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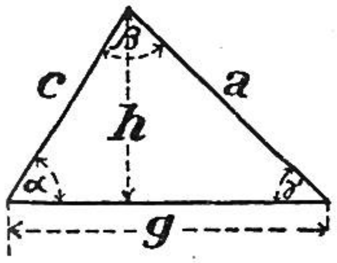
Rubrik: Geometrie

Geometrie

Formeln zur Inhaltsberechnung von Flächen u. Körpern

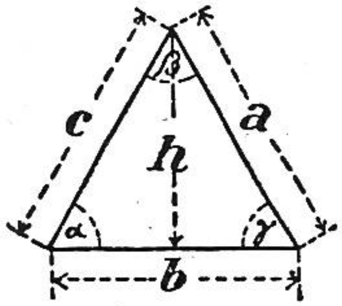


Dreieck:

Grundlinie = g, Höhe = h, Fläche = F,

$$F = \frac{g \cdot h}{2}; \quad g = \frac{2F}{h}; \quad h = \frac{2F}{g};$$

$$\sphericalangle \alpha + \sphericalangle \beta + \sphericalangle \gamma = 180° = 2 \text{ rechte } \sphericalangle$$



Gleichseitiges Dreieck:

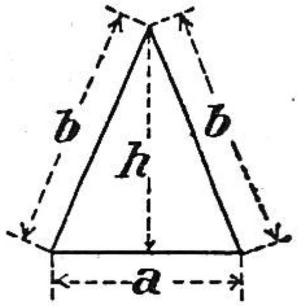
Seiten = $a = b = c$, $\sphericalangle\alpha = \sphericalangle\beta = \sphericalangle\gamma = 60°$,

$$F = \frac{a^2}{4}\sqrt{3} = 0{,}433\, a^2; \quad \sqrt{3} = 1{,}73205.$$

genauer $0{,}4330125 \,.\, a^2$

$$h = \sqrt{a^2 - \frac{b^2}{4}} = \sqrt{a^2 - \frac{c^2}{4}};$$

$$h = \frac{a}{2}\sqrt{3} = a \,.\, 0{,}866025.$$

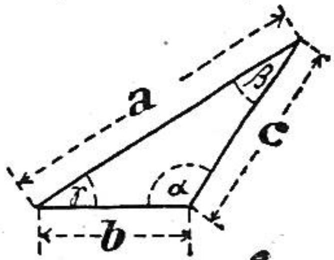


Gleichschenkliges Dreieck:

Grundlinie = a, gleiche Seiten = b

$$F = \frac{a}{4}\sqrt{(2b+a).(2b-a)} = \frac{a}{2}\sqrt{\left(b+\frac{a}{2}\right).\left(b-\frac{a}{2}\right)}$$

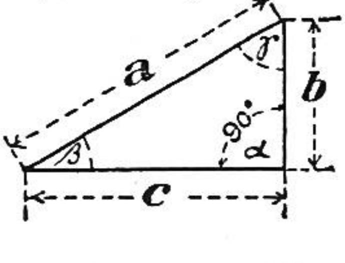
$$h = \sqrt{b^2 - \frac{a^2}{4}}, \quad h = \sqrt{\left(b+\frac{a}{2}\right).\left(b-\frac{a}{2}\right)};$$



Ungleichseitiges Dreieck:

Seiten a, b und c, $s = \dfrac{a+b+c}{2}$

$$F = \sqrt{s(s-a).(s-b).(s-c)}.$$

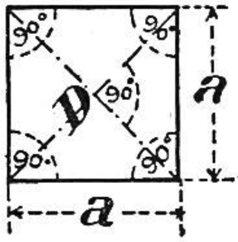


Rechtwinkliges Dreieck: $\sphericalangle a = 90°$;

Hypotenuse $= a$, Katheten $= b, c$,

$$F = \frac{b.c}{2}; \quad a^2 = b^2 + c^2; \quad a = \sqrt{b^2+c^2};$$

$$b = \sqrt{a^2 - c^2}, \quad c = \sqrt{a^2 - b^2}.$$



Quadrat: Seite = a, Diagonale = D,

$$F = a.a = a^2, \qquad a = \sqrt{F}.$$

$$D = \sqrt{a^2 + a^2} = \sqrt{2a^2} = a\sqrt{2} = a\ 1{,}4142.$$

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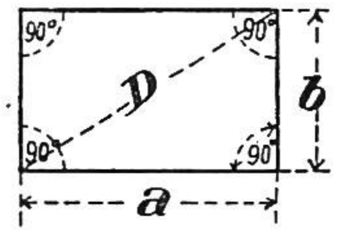
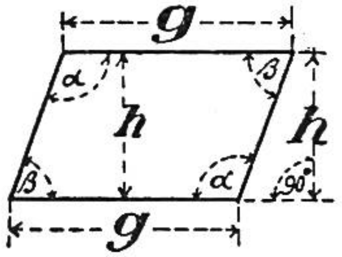


Rechteck:

Seiten a und b, Diagonale $= D$,

$$F = a \cdot b;\quad a = \frac{F}{b};\quad b = \frac{F}{a};$$

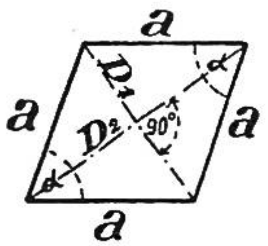
$$D = \sqrt{a^2 + b^2};$$



Parallelogramm:

Grundlinie $= g$, Höhe rechtwinklig auf Grundlinie $= h$,

$$F = g \cdot h;\quad g = \frac{F}{h};\quad h = \frac{F}{g}.$$

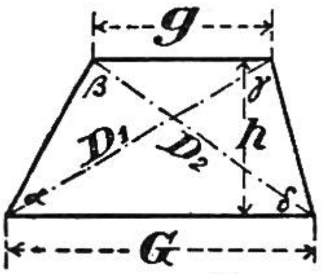


Rhombus:

Gleiche Seiten $= a$, Diagonalen D_1 u. D_2

$$F = a^2 \sin\alpha;\quad F = \frac{D_1 \cdot D_2}{2};$$

$$D_1 = \frac{2F}{D_2};\quad D_2 = \frac{2F}{D_1}.$$

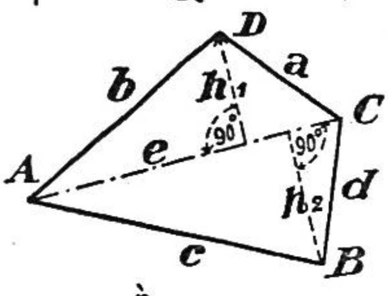


Trapez:

Parallelseiten $= G$ und g, Höhe $= h$,

Diagonalen $= D_1$ und D_2,

$$F = \frac{G+g}{2}\, h;\quad \sphericalangle\, \alpha+\beta+\gamma+\delta = 4R;$$

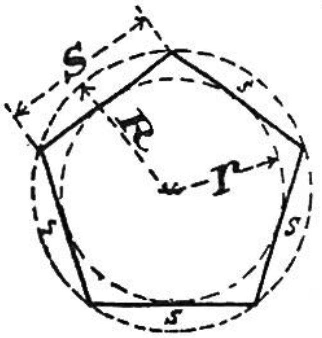


Trapezoid:

Diagonale AC, rechtwinklig darauf die Höhen h_1 und h_2

$$F = \frac{AC}{2} \cdot h_1 + h_2; = \frac{e}{2} \cdot h_1 + h_2.$$

Reguläre Vielecke (Polygone):



Seite $= S$,
Radius des umschriebenen Kreises $= R$
Radius d. eingeschriebenen Kreises $= r$.

Polygon	R	r	S	F
Dreieck	0.577 S	0.289 S	1.732 R od. 3.463 r	0.433 S² od. 1.299 R²
Quadrat	0.707 S	0.500 S	1.414 R „ 2.000 r	1.000 S² „ 2.000 R²
Fünfeck	0.851 S	0.688 S	1.176 R „ 1.453 r	1.721 S² „ 2.378 R²
Sechseck	1.000 S	0.866 S	1.000 R „ 1.155 r	2.598 S² „ 2.598 R²
Siebeneck	1.152 S	1.038 S	0.868 R „ 0.963 r	3.634 S² „ 2.736 R²
Achteck	1.307 S	1.208 S	0.765 R „ 0.828 r	4.828 S² „ 2.828 R²
Neuneck	1.462 S	1.374 S	0.684 R „ 0.728 r	6.182 S² „ 2.892 R²
Zehneck	1.618 S	1.540 S	0.618 R „ 0.649 r	7.694 S² „ 2.939 R²
Elfeck	1.775 S	1.704 S	0.563 R „ 0.587 r	9.366 S² „ 2.973 R²
Zwölfeck	1.932 S	1.866 S	0.518 R „ 0.536 r	11.196 S² „ 3.000 R²

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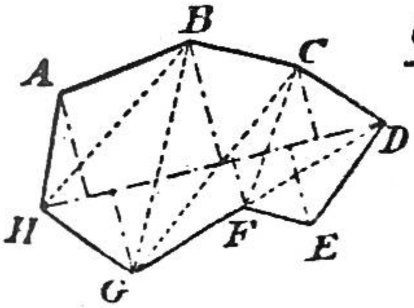
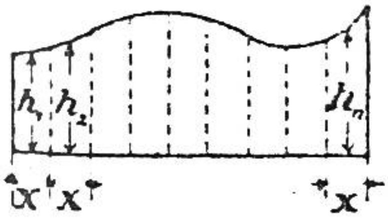
Unregelmässige Vielecke od. Flächen:



Die Fläche kann berechnet werden durch Zerlegung des Vielecks in Dreiecke mittels Diagonalen u. Summierung der ermittelten Dreiecksflächen, oder auch durch Einteilung in Trapeze u. Dreiecke vermittels einer passend gewählten Abscisse und rechtwinklig auf diese errichteten Koordinaten der Eckpunkte, Summierung des ermittelten Jnhalts dieser Trapeze u. Dreiecke.

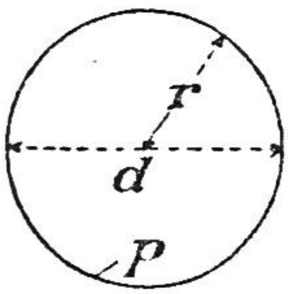


Durch Zerlegung in parallele Streifen von gleicher Breite x und mittl. Höhen $h_1 h_2 \ldots h_n$.

$$F = x.(h_1 + h_2 \ldots\ldots + h_n).$$

Kreis:

Durchmesser = d, Radius = r,
Umfang = p, Inhalt = F.



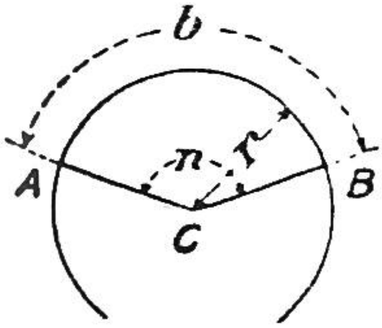
$$p = 2r.\pi = d\pi = d.3{,}14159,$$

$$F = r^2\pi = \frac{d^2\pi}{4} = 0{,}785.d^2.$$

$$r = \sqrt{\frac{F}{\pi}} = 0{,}564.\sqrt{F}.$$

$$d = 2.\sqrt{\frac{F}{\pi}} = 1{,}128\sqrt{F}.$$

Kreissektor: (ABC)



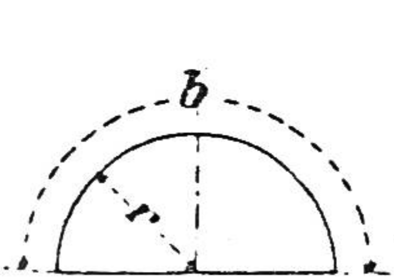
Radius = r, Bogen = b Zentriwinkel = n

$$F = \frac{r^2\pi.n}{360} = \frac{b.r}{2}$$

$$r = \sqrt{\frac{F.360}{\pi.n}} = \frac{b}{\pi}.\frac{180}{n};$$

$$n = \frac{360.F}{r^2\pi}, = \frac{b}{r.\pi}.180;$$

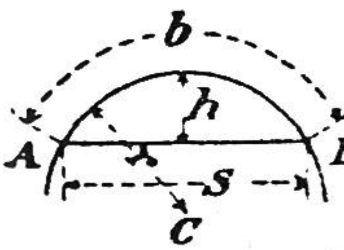
$$b = 2r.\pi.\frac{n}{360}. = r.\pi.\frac{n}{180},$$



Halbkreis: Bogen = $b = \pi r$; Fläche = $F = \frac{\pi r^2}{2}$.

Viertelkreis: » » $b = \frac{\pi r}{2}$; $F = \frac{\pi r^2}{4}$;

Kreisabschnitt:



Sehne = s, Höhe = h, $F = \frac{2}{3} s.h.$

genau $F = \frac{r^2\pi.n}{360} \; \frac{1}{2}s\sqrt{r^2 - \frac{1}{4}s^2} = \frac{br - s(r-h)}{2}.$

$$s = 2\sqrt{h(2r-h)}; \quad r = \frac{s^2}{8h} + \frac{h}{2}.$$

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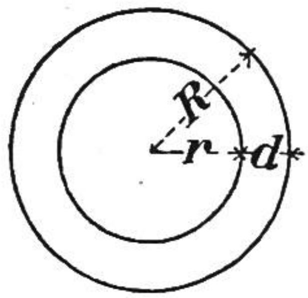
Kreisring:



Aeusserer Radius $= \mathbf{R}$,

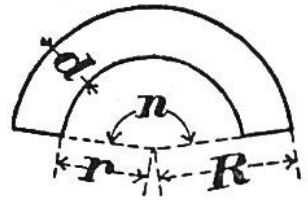
Innerer Radius $= \mathbf{r}$,

$F = R^2\pi - r^2\pi. = \pi.(R+r).(R-r).$

wenn d = radiale Breite des Kreisrings

so ist $F = \pi.(2r+d).d$;

Kreisringstück (Konzentrisch)



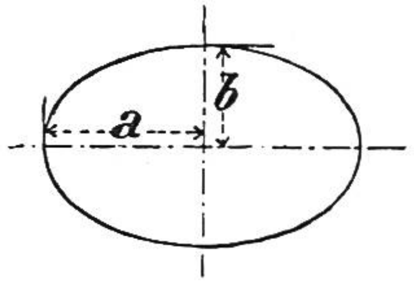
Aeusserer Radius $= \mathbf{R}$,

Innerer Radius $= \mathbf{r}$,

Zentriwinkel $= n$, radiale Breite $= d$,

$$F = (R^2\pi - r^2\pi)\frac{n}{360} = (R^2 - r^2).\frac{\pi n}{360};$$

$$= (R+r)d\frac{\pi n}{360} = (R+r)d.n.0.0087;$$



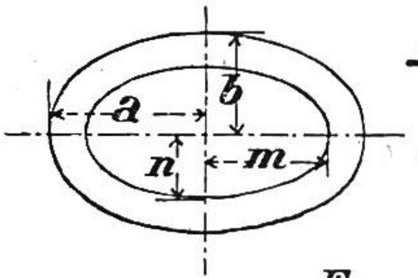
Ellipse:

Halbe grosse Achse $= a$,

Halbe kleine Achse $= b$,

Fläche $F = a.b.\pi$;

Elliptischer Ring:

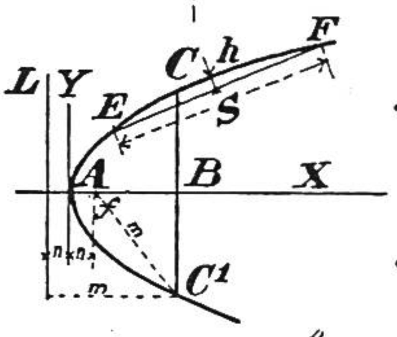


Halbe Achsen der äussern Ellipse $= a, b$,

Halbe Achsen der innern Ellipse $= m, n$,

Fläche: $F = \pi.(ab - mn)$

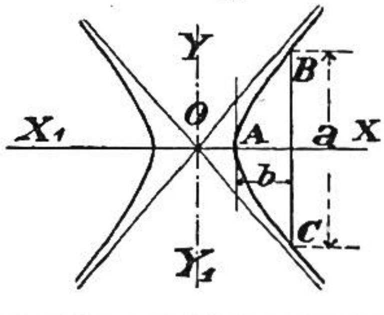
Parabelsegment ECFE:



$F = \frac{2}{3} s.h$; $s = \overline{EF}$;

Parabelfläche CAC¹C:

$F = \frac{2}{3}\ \overline{CC^1}.\overline{AB}$;



Hyperbelsegment ABCA:

Sehne $= a$; Höhe $= b$,

F (annähernd) $= \frac{3}{5} a.b$;

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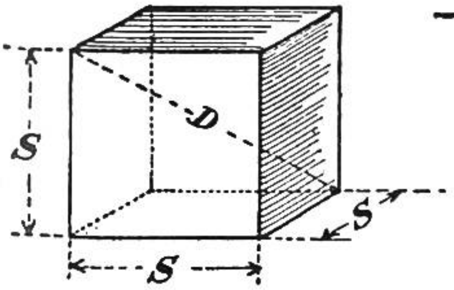
Würfel:



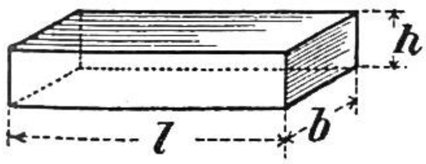
Seite = S, Inhalt = K; Oberfläche = O

$K = S^3$; $O = 6S^2$;

$S = \sqrt[3]{K}$

Diagonale $= D = \sqrt{3S^2} = S\sqrt{3} = S \cdot 1{,}732050$.

Parallelepipedon:

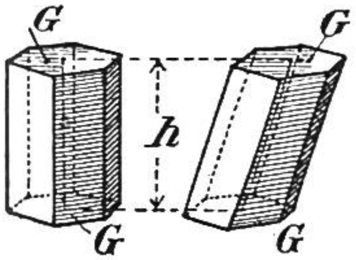


Länge = l, Breite = b, Höhe = h,

Inhalt = $K = l.b.h$.

Oberfläche $= O = 2.(l.b + l.h + b.h)$

Prisma:

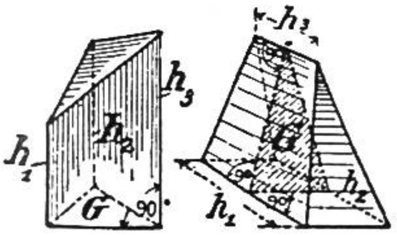


Grundfläche = G, Höhe = h,

Inhalt = $K = G.h$.

Oberfläche O = Umfang der Grundfläche $U \times h + 2G$.

Schiefabgeschnittenes Prisma:



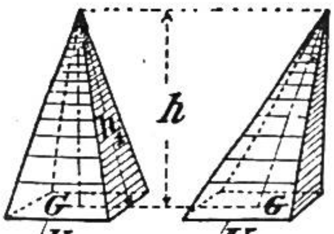
Flächeninhalt des senkrechten Querschnittes = G.

Länge der Kanten = $h_1 h_2 h_3 \ldots\ldots h_n$,

Inhalt $K = G.\dfrac{h_1 + h_2 + h_3 + \ldots\ldots h_n}{n}$.

Bei mehr als 3 Kanten ist das Prisma regelmässig oder hat dasselbe wenigstens Axensymetrie.

Pyramide:



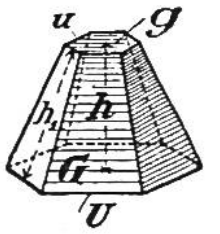
Grundfläche = G, Höhe = h,

$K = \dfrac{G.h}{3}$; $h = \dfrac{3K}{G}$; $G = \dfrac{3K}{h}$;

Mantel M = Umfang der Grundfläche $U \times \dfrac{h_1}{2}$;

Oberfläche $O = M + G$.

Abgestumpfte Pyramide:

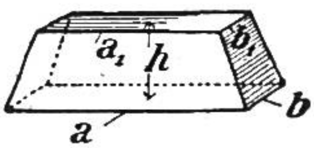


Parallele Endflächen = G, g, ihr Abstand = h,

ihre Umfänge U, u, Mantel $M = \dfrac{U+u}{2}.h_1$,

$K = \dfrac{h}{3}\left(G + g + \sqrt{Gg}\right)$; $O = M + G + g$.

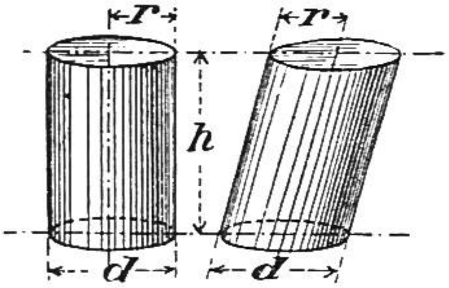
Obelisk, Wall, (regelmässig aufgeschütteter Haufen)



$K = \dfrac{1}{6}h\left[(2a + a_1)\,b + (2a_1 + a)\,b_1\right]$.

Tafel VI

Cylinder (Walze)



Radius = r, Durchmesser = d, Höhe = h

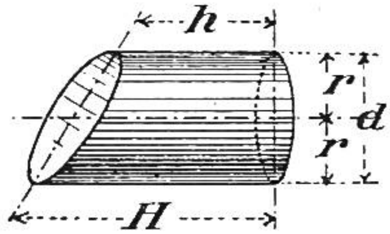
Inhalt $K = r^2\pi.h$ oder $\frac{d^2}{4}\pi.h$

$r = \sqrt{\frac{K}{\pi h}}$; $h = \frac{K}{r^2\pi}$.

Mantel = $2r\pi.h$ oder $d\pi.h$.

Oberfläche = $2r\pi.(r+h)$ oder $d\pi.(\frac{d}{2}+h)$

Schiefabgeschnittener Cylinder:

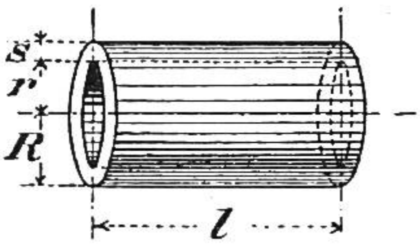


Grösste Höhe = H, kleinste Höhe = h,

Inhalt $K = r^2\pi.\frac{H+h}{2}$ oder $\frac{d^2}{4}\pi.\frac{H+h}{2}$

Mantel = $r\pi(H+h)$

Hohlcylinder (Rohr):



Innerer Radius = r,

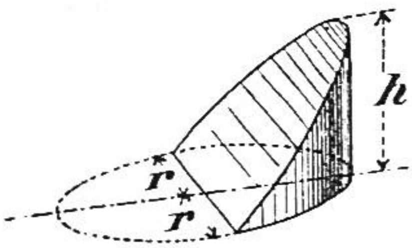
Aeusserer Radius = R, Länge = l,

Wandstärke = s = $R - r$,

Inhalt $K = \pi.l.(R^2 - r^2)$, oder

$K = \pi.l.s\,(2R - s)$ oder $\pi.l.s\,(2r + s)$.

Cylinderhuf:

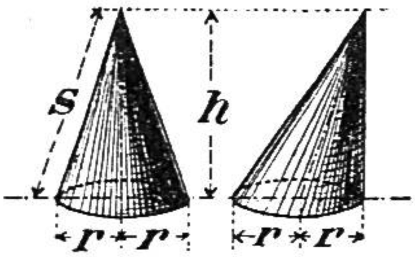


Radius der Grundfläche = r,

Höhe des Hufes = h, Mantel = $2rh$.

Inhalt: $K = \frac{2}{3}r^2 h$.

Kegel:



Radius der Grundfläche = r.

Höhe = h, Seite = s = $\sqrt{r^2+h^2}$,

Mantel $M = \pi r\sqrt{r^2+h^2}$ oder $\pi.r.s$,

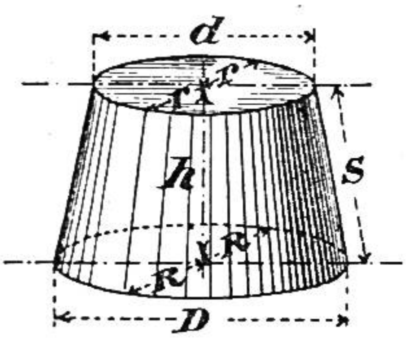
Oberfläche = $\pi.r^2 + r\pi.s$ oder $r\pi(r+s)$

oder = $\pi r.(r+\sqrt{r^2+h^2})$.

Inhalt $K = \frac{1}{3}r^2\pi.h$;

$r = \sqrt{\frac{3K}{\pi.h}}$; $h = \frac{3K}{r^2\pi}$.

Abgestumpfter Kegel:



Radien der parallelen Endflächen = R und r,

Durchmesser = D und d, Höhe = h, Seite = s,

Inhalt $K = \frac{1}{3}\pi h.(R^2 + Rr + r^2)$

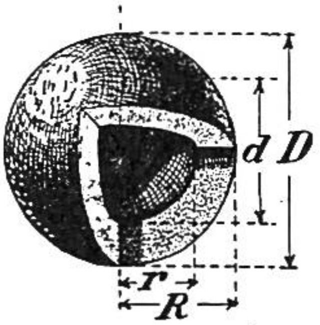
Mantel $M = \pi s.(R + r)$.

Oberfläche $O = \pi.[R^2 + r^2 + (R+r)s]$.

E. Poehon

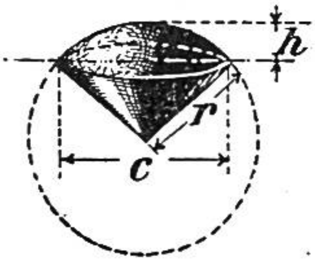
Kugel:

Radius $= \mathbf{r}$, Durchmesser $= \mathbf{d}$,
Oberfläche $O = 4r^2\pi = 12{,}566\,r^2$, oder $d^2\pi$.
Inhalt $K = \frac{4}{3}r^3\pi = 4{,}189\,r^3$, $K = \frac{O.r}{3}$,
„ $K = \frac{d^3\pi}{6} = 0{,}5236.d^3$,
Radius $r = \frac{1}{2}\sqrt{\frac{O}{\pi}}$; $r = \sqrt[3]{\frac{3K}{4\pi}}$.



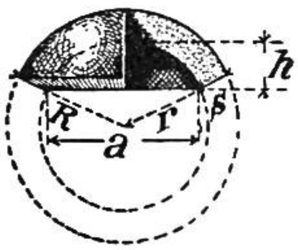
Hohlkugel:

Aeusserer Radius $= \mathbf{R}$, innerer $= \mathbf{r}$,
Aeusserer Durchmesser $= D$, innerer $= \mathbf{d}$,
Inhalt $K = \frac{4\pi}{3}\cdot(R^3 - r^3) = \frac{\pi}{6}(D^3 - d^3)$.



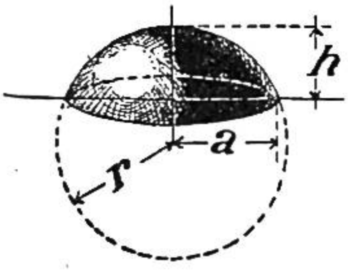
Kugelsektor:

Radius der Kugel $= \mathbf{r}$
Begrenzende Kalotte, Höhe $= \mathbf{h}$, Durchm. $= \mathbf{c}$,
Oberfläche $O = \frac{\pi r}{2}(4h + c)$
Inhalt $K = \frac{2}{3}r^2\pi.h = 2{,}0944\,r^2h$.



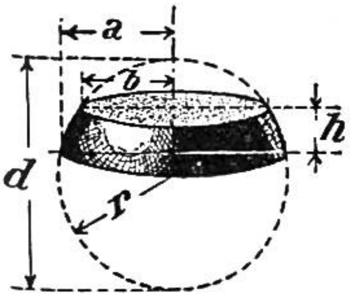
Hohlkugelsektor:

Aeusserer Radius $= \mathbf{R}$ innerer $= \mathbf{r}$
Wanddicke $= R - r = s$, $r = \frac{a^2 + 4h^2}{8h}$
Inhalt $K = 2{,}094\frac{h}{r}(R^3 - r^3)$.



Kugelsegment (Kugelkalotte):

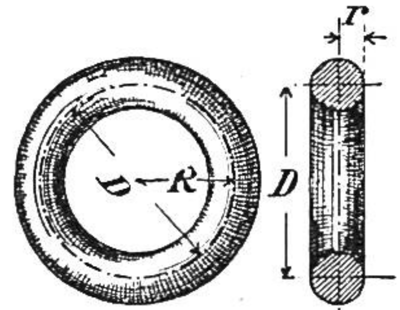
Radius der Kugel $= \mathbf{r}$,
Radius der Grundfläche $= \mathbf{a}$,
Höhe der Kalotte $= \mathbf{h}$,
Oberfläche $= O = 2\pi rh = \pi.(a^2 + h^2)$
Inhalt $K = \frac{1}{6}\pi.h\,(3a^2 + h^2)$ oder
$= \frac{1}{3}\pi h^2(3r - h)$.



Kugelzone:

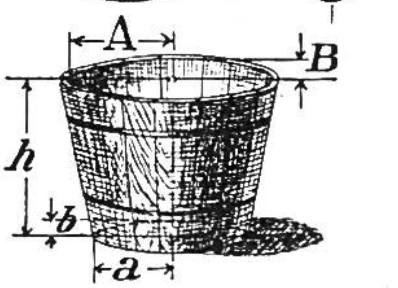
Höhe der Zone $= \mathbf{h}$, Radius der Kugel $= \mathbf{r}$
Radius der Endflächen $= \mathbf{a}$ und $\mathbf{b}$,
Mantel $M = 2r\pi h$, Oberfläche $O = M + a^2\pi + b^2\pi$;
Inhalt $K = \frac{1}{6}\pi h\,(3a^2 + 3b^2 + h^2)$.

E. Fochon



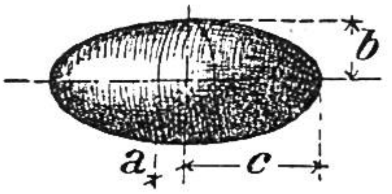
Cylindrischer Ring:

Radius des kreisförmigen Querschnittes $= r$,
Durchmesser des Ringes $= D$, *Radius* $= R$,
Inhalt $K = 2\pi^2 R r^2 = 2,467\, D d^2$.
Oberfläche $O = 4\pi^2 R r = 9,87\, D d$.



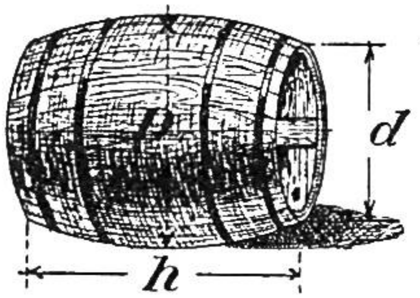
Kübel:

Die unter sich parallelen Endflächen sind Ellipsen mit den Halbachsen A, B *und* a, b,
Höhe zwischen den Endflächen $= h$.
Inhalt $K = \frac{1}{6}\pi h \left[2(AB + ab) + Ab + aB\right]$.



Ellipsoid:

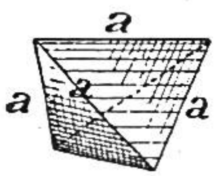
Bezeichnung der 3 Halbachsen $= a.\, b,\, c$,
Inhalt $K = \frac{4}{3}\pi . a . b . c = 4,189\, a b c$.



Fass:

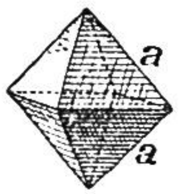
Spunddurchmesser $= D$, *Bodendurchmesser* $= d$,
Höhe (resp. Länge) $= h$,
Inhalt $K = \frac{1}{12}\pi h . (2D^2 + d^2)$,
$K = \frac{1}{15}\pi h . (2D^2 + Dd + \frac{3}{4} d^2)$,

Reguläre Polyeder:



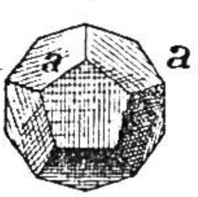
Tetraeder: (*4 gleichseitige Dreiecksflächen*)

Länge der Kante $= a$, *Oberfl.* $O = a^2\sqrt{3} = 1,732\, a^2$
Inhalt $K = \frac{1}{12} a^3 \sqrt{2} = 0,11785\, a^3$.



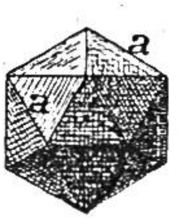
Oktaeder: (*8 gleichseitige Dreiecksflächen*)

Kante $= a$, *Oberfl.* $O = 2a^2\sqrt{3} = 3,4641016\, a^2$
Inhalt $K = \frac{a^3}{3}\sqrt{2} = 0,4714045\, a^3$.



Dodekaeder: (*12 regelmässige Fünfecke*)

Kante $= a$, $O = 3a^2\sqrt{25 + 10\sqrt{5}} = 20,645779\, a^2$
Inhalt $K = \frac{a^3}{4} . (15 + 7\sqrt{5}) = 7,663119\, a^3$.



Ikosaeder: (*20 gleichseitige Dreiecksflächen*)

Kante $= a$, $O = 5a^2\sqrt{3} = 8,6602545\, a^2$.
Inhalt $K = \frac{5}{12} a^3 (3 + \sqrt{5}) = 2,181695\, a^3$.

E. Pochon