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Autor: Redman, J.D.
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Holography

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by **J. D. Redman**, Aldermaston, Berkshire

Schluss von Heft 15

An Analysis of the Performance of a Representative Holographic System

Introduction

Potential users of holography are interested in *how big* an object they could holograph, *how near* the apparatus would have to be and in the *accuracy* and *sensitivity* of holographic strain measurement. In this communication answers to these questions are derived for a particular holographic arrangement. Though the analysis is very unsophisticated it is sufficient to show the interactions between the various holographic parameters and to indicate the scope for future improvement.

The main conclusions are summarised and then the analysis is presented.

Summary

1. The minimum strain measurable by holography is given by[1])

$k_{min} = [\lambda\, L/(2\, f^2)]\; [(1/k)\; (\partial\, k/\partial\, x)]^2$

2. The fractional error in measuring strain (if the strain is greater than k_{min}) is given by
fractional error $= [1.22\, \lambda\, L/(df)]\; [(1/k)\; (\partial\, k/\partial\, x)]$
or f (whichever is the larger)

3. The maximum area which can be holographed is given by $\hat{A} \approx J/(8.5 \times$ required signal-to-noise ratio)

where $\hat{A}$ is in cm² and J is in ergs and L, the distance from object to hologram plate, is large compared with the radius of the hologram plate.

4. There is no limit on the distance from which holograms can be made though the distance affects the resolution, the minimum measurable strain and the error in measuring strain.

The Analysis

The hologram making system

To simplify the analysis we consider the hologram arrangement shown in Fig. 8. This consists of a point source of coherent light located in the hologram plate. The hologram plate is orthogonal to a line joining its centre to a point in a plane perfectly diffusing object. The object plane is parallel to that of the hologram plate. A coherent off-axis reference beam also strikes the hologram plate.

The object is considered to be anchored at $x = 0$ and strained in the x direction[1]).

Estimate of sensitivity to strain

Clearly the optical path from the source to an object point at x is given by

$$p = \sqrt{(x^2 + L^2)}$$

When the object is strained the point originally at x moves to $x + \delta x$ and the optical path changes to $p + \delta p$ where

$$p + \delta p = \sqrt{[(x + \delta x)^2 + L^2]}$$

ie
$$\delta p = L \sqrt{[(x/L)^2 + (2\, x/L)\,(\delta x/L) + (\delta x/L)^2 + 1]} - L \sqrt{[(x/L)^2 + 1]}$$

and if $x/L \ll 1$ and $\delta x/L \ll 1$, $\delta p \approx x\, \delta x/L$

If a hologram is made with one exposure before strain and one after, a bright fringe will be seen whenever

$2\, \delta p = n\, \lambda$ where n is an integer or zero

ie when $2\, x\, \delta x/L = n\, \lambda$

If the strain in the x direction is constant

$\delta x = k\, x$

where k is the strain

ie there is a bright fringe when $2\, x^2\, k/L = n\, \lambda$

ie when $x = \sqrt{n\, \lambda\, L/(2\, k)}$

The interval between the n^{th} and the $(n + 1)^{th}$ fringe is therefore

$$\sqrt{[(n+1)\, \lambda\, L/(2\, k)]} - \sqrt{[n\, \lambda\, L/(2\, k)]} = \sqrt{\lambda\, L/(2\, k)}\, [\sqrt{n+1} - \sqrt{n}\,]$$

In particular when $n = 0$ the interval between fringes is $\sqrt{\lambda\, L/(2\, k)}$, ie

(1) $k = (\lambda\, L)/[2 \times (\text{fringe interval})^2]$

Other things being equal we would like the sampling gauge length to be small. If, however, the surface of the object has spatially varying reflectance or is unevenly illuminated the smallest allowable gauge length is equal to the fringe interval and we therefore require that the strain be substantially constant over this distance.

If x is a typical spatial co-ordinate and f is the permissible fractional change in strain over gauge length G we require

$$G\, \partial\, k/\partial\, x < f\, k$$

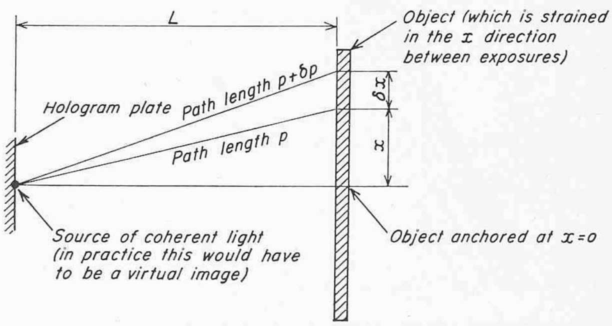


Figure 8. The hologram making arrangement (an off-axis reference beam is assumed but has been omitted from the sketch for clarity)

[1]) A full list of symbols is given at the end of the paper. We confine our analysis to the plane of Fig. 8.

If we substitute G for the fringe interval in (1) we obtain

$$k_{min} = \lambda\, L/(2\, G^2) = \lambda\, L\, (\partial\, k/\partial\, x)^2/(2\, f^2\, k^2)$$

ie $$k_{min} = (\lambda\, L)\, [(\partial\, k/\partial\, x)/k]^2/(2\, f^2)$$

where k_{min} is the minimum measurable strain.

This is plotted in Fig. 9 for $\lambda = 0.6943 \times 10^{-4}$cm (the ruby laser wavelength) and $f = 10^{-1}$ per gauge length (ie a change in strain of 10% over the gauge length).

Accuracy of strain measurement

The accuracy of strain measurement is determined by the resolution of the system. A rough guide to the resolution is given by the Rayleigh criterion which estimates the resolution as $1.22\, \lambda\, L/d$, where d is the diameter of the hologram plate[2]).

$\therefore$ The fractional error is therefore

$$1.22\, \lambda\, L/(d \times \text{fringe interval})$$

But fringe interval = gauge length $\equiv G$

$$= f\, k/(\partial\, k/\partial\, x) = f/[(\partial\, k/\partial\, x)\, (1/k)]$$

The fractional error is

$$[1.22\, \lambda\, L/(df)]\, [(\partial\, k/\partial\, x)\, (1/k)]$$

Note however that this is an estimate of the error on integrated strain over gauge length G. Where this is less than f it is more realistic to quote f as an estimate of error.

As an example if we put $f = 0.1$, $d = 10$ cm, $\lambda = 0.6943 \times \times 10^{-4}$ cm the fractional error is

$$0.847\, L\, [(1/k)\, (\partial\, k/\partial\, x)]$$

or 0.1, whichever is the larger.

Signal-to-noise ratio

We now calculate the signal-to-noise ratio in a holographic reconstruction. Consider the hologram to be illuminated by a reconstructing beam which deposits a total energy per unit time of F_o on to the finished hologram. If R is the ratio of total scattered power to total incident power then the amount of light scattered into area A at a distance L (ie into the reconstructed object) is

$$R\, F_o\, A/(\pi\, L^2)$$

(assuming that the angle Θ between the object beam and the normal to the hologram plate is small so that $\cos\, \Theta \approx 1$; see page 258 of reference [1]).

The energy per unit time diffracted into area A is

$$\beta\, F_o$$

where β is the fraction of the incident power diffracted into the primary reconstruction.

The signal-to-noise ratio is therefore given by

(2) $$S/N = \pi\, \beta\, L^2/\, (A\, R)$$

It can easily be shown that the depth of modulation of a hologram is

$$\frac{2\, \sqrt{(I_R\, I_o)}}{I_R + I_o} \equiv M$$

where I_R is the reference beam energy and I_o is the object beam energy both per unit area (see reference [4], page 35).

Furthermore it can be shown (reference [4], page 38) that the fraction of the energy in the reference beam which is diffracted into the primary reconstruction is given by

$$\beta = g^2\, (E_o)\, E_o^2\, M^2/4$$

where $g\, (E_o)$ is the derivative with respect to E_o of the amplitude transmittance of the hologram plate and E_o is the total energy per unit area falling on the hologram when it was made.

If B is the ratio of object beam energy to reference beam energy (both per unit area) then

$$I_o/I_R = B$$

and $$I_o + I_R = E_o$$

$$\therefore M = 2\, \sqrt{B}/(1 + B) \text{ and}$$

(3) $$\beta = g^2\, (E_o)\, B\, E_o^2/(1 + B)^2$$

Putting this in (2) we obtain

$$S/N = \{\pi\, L^2\, B/[A\, (1 + B)^2]\}\, [g^2\, (E_o)\, E_o^2/R]$$

Note that I_o must not exceed I_R (this is implied in the derivation of the expression for β) and therefore $B \leqslant 1$.

Fig. 10 shows the amplitude transmittance characteristic of Agfa Scientia 10E 75 plates and also the approximation

$$t_a = \exp\, (-0.0429\, E_o)$$

where E_o is in ergs cm^{-2}.

[2]) For simplicity we assume that the hologram plate is circular.

[3]) See page 225 of reference [2] where a value for Q is quoted for Kodak 649F plates. In the absence of specific information for Agfa Scientia 10E 75 the same value has been used.

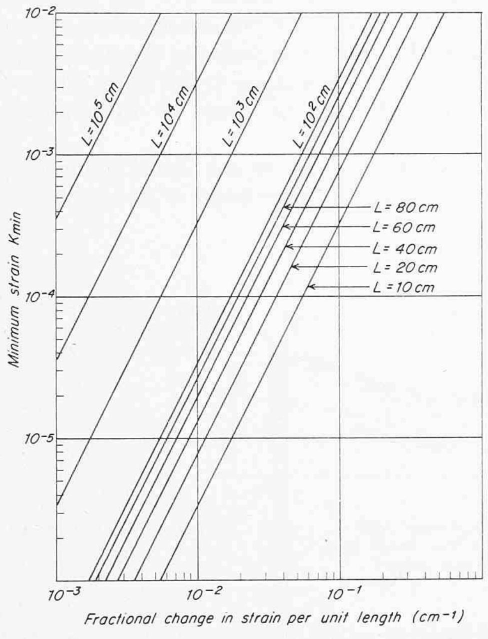


Figure 9. Effect of strain gradient and distance on the minimum strain measurable by holography

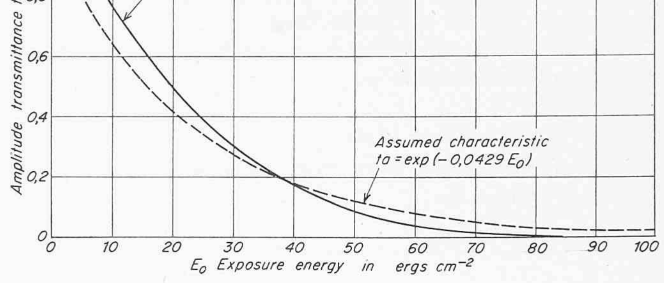


Figure 10. Amplitude transmittance characteristic for Agfa-Scientia 10 E 75

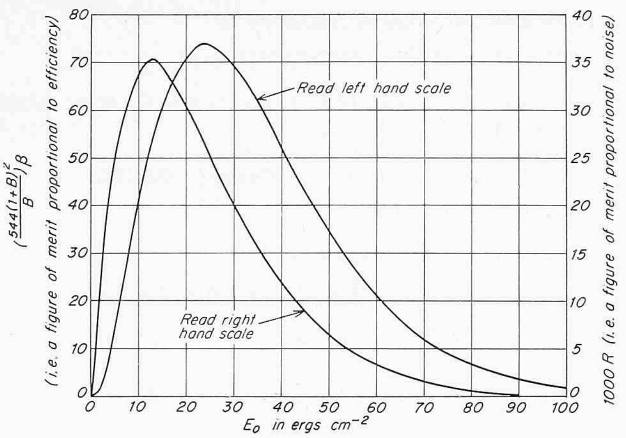


Figure 11. Efficiency and scattering as a function of exposure

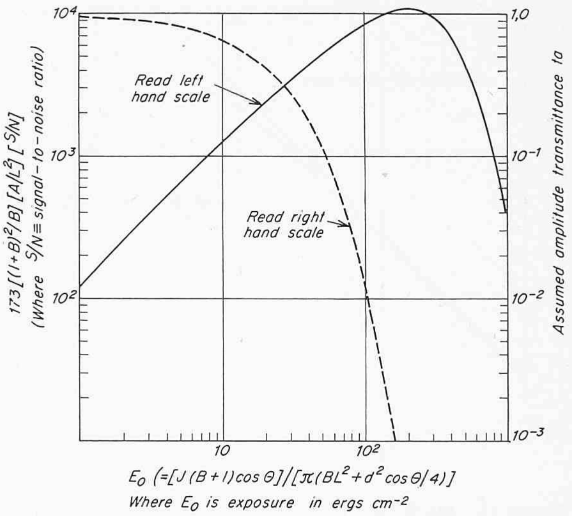


Figure 12. Signal-to-noise ratio and amplitude transmittance

Using the approximation and assuming $Q = 1.1^3$) (Q is the Callier Q, see reference [3])

$$R = \exp(-0.0780\,E_o) - \exp(-0.0858\,E_o)$$
$$= [\exp(-0.0780\,E_o)]\,[1 - \exp(-0.0078\,E_o)]$$

This is plotted in Fig. 11

$$g\,(E_o) = \partial t_a/\partial E_o = -0.0429\,\exp(-0.0429\,E_o)$$

so signal-to-noise ratio, S/N, is given by

$$S/N = \frac{\{\pi L^2 B/[A(1+B)^2]\}\,[1.84\times10^{-3}\exp(-0.0858\,E_o)]\,E_o^2}{[\exp(-0.0780\,E_o)]\,[1-\exp(-0.0078\,E_o)]}$$

$$= \frac{\{\pi L^2 B/[A(1+B)^2]\}\,[1.84\times10^{-3}\exp(-0.0078\,E_o)]\,E_o^2}{[1-\exp(-0.0078\,E_o)]}$$

$$= \frac{\{\pi L^2 B/[A(1+B)^2]\}\,(1.84\times10^{-3})\,E_o^2}{\exp(0.0078\,E_o)-1}, \text{ ie}$$

(4) $\therefore 173\,[(1+B)^2/B]\,(A/L^2)\,(S/N) =$

$$= E_o^2/[\exp(0.0078\,E_o) - 1] \equiv \varnothing$$

This is plotted in Fig. 12 (note, however, that B must be less than unity).

For small values of E_o equation (4) becomes

(4a) $\varnothing \approx E_o/0.0078$

and the percentage error resulting from the use of this expression is approximately $0.39\,E_o$ (ie less than 9% for practical purposes).

We now calculate E_o in terms of laser output energy J and beam balance ratio B.

E_o = object beam energy per unit area + reference beam energy per unit area

B = (object beam energy per unit area)/(reference beam energy per unit area)

$\therefore E_o =$ (object beam energy per unit area) $(1 + 1/B)$

Energy per unit area reaching object

$= [J$-(reference beam energy per unit area) $(\pi d^2/4)]/A$

$= [J$-(object beam energy per unit area) $(\pi d^2/4)(1/B)]/A$

$= [J - E_o\,(\pi d^2/4)\,(1/B)/(1+1/B)\,]/A$

$= \{J - E_o\,\pi\,d^2/[4\,(B+1)]\}/A$

The energy accepted by a hologram plate of diameter d at distance L

$$= \left[\begin{pmatrix}\text{Energy per unit area}\\ \text{reaching the object}\end{pmatrix}\times A\right]\,[(\cos\Theta)/\pi]\,\cdot$$
$$\cdot\,[(\pi d^2/4)\,(1/L^2)] + E_o(\pi d^2/4)/(B+1)$$

$$= J - E\,[(\pi d^2/4)/(B+1)]\,[(\cos\Theta)/\pi]\,[(\pi d^2/4)\,(1/L^2)] +$$
$$+ E_o\,(\pi d^2/4)/(B+1)$$

$$\therefore E_o = [J/(\pi d^2/4) - E_o/(B+1)]\,[(\cos\Theta)\,d^2/(4\,L^2)] +$$
$$+ E_o/(B+1)$$

ie $\quad E_o\,\{B/(B+1) + (d^2\cos\Theta)/[4\,L^2\,(B+1)]\} =$

$$= J(\cos\Theta)\,d^2/(\pi\,d^2\,L^2) = J\,(\cos\Theta)/(\pi\,L^2)$$

$$\therefore E_o = [J\,(\cos\Theta)\,(B+1)]/(\pi\,L^2)]/[B +$$
$$+ (d^2\cos\Theta)/(4\,L^2)]$$

(5) $= [J\,(B+1)\cos\Theta]/[\pi\,B\,L^2 + \pi\,d^2\,(\cos\Theta)/4]$

$$= [J/(\pi\,L^2)]\,[(B+1)/B] \quad \text{when} \quad \cos\Theta \approx 1$$

and $L \gg d/(2\sqrt{B})$

substitution of this in (4a) yields

$$173\,[(1+B)^2/B]\,(A/L^2)\,(S/N) = (J/(\pi\,L^2)]\,\cdot$$
$$\cdot\,[(B+1)/B]/0.0078$$

ie $\quad S/N = [(J/A(1+B)]\,[1/(173\times0.0078\,\pi)]$

$$= (J/A)/[(1+B)\times4.25]$$

Note that signal-to-noise varies only by factor of 2 over the range $B = 0$ to $B = 1$. In practice therefore one would choose a value of B which gives maximum efficiency. The next section shows that this means putting $B = 0.0273\,J/L^2$ where J is in ergs and L is in cms. If this value is greater than unity however the output energy from the laser should be reduced to $L^2/(0.0273\,J)$.

Hologram efficiency

Equation (3) has shown that hologram efficiency, β, is given by

$$\beta = g^2\,(E_o)\,BE_o^2/(1+B)^2$$

where β is the fraction of the incident energy which is diffracted into the primary reconstruction.

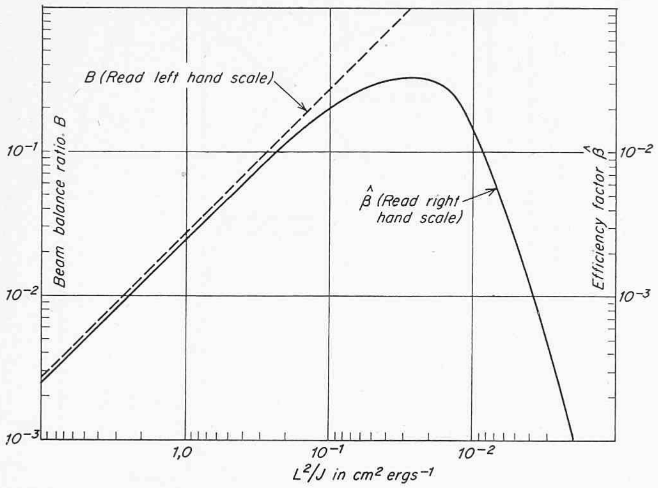


Figure 13. Maximum efficiency and corresponding beam balance ratio as a function of distance and energy

But $g = -0.0429 \exp(-0.0429\ E_o)$, $\therefore$

(6) $\beta = 1.84 \times 10^{-3} \exp(-0.0858\ E_o)\ BE_o^2/(1+B)^2$

ie $544\ \beta\ (1+B)^2/B = E_o^2 \exp(-0.0858\ E_o)$

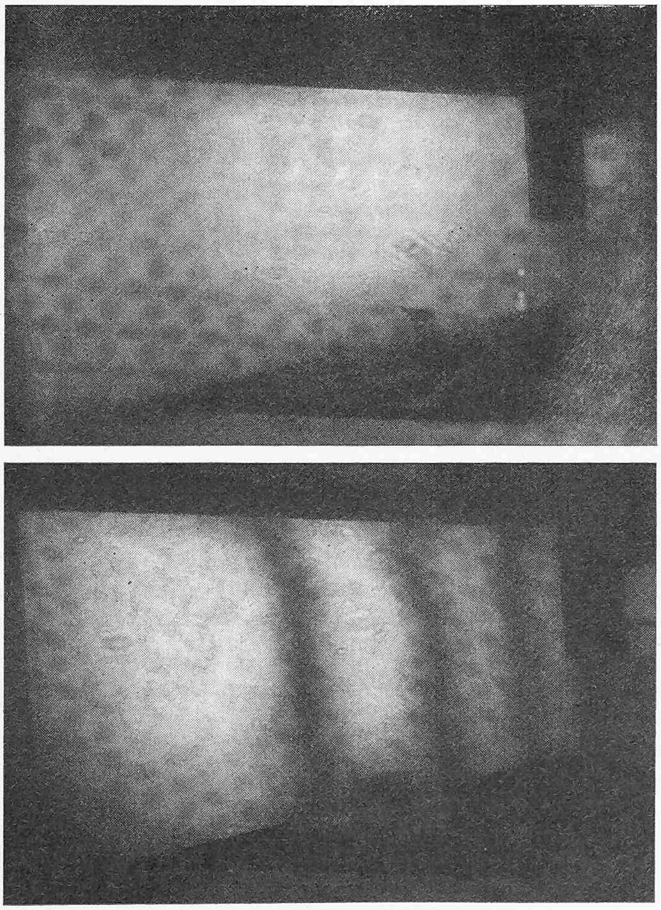
This is plotted in Fig. 11 together with R, the ratio of total scattered power to total incident power.

Note that for a given beam balance ratio, B, β is a maximum when

$$E_o = 2/0.0858 = 23.3 \text{ ergs cm}^{-2}$$

Figure 14. Reconstruction from a hologram of a large aluminium sheet

a (above) single exposure b (below) double exposure



The maximum value of β is given by

$$544\ \hat{\beta}\ (1+B)^2/B = [4/(0.0858)^2] \exp(-2) = 73.2$$

Substition of the value of E_o from (5) into (6) gives (when $L \gg d/(2\sqrt{B})$ and $\cos\Theta \approx 1$).

$$\beta = [(1.84 \times 10^{-3} J^2)/(\pi^2 L^4 B)] \exp[-0.0858\ J \cdot \cdot (B+1)/(\pi\ L^2\ B)]$$

For stationary values $\partial\beta/\partial B = 0$

For simplicity we put $\alpha \equiv 0.0858\ J/(\pi\ L^2)$ and hence

$$\beta\ \alpha/\exp[-\alpha\ (B+1)/B]/(B)$$

ie $$\frac{\partial\beta}{\partial B}\ \alpha\ [(\alpha/\beta - 1)/B^2] \exp[-\alpha\ (B+1)/B]$$

$\therefore$ For a maximum $\alpha = B$

ie $B = 0.0858\ J/(\pi\ L^2) = 0.0273\ J/L$

The maximum value of β is given by

$$\hat{\beta} = \frac{6.81 \times 10^{-3}\ J}{L^2} \exp\{-[0.0858\ J/(\pi\ L^2) + 1]\}$$

$$= 2.50 \times 10^{-3}\ (J/L^2) \exp(-0.0273\ J/L^2)$$

$\hat{\beta}$ is plotted against L^2/J in Fig. 13[4]). Also plotted is the corresponding value of B. Note that when $\beta = \hat{\beta}$

$$E_o = [1 + 0.0858\ J/(\pi\ L^2)]/0.0858$$
$$= 11.6 + J/(\pi\ L^2) \text{ ergs cm}^{-2}$$

Influence of reconstructing power available and object distance on the required efficiency.

When a hologram is reconstructed with a continuous wave laser of power P the power diffracted so that it appears to come from the object is $P\ \beta$.

We note that if the object were to be illuminated by light from a CW laser and viewed directly there would be a critical illuminating power below which an observer would experience difficulty in seeing the target. We let this critical power be σ. Note that σ depends on operational conditions but that in a dark room without prolonged dark adaptation $\sigma \approx 5$ ergs cm^{-2} sec^{-1} with a helium-neon Laser.

We require $P\ \beta$ to be greater than the energy which would be delivered to the hologram plate if the object were critically illuminated ie we require:

$$P\ \hat{\beta} \geqslant (\sigma/\pi)\ (\pi\ D^2/4)\ (\pi\ d^2/4)\ (1/L^2)$$

but $\hat{\beta} = 2.5 \times 10^{-3}\ (J/L^2) \exp(-0.0273\ J/L^2)$

$\therefore$ $2.5\ P\ (J/L^2) \exp(-0.0273\ J/L^2) \times \times 10^{-3} \geqslant \pi\ \sigma\ D^2\ d^2/(16\ L^2)$

ie $\exp(-0.0273\ J/L^2) \geqslant 25\ \sigma\ D^2\ d^2/(P\ J)$

Note that as L increases it becomes easier to meet this condition. In other words considerations of efficency do not limit the distance from which holograms may be made. Note that the analysis breaks down for several reasons when L is small (eg, $\cos\Theta$ does not then approximate to unity; also if L is not very much greater than $d/(2\sqrt{B})$ the approximation for β is invalid).

Discussion

It is important to remember the limitations on this analysis. These are stated explicitly where they arise. Briefly the analysis is restricted to large distances, low energies, large

[4]) $\hat{\beta}$ is itself a maximum when $J/L^2 = 1/0.0273$ and its value is then given by $\hat{\hat{\beta}} = 3.37 \times 10^{-2}$.

areas of object etc. We have also assumed a perfect diffuse scatterer as an object. In practice real engineering objects are very far from perfect diffuse scatterers except when they are painted white. If painting is not permitted the analysis is approximately correct if the value used for J is multiplied by the diffuse reflectance of the object.

The analysis predicts that a diffusely-reflecting white object of area 6×10^3 cm² could be holographed with a signal-to-noise ratio of 3 dB using a 10 mJ laser such as the Laser Associates Model 253H.

Though no experiments have been done to verify the analysis directly a re-examination of holograms made for other purposes gives results consistent with those predicted. Fig. 14a for example shows the reconstruction of an aluminium sheet painted white and illuminated with a 14 mJ laser beam diverged to fill an area of about 8×10^3 cm². The analysis predicts an average signal-to-noise ratio of 2.1. Note that the unpainted table on which the aluminium sheet was standing also reconstructed fairly well. Fig. 14b shows a double exposure hologram taken under the same conditions but with the object moved between exposures. The reconstructions could be viewed satisfactorily using a 10 mW laser.

Conclusion

An analysis of a particular holographic arrangement has resulted in expressions for the accuracy of strain measurement and the maximum area which can be holographed. These expressions are listed in the summary. One implication of the analysis is that a commercially available 10 mJ laser would be capable of holographing an area of 6×10^3 cm².

List of Symbols (NB cgs units are mandatory

- A Area of the object (assumed plane)
- $\hat{A}$ Maximum area which can be holographed
- B The ratio of the energy arriving at the hologram plate from the object to that in the reference beam.
- d The diameter of the hologram plate (assumed circular).
- D The diameter of the object (which is assumed to be plane and circular).
- E_0 Energy per unit area falling on the hologram plate when it was being made into a hologram.
- F_0 Energy per unit time falling on the hologram plate at reconstruction time.
- f The fractional change in strain over a gauge length G
- $g(E_0)$ The derivative with respect to E_0 of the amplitude transmittance of the hologram.
- G A gauge length: the distance over which a measurement of strain is integrated.
- J The laser output energy.
- k A typical component of strain; $\delta x/x$ from object anchored at $x = 0$ where the x direction is normal to the line of sight.
- k_{min} The minimum measurable value of k.
- L The distance from the hologram plate to the object.
- n An integer.
- p The optical path from source to a point on the object.
- R The ratio of the total scattered energy from a hologram plate to the total incident energy at reconstruction time.
- P The power output from the reconstructing laser.
- x Spatial Cartesian Co-ordinate transverse to the line of sight with its orgin on the line of sight.
- δx Small increment in x
- β Fraction of the energy in the reference beam which is diffracted into the primary reconstructions.
- $\hat{\beta}$ The maximum value of β for a given value of J/L^2.
- $\hat{\hat{\beta}}$ The maximum obtainable value of β.
- λ The wavelength of the light.
- σ The illuminating power below which an object could not be comfortably seen under particular operation conditions.
- Θ The angle between the object beam and the normal to the hologram plate[5]).

[5]) The analysis is restricted to cos $\Theta \approx 1$, ie, if we accept a 10% error, to values of Θ less than .45 radians. This means that $D/L < 0.9$.

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Address of the author: *J. D. Redman,* BSc, M Inst P, Applied Physics Division, Atomic Weapons Research Establishment, Building C15, Aldermaston, Berkshire, England.

Der schiefe Turm von St. Moritz im Vergleich zum schiefen Turm von Pisa

Von **R. Haefeli**, Zürich

DK 624.131.31

I. Vorwort

Der Schiefe Turm von St. Moritz (Bilder 1 und 2) ist nicht nur ein eindrückliches Wahrzeichen des berühmten Kurortes, sondern auch eine dauernde Erinnerung an den Brückenbauer *Robert Maillart,* dem es zu verdanken ist, dass das ehrwürdige Baudenkmal vor dem drohenden Einsturz bewahrt wurde [1]. Es bietet sich hier eine Gelegenheit, des im wörtlichen Sinne «bewegten» Schicksals des Turmes und dem Können seines Retters zu gedenken, in der Hoffnung, dass sich dabei neue Wege zur Behandlung und Erhaltung schiefer Türme ergeben werden, die auch für den berühmten Schiefen Turm von Pisa von Interesse sein könnten.

II. Zur Geschichte des schiefen Turmes von St. Moritz

Im Laufe des Mittelalters wurde der ursprünglich im romanischen Stil erbaute Turm erhöht. Ein erster Aufbau erfolgte nach *L. Bendel* im 16. Jahrhundert, ebenso der Einbau der Glocken und der Uhr. Das oberste Geschoss ist angeblich ein Werk der Spätrenaissance des 17. Jahrhunderts. Im Jahre 1797 berichtet *H. Lehmann,* dass ein Erdbeben den Turm, der mit seinem Einsturz drohe, verschoben habe. Im Jahre 1890 wurden die Glocken aus dem Turm entfernt und in einem besonderen Holzgerüst montiert, das neben der Kirche aufgestellt wurde. Während der Turm nach der Talseite überhing, neigte sich die hangwärts anschliessende, später abgebrochene