K = \frac{a^3}{4} \cdot (15 + 7\sqrt{5}) = 7.663119\, a^3$

Fig. 50



Ikosaeder 20 gleichseitige Dreiecksflächen.

Kante $= a$, $O = 5a^2\sqrt{3} = 8.6602545\, a^2$

$K = 5/12\, a^3 (3 + \sqrt{5}) = 2.181695\, a^3$.